Homogenization of reaction-diffusion system in poroelastic media

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The aim of this talk is to study a microstructure problem modeling the diffusion of solutes in a poro-elastic material considered as a periodic composite medium. The two scale convergence technique is used to determine the macro-model as the period of the medium tends to 0.

Homogenization of composite media with active interfaces and singular sources

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We prove existence and homogenization results for a family of elliptic problems depending on two parameters $\epsilon > 0$ and $\alpha \geq -1$. The small parameter ϵ represents the characteristic length of the microstructure underlying the model and will let tend to 0 in the homogenization procedure. The parameter $\alpha \geq -1$ takes into account different scalings and, therefore, different physical properties appearing in the model and it will be kept fixed in the limit $\epsilon \to 0$. We focus our attention on the differences of the limit equations (characterizing the properties of the material from the macroscopic point of view), with respect to the parameter α (see, [3, 1]).

Such problems could be useful to describe the equilibrium energy for the heat conduction in composite materials (for details on the related physical models, see, for instance, [5, 6] and the reference therein).

In our model, we assume that the microscopic array of the region occupied by the material is made by two phases separated by an active interface having a possibly nonlinear response. The mathematical description is given by two non-homogeneous elliptic equations in each diffusive phase, complemented with the assumption that the heat flux is continuous across the interface and proportional to a possibly nonlinear function of the jump of the temperature. Moreover, we assume that in both phases the rate of the heat generation is given by a singular source, that is a function which blows up when the solution approaches zero.

Similar kinds of energy can also be useful to study the electrical conduction in biological tissues (see for instance [1, 2], where the related dynamical problems without singular source are studied).

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Weighted multilayer competition fosters structural stability of mutualistic ecosystems

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Species rarely live in isolation, but constantly interact with other species forming different types of ecological interactions, such as prey-predation, competition and mutualism [1]. Mutualism, in which different species interact to their mutual benefit, is ubiquitous in terrestrial ecosystems. Examples include, but not limited to, plants receiving effective pollination or seed-dispersion by offering rewards of nutrients to their visiting animals, plants gaining resistance to insect herbivores by offering nutrients and shelter to fungi or ant and leguminous plants obtaining nitrogen by rewarding nitrogen-fixing bacteria.

Diverse species and complex interactions between species form a network of interdependence [2, 3]. Current single network representation encodes information of who interacting with who, but skirts the types of effect on species through interactions with other species.

We uncover that homogeneous and heterogeneous competitions yield different feasibility patterns, showing that even subtle changes in the network structure have strong implications on biodiversity. Next, we develop approximations in order to analytically assess the feasible area of observed mutualistic networks. We show that the proposed theoretical framework is able to accurately predict the dynamical behavior of a series of real-world mutualistic networks under the soft mean field competition coupling. General trends of the dependence of the feasible area on parameters are also captured in the multilayer coupling. Finally, extensive simulations and a multiple linear regression analysis reveal that the total number of species and the maximum inter-species degree are more likely to impact biodiversity than nestedness.

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Homogenization of fibre-reinforced hydrogel models

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Fibre-reinforced hydrogels (FIHs), composites of micro-fibre scaffolds and hydrogel, are a promising concept in tissue engineering and they are constructed such that they mimic the natural composite structure of soft tissue. The filament spacing of the scaffold is usually in the range of μm while the overall size of an FIH is in the range of mm to cm. Due to this scale heterogeneity, the mechanical properties of FIHs are not yet fully understood and, as a consequence, there is an interest in investigating their effective properties.

In this talk, we consider highly heterogeneous two-component media composed of a connected fibre-scaffold with periodically distributed inclusions of hydrogel. While the fibres are assumed to be elastic, the hydromechanical response of hydrogel is modeled via *Biot's poroelasticity*.

We show that the resulting mathematical problem admits a unique weak solution and investigate the limit behavior of the solutions with respect to a scale parameter characterizing the heterogeneity of the medium. This is done in the context of the two-scale convergence method.

Pressure-driven flow in thin domains

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We will discuss the asymptotic behavior of pressure-driven Stokes flow in a thin domain. By letting the thickness of the domain tend to zero we prove a generalized form of the classical Reynolds–Poiseuille law, i.e. the limit velocity field is a linear function of the pressure gradient. The proof is based on the method "two-scale convergence for thin domains" introduced in [4]. The choice of boundary conditions is crucial for this type of problem. By prescribing the external pressure as a normal stress condition, we recover a Dirichlet condition for the limit pressure. In contrast, a Dirichlet condition for the velocity yields a Neumann condition for the limit pressure.

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Numerical investigation of a fully coupled micro-macro model for mineral dissolution and precipitation

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We investigate multiscale models describing mineral dissolution and precipitation processes. Such multiscale models may be derived from detailed pore-scale models applying upscaling techniques. Since mineral reactions alter the porous medium's structure and its bulk properties, the models comprise several levels of couplings. Our model consists of transport equations at the scale of the porous medium (macroscale) while taking the processes of convection, diffusion, and reaction into account. They include averaged time- and space-dependent coefficient functions which are in turn explicitly computed by means of auxiliary cell problems (microscale). Structural changes due to dissolution and precipitation reactions result in a time- and space-dependent domain, on which cell problems are defined. The interface between the mineral and the fluid, and consequently the explicit geometric structure, is characterized by means of a level set. Here, information from the transport equations' solutions is taken into account (micro-macroscale). A numerical scheme has been developed which enables evaluating such complex settings. For the level-set equation, an upwind scheme by Rouy and Tourin is applied. An extended finite element method is used for the evaluation of the cell diffusion problems. For the Stokes-type cell problems local mesh refinement strategies are applied to enhance accuracy. Within this framework the potentially degenerating bulk properties of the medium such as porosity, diffusivity and permeability could be investigated. The transport equations on the macroscale are solved applying mixed finite elements. Likewise, Darcy's law is treated by this means. To reduce computational costs, adaptive methods for controlling the macroscopic steps and evaluating the cell problems are investigated. Moreover, we applied our approach to the dissolution of an array of calcite grains in the multiscale context and validated our numerical scheme.

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A Multiscale Method for Discrete Network Models

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In this work, a multiscale method for discrete network models is presented [1]. The method is based on the Localized orthogonal decomposition method (LOD) [2, 3], which is a generalized finite element multiscale method for elliptic partial differential equations with highly varying coefficients. The novel discrete method is suitable for solving matrix equations of the form

$$Ku = F \tag{1}$$

which could for example describe the mechanics of a fiber network, where $K \in \mathbb{R}^{n\times n}$ is the connectivity matrix (symmetric and positive definite), $u \in \mathbb{R}^n$ is the node displacements of the fibers, and $F \in \mathbb{R}^n$ is applied load forces.

The matrix problem is reformulated on variational form as

$$\text{Find } u \in V: \quad v^T K u = v^T F, \quad \forall v \in V, \tag{2}$$

where $V \subset \mathbb{R}^n$ is the high dimensional solution space. The idea of the method is to construct a low dimensional solution space, V^{ms}, with good approximation properties. This is achieved by first representing the network by a coarse FEM grid, for example with bilinear elements. The bilinear shape functions define a low dimensional solution space with poor approximation properties for complex networks. By solving localized subproblems of small size, resulting in modifications of the bilinear basis functions, a new basis is attained, which spans the multiscale space V^{ms}. The method has optimal convergence rates for unstructured networks with highly varying coefficients.

This new method is useful for reducing the size of problems with large n, since the local subproblems are small in size, and the multiscale solution space has low dimension. Moreover, the local subproblems are solved independent of the load vector F, meaning that problems with the same K matrix and different load vectors can be solved efficiently. Additionally, problems where the matrix is modified locally can be solved with low cost since only affected subproblems have to be recalculated. For example this is the case when investigating crack propagation of fiber networks.

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Hexagonal snow crystal growth model with singularities

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We propose a generalized crystalline motion in 2D with hexagonal facets. The motion of the hexagonal polygon is governed by a surface tension and a driving force. We allow the polygonal motion to have several types of singularities such as facet collision, facet generation, facet merging, and facet breaking.

Introducing a weak notion of the solution to it, we show global existence and uniqueness of the solution for a simplified snow crystal growth model. We also consider a more realistic model which includes the vapor saturation and the Gibbs-Thomson effect and show several numerical examples of snow flake growth as an application of our model.

This is a joint work with Tomoe Tanaka (Kanazawa Univ.) and Ryohei Yamaoka (Kanazawa Univ.).

Multiscale Finite Element Methods for heterogeneous plates

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In this talk, we consider composite plates. These are materials occupying a three-dimensional domain $\Omega \subset \mathbb{R}^3$ which is very thin in the third direction: $\Omega = \omega \times (0, \varepsilon)$ with $\omega \subset \mathbb{R}^2$ and $\varepsilon \ll 1$. They are modelled by elliptic partial differential equations (say, linear diffusion problems, or linear elasticity) with coefficients that are highly oscillatory in the in-plane directions. There are hence two small scales in the problem, the thickness of the plate and the characteristic dimension of the heterogeneities in the in-plane directions.

We consider first the case when heterogeneities are periodic in the (x_1, x_2) directions. For various choices of regime between the two small scales, the homogenized limit of such problems has been identified in the literature: it is a partial differential equation with constant coefficients posed in ω. The work we present here aims at establishing two-scale expansions results, leading to an accurate description of the oscillatory solution.

We next turn to non-periodic situations. To handle these numerically, an idea is to use the Multiscale Finite Element method, which is a Finite Element type approach for multiscale problems, where the basis functions used to generate the approximation space are precomputed. They encode fine-scale details of the microstructure, and are thus specifically adapted to the problem at hand. The computation is performed in a two-stage procedure: (i) a offline stage, in which basis functions are computed as solutions to local fine scale problems, and (ii) a online stage, in which the global problem is solved using an inexpensive Galerkin approximation using a coarse mesh. We will discuss the local problems to be solved, and present preliminary numerical results.

This talk is based on joint works with V. Ehrlacher, A. Lebée and A. Lesage.

Homogenization and numerics for the Landau-Lifshitz equation with highly oscillatory coefficient

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We use a simplified version of the Landau-Lifshitz equation, to study a composite ferromagnetic object that consists of magnetic materials with different interaction behavior. In the model this is represented by a rapidly varying material coefficient a^ϵ, $\varepsilon \ll 1$. More precisely, we study

$$\partial_t \mathbf{m} = -\gamma \mathbf{m} \times \mathbf{H} - \alpha \mathbf{m} \times \mathbf{m} \times \mathbf{H}\,, \tag{3}$$

where

$$\mathbf{H} = \boldsymbol{\nabla} \cdot (a^\varepsilon \boldsymbol{\nabla} \mathbf{m})\,.$$

Direct numerical simulation of the problem is expensive as the small ε-scale must be resolved; however, the small scale cannot be ignored either, since it has a significant influence on the magnetization behavior on the coarse scale. We aim to design an efficient numerical method for this case using the framework of heterogeneous multiscale methods (HMM)

In order to understand which behavior to expect from the solution and to obtain a good reference solution, we derive a homogenized equation where $\mathbf{H}$ in (3) is replaced by $\bar{\mathbf{H}} = \boldsymbol{\nabla} \cdot (A \boldsymbol{\nabla} \mathbf{m})$ with a constant matrix A. We also derive equations for higher order correction terms to the homogenized solution $\mathbf{m_0}$. We show estimates for the difference $\mathbf{m_0} - \mathbf{m}$ in terms of ε. Investigating the correction terms, we find that there is a rapidly oscillating contribution that dominates the short-term behavior but decays exponentially with time t/ε^2. This is especially important in the context of HMM where we average over short time intervals in order to approximate the effective long time behaviour.

We use these results to analyze an HMM method and also show numerical results that confirm the accuracy of the method, as well as the structure of the homogenized solution and its correction terms.

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A semidiscrete Galerkin scheme for a coupled two-scale elliptic-parabolic system: well-posedness and convergence rates

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We study the numerical approximation of a coupled system of elliptic-parabolic equations posed on two separated spatial scales. The model equations describe the interplay between macroscopic and microscopic pressures in an unsaturated heterogeneous medium with distributed microstructures as they often arise in modeling reactive flow in cementitious-based materials. Besides ensuring the well-posedness of our two-scale model, we design two-scale convergent numerical approximations and prove *a priori* error estimates for the semidiscrete case. We complement our analysis with simulation results illustrating the expected behaviour of the system.

Multiscale methods for time dependent and high contrast diffusion problems

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I will discuss two challenges in solving partial differential equations with heterogeneous diffusion coefficient. First, I will consider the problem of high contrast data with thin highly conductive structures, which lead to global couplings only seen on the finer scales. This is challenging for numerical methods since it leads to slow convergence of iterative methods or dense coarse scale representations for multiscale methods. Second, I will consider changing coefficients in a random or time dependent setting. I will discuss how error indicators can be used to update the multiscale basis functions locally in an efficient way.

Asymptotic analysis of an ϵ-Stokes problem with several pressure boundary conditions

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We consider an ε-Stokes problem [1, 2];

$$\begin{cases} -\Delta u^\epsilon + \nabla p^\epsilon = F & \text{in } \Omega, \\ -\epsilon \Delta p^\epsilon + \nabla \cdot u^\epsilon = -\epsilon \nabla \cdot F & \text{in } \Omega, \\ u^\epsilon = u^b & \text{on } \Gamma := \partial\Omega, \\ + \text{ boundary condition for } p^\epsilon, \end{cases}$$

connecting the classical Stokes problem and the corresponding pressure-Poisson problem using one parameter $\varepsilon > 0$. We show that the solution to the ϵ-Stokes problem converges to the one for the Stokes problem as ϵ tends to 0, and to the one for the pressure-Poisson problem as ϵ tends to ∞. Several numerical examples illustrate the expected behavior of the solutions. In addition, we show that the solution to the ϵ-Stokes problem has a nice asymptotic structure.

This research was a joint work with Professor Masato Kimura at Kanazawa University, Professor Adrian Muntean at Karlstad University, and Professor Hirofumi Notsu at Kanazawa University.

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Analysis of a Nonlinear, Moving-Boundary, 3d Fluid-Mesh-Shell Interaction Problem

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We consider a nonlinear, moving-boundary, fluid-structure interaction problem between an incompressible, viscous fluid flow, and an elastic structure composed of a cylindrical shell supported by a mesh-like elastic structure. The fluid flow is modeled by the time-dependent Navier-Stokes equations in a three-dimensional cylindrical domain, while the cylindrical shell is described by a two-dimensional linearly elastic Koiter shell equations allowing displacements in all three spatial directions. The mesh-like structure is modeled as a one-dimensional hyperbolic net composed of linearly elastic curved rods. The rods are coupled at net's vertices via continuity of displacement and infinitesimal rotation, and through balance of forces and contact moments. The fluid and the mesh-supported structure are coupled via the kinematic and dynamic boundary coupling conditions describing continuity of velocity and balance of contact forces at the fluid-structure interface. We prove the existence of a weak solution to this nonlinear, moving-boundary problem by using the time discretization via Lie operator splitting method, Arbitrary Lagrangian-Eulerian mapping and a non-trivial extension of the Aubin-Lions-Simon compactness result to problems on moving domains.

Singular limits in thin domains - Low Mach Number limits /An accretion disk

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We study the 3-D compressible barotropic radiation fluid dynamics system describing the motion of the compressible rotating viscous fluid with gravitation and radiation confined to a straight layer $\Omega_\epsilon = \omega \times (0, \epsilon)$, where ω is a 2-D domain. We show that weak solutions in the 3-D domain converge to the strong solution of the rotating 2-D Navier–Stokes–Poisson system with radiation in ω as $\epsilon \to 0$ or to the rotating pure 2-D Navier–Stokes system with radiation.

Secondly, we consider the compressible Navier-Stokes system describing the motion of a viscous fluid confined to a straight layer $\Omega_\delta = (0, \delta) \times \mathbb{R}^2$. We show that the weak solutions in the 3D domain converge strongly to the solution of the 2D incompressible Navier-Stokes equations (Euler equations) when the Mach number ϵ tends to zero as well as $\delta \to 0$ (and the viscosity goes to zero).

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Mathematical modeling and multiscale analysis of transport processes through membranes

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In this presentation, we develop multiscale methods for the derivation and analysis of effective models in environments containing membranes. At the microscopic level, where membranes are modeled as thin heterogeneous layers, the model consists of nonlinear reaction-diffusion equations within each subdomain. At the macroscopic level, membranes are reduced to interfaces, and effective transmission conditions and/or effective equations at these interfaces are derived. It turns out that the form of the effective laws at the interface depends on the scaling of the microscopic system as well as of the type of microscopic transmission conditions imposed at the bulk-layer interface. For the derivation of macroscopic (effective) models, we first generalize the concept of weak and strong two-scale convergence to thin layers with periodic structure. This allows to derive cell-problems, which approximate at zeroth order the processes in the membrane. By adding corrector terms to the macroscopic solution, we obtain higher order approximations. To validate these approximations, we prove error estimates with respect to the scaling parameter ϵ. This talk is based on a series o papers obtained in collaboration with Markus Gahn (Hasselt), Willi Jäger (Heidelberg) and Peter Knabner (Erlangen).

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Exponential decay of the resonance error in numerical homogenization via a modified elliptic approach.

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Numerical homogenization methods are widely employed for approximating the macroscopic behaviour of multiscale differential problems. These methods aim at solving a *homogenized* model derived by averaging the solutions of *correctors equations* over local representative volume elements (RVEs) of size $\delta = \mathcal{O}(\varepsilon)$, where ε is the finest oscillation length. A resonance error, proportional to ε/δ, occurs if the period of the heterogeneities is not perfectly captured, or if the medium is not periodic (e.g., random), as proved in [2] and shown in [3] with several boundary conditions. In many applications, such an error dominates other discretization errors, but reducing it by enlarging the RVE size can be computationally too expensive. Therefore, it is fundamental to design micro-macro coupling techniques with higher order rates for the resonance error.

We propose a novel modified corrector equation to compute the homogenized coefficients, with a forcing term that is the solution of a diffusion equation at a finite time T. In the limit for $T \to +\infty$, the forcing term vanishes and the standard corrector equation is recovered. By use of Green's function estimates we prove the exponential convergence of the resonance error w.r.t. δ/ε, provided that some modelling parameters are properly chosen, as described in [1]. The results are proved rigorously in the smooth, periodic setting, but numerical experiments show that the method is also applicable to piecewise discontinuous and stochastic coefficients.

Finally, instead of solving a costly time-dependent diffusion equation up to time T, the forcing term can be cheaply pre-computed by a proper discretization of the operator e^{-AT}, where A is the second order differential operator defined by $A := \nabla \cdot (a(x)\nabla)$. The operator e^{-AT} is approximated by computing the exponential function of a low-rank approximation of the stiffness matrix, obtained by Arnoldi decomposition.

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Simulating Structure Formation in Soils across Scales using Discontinuous Galerkin Methods

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We discuss mathematical modeling, analysis, and numerical realization of microaggregates in soils. These microaggregates have the size of a few hundred micrometers and can be understood as the fundamental building units of soil. Thus, understanding their dynamically evolving, three-dimensional structure is crucial for modeling and interpreting many soil parameters such as diffusivities and flow paths that come into play in CO_2-sequestration or oil recovery scenarios.

Among others, the following aspects of the formation of microaggregates should be incorporated into a mathematical model and investigated in more detail: the spatial heterogeneity of the temporally evolving structure of microaggregates and the different processes that take place on different scales — temporal and spatial — within the so-called micro-scale itself. The presentation aims at formulating a process-based pore-scale model, where all chemical species are measured in concentrations. That is, we have a continuous model for reactive transport mainly in terms of partial differential equations (PDEs) with algebraic constraints. This continuous model is defined on a discrete and discretely moving domain whose geometry changes according to the rules of a cellular automaton method (CAM). These rules describe the restructuring of the porous matrix, growth and decay of biomass, and the resulting topological changes of a wetting fluid and a gas phase. The cellular automaton rules additionally imply stochastic aspects that are important on the pore-scale.

Moreover, effects and knowledge deduced from the model are transferred to scales which are more relevant for applications. The quality of these averaged models is of general interest, since simulations for the field-scale that resolve the pore-scale are not applicable for economical reasons. Thus, we compare parameterizations of diffusivities with mathematically rigorous results and give suggestions to improve the formulas that can be found in the literature.

The discrete movement of the microaggregates' geometry at the micro-scale poses mathematical problems. The following question arises: Can the averaged quantities deduced from the pore-scale really be used for models on other scales or are the impacts of the artificial temporal jumps too detrimental for the solutions on other scales to be accurate? In the following, this problem is also dealt with, and the reliability of the obtained parameters is underlined.

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Semi-Lagrangian Reconstruction Finite Elements for Advection-dominated Problems with Rough Data

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Long time scales in climate simulations, e.g., in simulations of paleo climate, require coarse grids due to computational constraints. Coarse grids, however, leave important smaller scales unresolved. Thus small scale processes that significantly influence the resolved scales need to be taken care of by different means. Such subgrid processes include (slowly) moving land-sea interfaces or ice shields as well as flow over urban areas or biogeochemistry. State-of-the-art dynamical cores represent the influence of subscale processes typically via subscale parametrizations and often employ a rather heuristic coupling of scales.

We aim to improve the mathematical consistency of the upscaling process that transfers information from the subgrid to the coarse prognostic and diagnostic quantities (and vice-versa). We investigate new bottom-up techniques for advection-dominated problems whose main motivation are climate simulations [1]. Our tools are based on ideas for multiscale finite element methods for elliptic problems that play a role, in oil reservoir modeling and porous media in general [2, 3]. These ideas, however, fail in advection-dominated scenarios (which are typical for flows encountered in climate models) since they are not based on a suitable decomposition of the computational domain.

We present new Garlerkin based ideas to account for the typical difficulties in climate simulations. Our idea is based on a previous work that employs a change of coordinates based on a coarse grid characteristic transform induced by the advection term to make its effect on coarse scales milder. This also accounts for appropriate subgrid boundary conditions for the multiscale basis functions. Boundary conditions are essential for such approaches. This is the starting point of a set of semi-Lagrangian techniques that locally in time reconstruct subgrid variability in a Galerkin basis through many local inverse problems. We discuss extensions and drawbacks of this approach and present examples with rapidly varying coefficients on several scales.

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A nonlocal terms arising in effective problems in critical scale homogenization of a different boundary-value problems with dynamic boundary conditions

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In present talk we study a homogenization of problems for a parabolic and elliptic equations in a domains depending on a small positive parameter ϵ with a dynamic boundary conditions containing a parameter $\beta(\epsilon)$, which has the critical value. The main goal of our work is to construct the effective problems and to prove a weak convergence of the sequence of solutions of the original problem to the solution of the homogenized problem as $\epsilon \to 0$.

Multiscale Wavelet Structures on the Sphere

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It is well known that adaptive wavelet methods can be used to effectively solve operator equations of the form

$$\mathcal{A}u = f$$

where $\mathcal{A}$ can be either pseudodifferential operators[2] or integral operators[3]. Therefore constructing wavelets systems with good properties on manifolds becomes especially important to understand the behaviour and to approximate the solutions of PDEs.

In this talk I would like to quickly introduce some successful methods for wavelet construction on the sphere, since spheres have the broadest application next to the Euclidean planes among manifolds, and expose their different merits and probably as well as some insufficiency. In particular, I exhibit the multiscale structures arising from those methods. Afterwards I propose a new wavelet system consisting of spherical α-wavelets/shearlets, generalizing and improving some of the previous ideas, while having the potential to deal with both isotropic and anisotropic singularities.

Furthermore, according to a new result from [1], for a large class of admissible generating functions, the corresponding spherical α-wavelets form a tight frame for square integrable functions, namely they can be utilized to decompose and reconstruct a function in a stable way. Consequently, Galerkin-scheme can be applied to solve operator equations on the sphere in a multiscale way.

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Macroscopic models for reaction-diffusion processes in composite media with imperfect interfaces

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In this talk, we shall present some recent homogenization results for a class of reaction-diffusion problems in a highly heterogeneous periodic composite material formed by two constituents, separated by an imperfect interface where the solution and its flux exhibit jumps. Depending on the geometry of the composite medium, on the properties of its two constituents, and on the interplay between the scaling parameters and the form of the functions describing the jumps involved in the microscopic problems, several macroscopic models are derived via periodic homogenization techniques and compared to some existing models in the literature (see, e.g., [2, 3, 4, 5, 7]). The influence of the above mentioned discontinuities is captured at the limit in various ways: in the macroscopic solution, in additional source terms, or in the correctors. This setting can be also relevant for modeling the electrical conduction or the calcium dynamics in biological tissues [1, 6, 8].

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On the role of asymptotic methods in lubrication

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Lubrication plays a crucial role in many engineering areas. For example, in almost all machines there are surfaces in proximity and moving relative to each other, see Fig. 1. Friction and wear can be significantly reduced by lubricating the surfaces with oil or

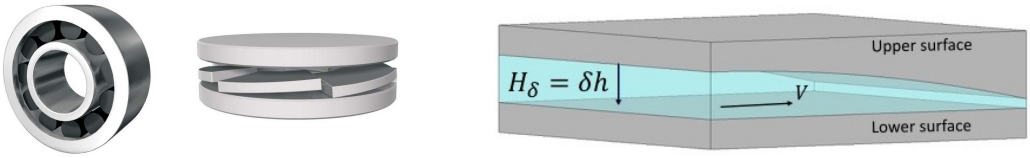

FIGURE 1. Two kinds of bearings and an illustration of how a small parameter δ naturally appears in the modelling of thin film flow.

grease. It is therefore important to develop applicable mathematical models which can be used to optimise the lubrication. This leads us to modelling of thin film flow.

Assume that the distance between the surfaces is of the form $H_\delta = \delta h$, see Fig. 1. Hence δ is a parameter which describes the thickness of the fluid film. The velocity u_δ, the pressure p_δ and the temperature T_δ can be modeled by the continuity equation, Navier-Stokes equations and an energy equation. Obviously, this system is very complex and it is therefore desirable to find simplified models.

In the mathematical modelling of lubrication it has turned out to be very useful to use fact that the fluid domain is thin to reduce the original three dimensional problem to a two dimensional one. This is done by various types of asymptotic analysis as the film thickness goes to zero, i.e. $\delta \to 0$. The classical result by Reynolds states that $p_\delta \approx p$ for small values of δ, where p is the solution of the Reynolds equation

$$\frac{\partial(\rho H)}{\partial t} - \nabla \cdot \left(\frac{\rho H^3}{12\mu} \nabla p \right) = -\nabla \cdot \left(\frac{\rho H}{2} V \right). \tag{4}$$

The equation (4) is only rigorously derived for simple types of flows. However, in lubrication theory it is common to assume that it also is valid for much more general flow situations. For example it is common to incorporate different physical phenomena into the Reynolds equation such as e.g. cavitation, surface roughness, elastohydrodynamic effects, temperature and pressure dependent viscosity etc. To have a satisfactory theory it would therefore be important to include such effects already in the Navier-Stokes equations and thereafter consider the asymptotic behavior as $\delta \to 0$. In particular, this is interesting when surface roughness is taken into account, since then there are two small parameters involved, ε representing the wavelength of the roughness and δ.

In this talk it will be shown that in some situations the error can be substantial if the effect of surface roughness is just added to the Reynolds equation, see [1]. Moreover, we will discuss some known results and point out some open questions in the field [2]. Leakage in seals and its relation to flow in thin porous media will also be considered.

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