Equadiff 2024

Karlstad, Sweden

10-14 June, 2024

Contents

Welcome

Dear participant,

we welcome you to Equadiff 2024 and to Karlstad University. We truly hope that the event will be interesting and scientifically fruitful.

In summary, 484 participants from more than 40 countries have decided to come to Karlstad and engage with 10 plenary talks, 33 invited mini-symposia, 27 special sessions with contributed talks, and 19 posters.

We hope that your week is filled with much science and social interaction!

Best wishes,

Adrian Muntean, on behalf of Equadiff 2024

Useful Information

Length of talks: All plenary talks will be an hour. All other talks will be 25 minutes with 5 minutes for questions. All rooms are equipped with both projectors and whiteboards. Poster presentations will take place during the first fika breaks on Tuesday, Wednesday, and Thursday in the Aula Foyer. The poster presenters are asked to be with their posters during the indicated time slots. Additionally, the posters will be displayed throughout the week.

Registration: The registration desk is located in the Aula foyer and will be open during the conference starting **Monday 8:15**. Here, you will get your name tag and can request help with installing the EquaDiff app *Kau Event* as well.

Location: The conference takes place at Karlstad university in the Aula, Building 1, and Building 21 (see map). There will be signs guiding you to the different rooms. We also recommend the MazeMap[1] to find your way around.

App: Please donwload and install the EquaDiff app **Kau Event** (available on Google Play and Apple Store) where you can access the schedule, abstracts, bus tickets, lunch menus and further useful information. Also, you can connect on LinkedIn with the handle *The Equadiff Conference 2024*.

Connection: The bus stop *Universitetet* (see map) is served from Karlstad centrum (*Stora Torget*) by bus lines 1, 2, 3 (with 1 being the fastest and 3 being the slowest route). Complimentary bus tickets are available via the EquaDiff app *Kau Event*.

Fika: A Swedish coffee break with socialization!

Lunch: Lunch is offered in the restaurant *Solsta Inn* and the *Transformum* (both in building 1A). Lunch there is free-of-charge upon showing your name tag. The menu will be available in advance via the app *Kau Event*.

[1] Available as an app or at https://www.kau.se/campuskartor.

Wi-Fi: Access is provided via Eduroam throughout the whole conference area. If you do not have Eduroam at your University, please inform the staff when registering and they will arrange a guest account for the week.

Schedule

Time	Monday	Tuesday	Wednesday	Thursday	Friday
08:00					
		08:45-10:45 MS: 2, 4, 6, 7, 10, 16, 18, 24, 26, 28, 31 CT: 9, 10, 11	**08:45-10:45** MS: 1, 3, 6, 10, 12, 14, 15, 21, 27, 32, 35 CT: 15, 16, 17	**08:45-10:45** MS: 1, 2, 4, 5, 9, 13, 15, 24, 25, 30, 32, 20 CT: 20, 22	**08:45-10:45** MS: 3, 5, 8, 14, 16, 17, 18, 19, 24 CT: 26, 27
09:00					
09:30	**OPENING**				
	09:45-10:45 **Kühn**				
10:00					
	FIKA	**FIKA** **Posters**	**FIKA** **Posters**	**FIKA** **Posters**	**FIKA**
11:00					
	11:15-12:15 **Karlsen**	**11:15-12:15** **Vicol**	**11:15-12:15** **Baladi**	**11:15-12:15** **Sakajo**	**11:15-12:15** **Mielke**
12:00					
	12:15-13:45 LUNCH	**12:15-13:45** LUNCH	**12:15-13:45** LUNCH	**12:15-13:45** LUNCH	**12:15-13:45** LUNCH
13:00					
	13:45-14:45 MS: 28, 31, 33, 34 CT: 1, 2, 3, 4	**13:45-14:45** **Świerczewska-Gwiazda**	**13:45-14:45** **Dalibard**	**13:45-14:45** **Hoffmann**	**13:45-14:45** **Bedrossian**
14:00					
	FIKA	**FIKA**	**FIKA**	**FIKA**	**CLOSING**
15:00					**FIKA** **TO-GO**
	15:15-17:15 MS: 6, 9, 16, 17, 25, 27, 28, 30, 33, 35 CT: 5, 6, 7, 8	**15:15-17:15** MS: 1, 6, 12, 13, 19, 20 21, 22, 25, 27, 34 CT: 12, 13, 14	**15:15-16:15** MS: 5, 14, 15, 31, 34 CT: 18, 19, 21	**15:15-17:15** MS: 1, 5, 7, 8, 10, 12, 14, 18, 22, 26 CT: 23, 24, 25	
16:00					
17:00					
	MS: 28, 33 (until 18:00)				
18:00					
19:00	Mingle Bazar		Conference Dinner KCCC		

List of Minisymposia

MS1: Analysis of nonlocal PDEs.

Organizers: Nikos Kavallaris (Karlstad University) and Kateryna Marynets (Delft University of Technology).

MS2: Around boundary layers and singular limits in fluid mechanics.

Organizers: Roberta Bianchini (IAC CNR, Rome), Anna Mazzucato (Penn State University), and Marco Sammartino (University of Palermo).

MS3: Collective phenomena.

Organizers: Roman Shvydkoy (University of Illinois at Chicago), Trevor Leslie (Illinois Institute for Technology), and Changhui Tan (University of South Carolina).

MS4: Compressible fluids (singular limits).

Organizers: Milan Pokorny (Charles University, Prague) and Aneta Wroblewska-Kaminska (Polish Academy of Sciences).

MS5: Computer assisted proofs in dynamics.

Organizers: Alex Haro (Universitat de Barcelona & CRM) and Javier Gomez Serrano (Brown University).

MS6: Delay differential equations and applications.

Organizers: Teresa Faria (University of Lisbon), Abel Garab (University of Szeged), and Anatoli Ivanov (Pennsylvania State University).

MS7: Diffusion processes in population dynamics and biology.

Organizers: Luisa Malaguti (University of Modena and Reggio Emilia) and Torsten Lindström (Linnaeus University).

MS8: Dynamics of localized patterns- the interplay between intrinsic and extrinsic instabilities.

Organizers: Arjen Doelman (Leiden University) and Yasumasa Nishiura (Hokkaido University).

MS9: Dynamics of vorticies and filaments.

Organizers: Monica Musso (University of Bath) and Thierry Gallay (Université Grenoble

Alpes).

MS10: Emerging problems in the homogenization of multiphysics systems.

Organizers: Grigor Nika (Karlstad University) and Mariya Ptashnyk (Heriot-Watt University).

MS12: Fluid-structure interactions.

Organizers: Boris Muha (University of Zagreb), Irena Lasiecka (University of Memphis), Ana Silvestre (Instituto Superior Técnico, Universidade de Lisboa).

MS13: Functional inequalities and nonlinear PDEs.

Organizers: Erik Lindgren (KTH), Lorenzo Brasco (Università di Ferrara), Simon Larson (University of Gothenburg)

MS14: Higher-dimensional dynamics: Invariant sets and stability properties.

Organizers: Tali Pinsky (Technion), Warwick Tucker (Monash University), Dmitry Turaev (Imperial College London).

MS15: Hypercoercivity in evolutionary PDEs.

Organizers: Eric Carlen (Rutgers University), Anton Arnold (Vienna University of Technology), Arnaud Guillin (Clermont-Auvergne) and Frederic Herau (Nantes University).

MS16: Hyperbolic PDEs: Models and theorems.

Organizers: Rinaldo M. Colombo (University of Brescia) and Katrin Grunert (NTNU).

MS17: Mathematical models and methods in materials science.

Organizers: Toyohiko Aiki (Japan Women's University) and Michael Eden (Karlstad University).

MS18: Mathematics of porous media.

Organizers: Christian Rohde (University of Stuttgart) and Iuliu Sorin POP (Hasselt University).

MS19: Measure-valued equations and applications.

Organizers: Sander Hille (Leiden University) and Piotr Gwiazda (University of Warsaw).

MS20: Nonautonomous dynamical systems and oscillation.

Organizers: Roberta Fabbri (Università di Firenze) and Roman Simon Hilscher (Masaryk University).

MS21: Nonlinear waves in dispersive equations.

Organizers: Louise Gassot (Université de Rennes) and Changzhen Sun (University of Franche-Comte).

MS22: Nonlinear waves in reaction-diffusion systems.

Organizers: Gabriela Jaramillo (University of Houston), Stephanie Dodson (Colby College), and Peter van Heijster (Wageningen University & Research).

MS24: Pattern-forming systems and asymptotic models.

Organizers: Bastian Hilder (Technical University of Munich), Jonas Jansen (Lunds universitet), and Guido Schneider (Stuttgart University).

MS25: Random dynamical systems.

Organizers: Nils Berglund (IDP, University of Orleans) and Maximilian Engel (University of Amsterdam).

MS26: Recent developments in mathematical fluid dynamics.

Organizers: Christian Seis (Universität Münster) and Michele Coti Zelati (Imperial College London).

MS27: Singularly perturbed systems.

Organizers: Paul Carter (University of California, Irvine), Peter Szmolyan (Technische Universität Wien), and Kristian Uldall Kristiansen (Technical University of Denmark).

MS28: Stochastic PDEs.

Organizers: Benjamin Gess (Bielefeld University), Mate Gerencser (TU Wien), and Daniel Heydecker (Max Planck Institute for Mathematics in the Sciences).

MS30: Synchronisation and reduction techniques of coupled oscillators.

Organizers: Peter Langfield (Inria Bordeaux), Anne Skeldon (University of Surrey), and Hildeberto Jardon Kojakhmetov (University of Groningen).

MS31: Systems with multiple timescales.

Organizers: Roberto Barrio (University of Zaragoza), Annalisa Iuorio (Parthenope University of Naples), and Christian Kühn (TU München).

MS32: Time-asymptotic stability in hyperbolic equations.

Organizers: Miguel Rodrigues (Université de Rennes) and Moon-Jin Kang (KAIST).

MS33: Topological data analysis.

Organizers: Alexandria Volkening (Purdue University), Daniel Cruz (University of Florida), and Sarah Tymochko (UCLA).

MS34: Topological fluid dynamics.

Organizers: Renzo Ricca (University of Milano-Bicocca), Takashi Sakajo (Kyoto University), and Jean-Luc Thiffeault (University of Wisconsin - Madison).

MS35: Water waves and free boundary problems in fluid mechanics.

Organizers: Samuel Walsh (University of Missouri) and Ian Tice (Carnegie Mellon University)

Classifications of Contributed Talks

Dynamical systems: CT 1, 5, 10, 13, 17

Applied partial differential equations: CT 2, 3, 6, 11, 12, 16, 18, 19, 21, 22, 23, 24, 25, 26

Computational mathematics: CT 7, 14, 27

Model reduction: CT 20

Ordinary differential equations: CT 9, 15

Variational methods and other topics: CT 4, 8

Schedule by day

Monday, 13:45 - 14:45

[**MS28:**] *Stochastic PDEs.* Room: 1A305

[**MS31:**] *Systems with multiple timescales.* Room: 1B306

[**MS33:**] *Topological data analysis.* Room: 1B309

[**MS34:**] *Topological fluid dynamics.* Room: 1B364

[**CT1:**] *Dynamical Systems.* Room: 1D222

[**CT2:**] *Applied PDEs.* Room: 1D226

[**CT3:**] *Applied PDEs.* Room: 1D236

[**CT4:**] *Variational methods and other topics.* Room: 1D237

Monday, 15:15 - 17:15

[**MS6:**] *Delay differential equations and applications.* Room: 1B364

[**MS9:**] *Dynamics of vorticies and filaments.* Room: 1B306

[**MS16:**] *Hyperbolic PDEs: Models and theorems.* Room: 1D222

[**MS17:**] *Mathematical models and methods in materials science.* Room: 1D226

[**MS25:**] *Random dynamical systems.* Room: 1D236

[**MS27:**] *Singularly perturbed systems.* Room: 1D237

[**MS28:**] *Stochastic PDEs.* Room: 1A305

[**MS30:**] *Synchronisation and reduction techniques of coupled oscillators.* Room: 1D327

[**MS33:**] *Topological data analysis.* Room: 1B309

[**MS35:**] *Water waves and free boundary problems in fluid mechanics.* Room: 1D328

[**CT5:**] *Dynamical systems.* Room: 1D340

[**CT6:**] *Applied PDEs.* Room: 1D341

[**CT7:**] *Computational mathematics.* Room: 21A342

[**CT8:**] *Variational methods and other topics.* Room: 1D227

Tuesday, 8:45 - 10:45

[**MS2:**] *Around boundary layers and singular limits in fluid mechanics.* Room: 1B309

[**MS4:**] *Compressible fluids (singular limits).* Room: 1D222

[**MS6:**] *Delay differential equations and applications.* Room: 1B364

[**MS7:**] *Diffusion processes in population dynamics and biology.* Room: 1D226

[**MS10:**] *Emerging problems in the homogenization of multiphysics systems.* Room: 1D236

[**MS16:**] *Hyperbolic PDEs: Models and theorems.* Room: 1A305

[**MS18:**] *Mathematics of porous media.* Room: 1B306

[**MS24:**] *Pattern-forming systems and asymptotic models.* Room: 1D237

[**MS26:**] *Recent developments in mathematical fluid dynamics.* Room: 1D227

[**MS28:**] *Stochastic PDEs.* Room: 1D327

[**MS31:**] *Systems with multiple timescales.* Room: 1D328

[**CT9:**] *ODEs.* Room: 1D340

[**CT10:**] *Dynamical systems.* Room: 1D341

[**CT11:**] *Applied PDEs.* Room: 21A258

Tuesday, 15:15 - 17:15

[**MS1:**] *Analysis of nonlocal PDEs.* Room: 1A305

[**MS6:**] *Delay differential equations and applications.* Room: 1B364

[**MS12:**] *Fluid-structure interactions.* Room: 1B306

[**MS13:**] *Functional inequalities and nonlinear PDEs.* Room: 1D222

[**MS19:**] *Measure-valued equations and applications.* Room: 21A342

[**MS20:**] *Nonautonomous dynamical systems and oscillation.* Room: 1D226

[**MS21:**] *Nonlinear waves in dispersive equations.* Room: 1D236

[**MS22:**] *Nonlinear waves in reaction-diffusion systems.* Room: 1D237

[**MS25:**] *Random dynamical systems.* Room: 1D328

[**MS27:**] *Singularly perturbed systems.* Room: 1D340

[**MS34:**] *Topological fluid dynamics.* Room: 1D341

[**CT12:**] *Applied PDEs.* Room: 1B309

[**CT13:**] *Dynamical systems.* Room: 1D327

[**CT14:**] *Computational mathematics.* Room: 1D227

Wednesday, 8:45 - 10:45

[**MS1:**] *Analysis of nonlocal PDEs.* Room: 1A305

[**MS3:**] *Collective phenomena.* Room: 1B309

[**MS6:**] *Delay differential equations and applications.* Room: 1B364

[**MS10:**] *Emerging problems in the homogenization of multiphysics systems.* Room: 21A342

[**MS12:**] *Fluid-structure interactions.* Room: 1D222

[**MS14:**] *Higher-dimensional dynamics: Invariant sets and stability properties.* Room: 1D236

[**MS15:**] *Hypercoercivity in evolutionary PDEs.* Room: 1D237

[**MS21:**] *Nonlinear waves in dispersive equations.* Room: 1D327

[**MS27:**] *Singularly perturbed systems.* Room: 1D328

[**MS32:**] *Time-asymptotic stability in hyperbolic equations.* Room: 1B306

[**MS35:**] *Water waves and free boundary problems in fluid mechanics.* Room: 1D226

[**CT15:**] *ODEs.* Room: 1D340

[**CT16:**] *Applied PDEs.* Room: 1D341

[**CT17:**] *Dynamical systems.* Room: 1D227

Wednesday, 15:15 - 16:15

[**MS5:**] *Computer assisted proofs in dynamics.* Room: 1D222

[**MS14:**] *Higher-dimensional dynamics: Invariant sets and stability properties.* Room: 1D226

[**MS15:**] *Hypercoercivity in evolutionary PDEs.* Room: 1B306

[**MS31:**] *Systems with multiple timescales.* Room: 1B309

[**MS34:**] *Topological fluid dynamics.* Room: 1A305

[**CT18:**] *Applied PDEs.* Room: 1D236

[**CT19:**] *Applied PDEs.* Room: 1D237

[**CT21:**] *Applied PDEs.* Room: 1D327

Thursday, 8:45 - 10:45

[**MS1:**] *Analysis of nonlocal PDEs.* Room: 21A342

[**MS2:**] *Around boundary layers and singular limits in fluid mechanics.* Room: 1A305

[**MS4:**] *Compressible fluids (singular limits).* Room: 1B309

[**MS5:**] *Computer assisted proofs in dynamics.* Room: 1B364

[**MS9:**] *Dynamics of vorticies and filaments.* Room: 1D222

[**MS13:**] *Functional inequalities and nonlinear PDEs.* Room: 1D226

[**MS15:**] *Hypercoercivity in evolutionary PDEs.* Room: 1D236

[**MS20:**] *Nonautonomous dynamical systems and oscillation.* Room: 1D341

[**MS24:**] *Pattern-forming systems and asymptotic models.* Room: 1D237

[**MS25:**] *Random dynamical systems.* Room: 1D327

[**MS30:**] *Synchronisation and reduction techniques of coupled oscillators.* Room: 1D328

[**MS32:**] *Time-asymptotic stability in hyperbolic equations.* Room: 1D340

[**CT20:**] *Model reduction.* Room: 1B306

[**CT22:**] *Applied PDEs.* Room: 1D227

Thursday, 15:15 - 17:15

[**MS1:**] *Analysis of nonlocal PDEs.* Room: 21A342

[**MS5:**] *Computer assisted proofs in dynamics.* Room: 1B364

[**MS7:**] *Diffusion processes in population dynamics and biology.* Room: 1A305

[**MS8:**] *Dynamics of localized patterns- the interplay between intrinsic and extrinsic instabilities.* Room: 1B309

[**MS10:**] *Emerging problems in the homogenization of multiphysics systems.* Room: 1D222

[**MS12:**] *Fluid-structure interactions.* Room: 1D226

[**MS14:**] *Higher-dimensional dynamics: Invariant sets and stability properties.* Room: 1D236

[**MS18:**] *Mathematics of porous media.* Room: 1D237

[**MS22:**] *Nonlinear waves in reaction-diffusion systems.* Room: 1D328

[**MS26:**] *Recent developments in mathematical fluid dynamics.* Room: 1D340

[**CT23:**] *Applied PDEs.* Room: 1D341

[**CT24:**] *Applied PDEs.* Room: 1B306

[**CT25:**] *Applied PDEs.* Room: 1D327

Friday, 8:45 - 10:45

[**MS3:**] *Collective phenomena.* Room: 1A305

[**MS5:**] *Computer assisted proofs in dynamics.* Room: 1A306

[**MS8:**] *Dynamics of localized patterns- the interplay between intrinsic and extrinsic instabilities.* Room: 1B364

[**MS14:**] *Higher-dimensional dynamics: Invariant sets and stability properties.* Room: 1D222

[**MS16:**] *Hyperbolic PDEs: Models and theorems.* Room: 1D226

[**MS17:**] *Mathematical models and methods in materials science.* Room: 1D236

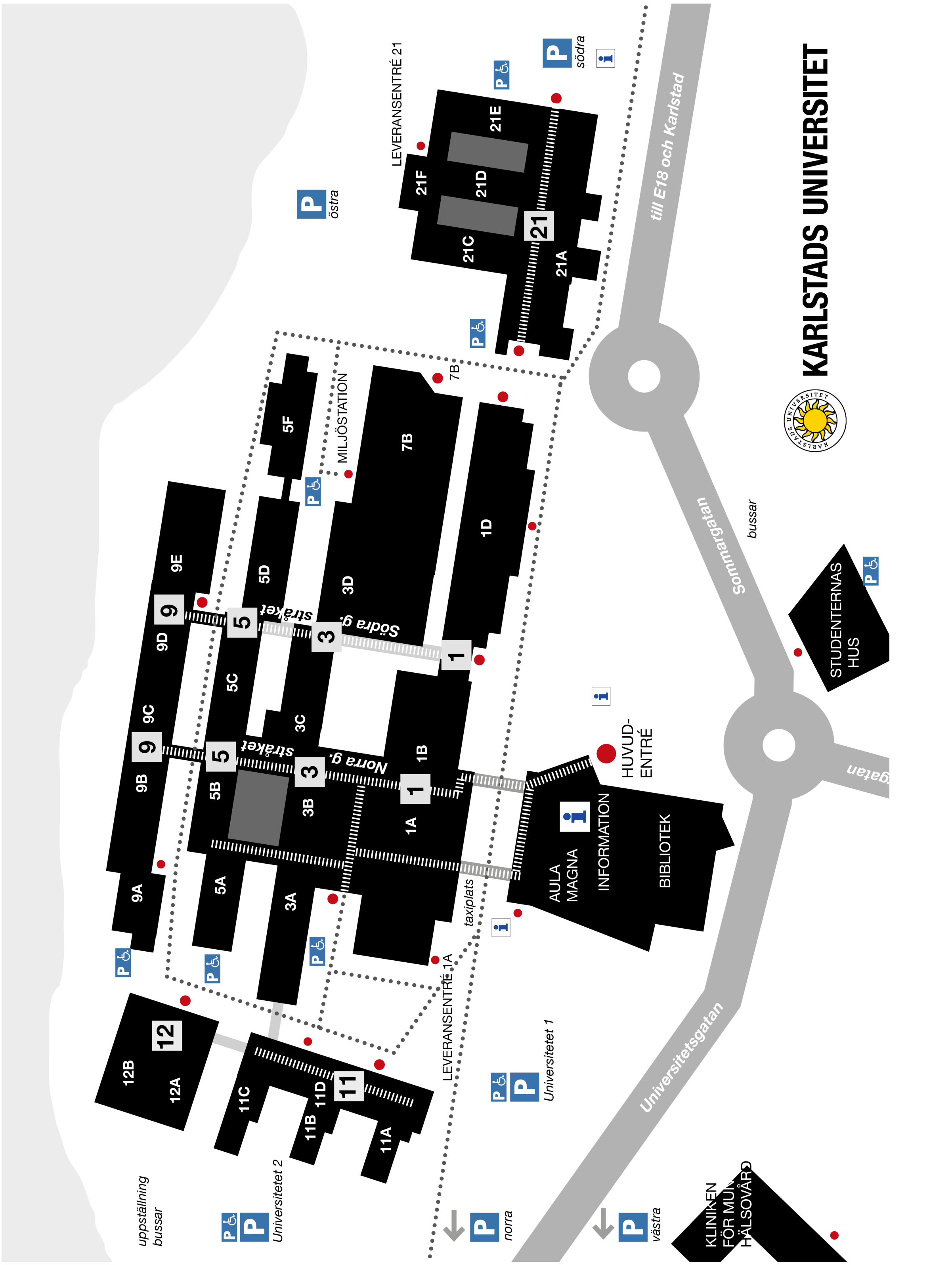
[**MS18:**] *Mathematics of porous media.* Room: 1D237

[**MS19:**] *Measure-valued equations and applications.* Room: 21A342

[**MS24:**] *Pattern-forming systems and asymptotic models.* Room: 1D324

[**CT26:**] *Applied PDEs.* Room: 1D328

[**CT27:**] *Computational mathematics.* Room: 1D341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Local Organizing Chairs

- Adrian Muntean (Karlstad University)
- Rainey Lyons (Karlstad University)
- Michael Eden (Karlstad University)
- Erik Wahlén (Lund University)

International Scientific Committee

- Arjen Doelman (Leiden University)
- Alessio Figalli (ETH Zurich)
- Marek Fila (Comenius University of Bratislava)
- Thierry Gallay (University of Grenoble I)
- Martin Hairer (EPFL)
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- Yasumasa Nishiura (Hokkaido University)
- Hinke Osinga (University of Auckland)
- Björn Sandstede (Brown University)
- Kevin Zumbrun (Indiana University)

Abstracts

Plenary Talks

Anisotropic Banach Spaces and Dispersive Billiards

Viviane Baladi

CNRS and Sorbonne University

Wednesday, June 12, 11:15-12:15, Aula Magna

Joint work with J. Carrand (Centro di Giorgi, Pisa, Italy) and M. Demers (Fairfield University, USA).

It has been a quarter century now that Banach spaces of anisotropic distributions have been introduced to study statistical properties of chaotic dynamical systems via Ruelle transfer operators. This approach gives both new proofs of classical results and new results. The first successes were obtained for smooth hyperbolic dynamics. However, some natural dynamical systems, such as dispersive (Sinai) billiards are not smooth. The singularities cause challenging technical difficulties. We shall survey new results on dispersive (discrete or continuous time) dispersive billiards obtained in the past five years using anisotropic Banach spaces, ending with a very recent construction of the measure of maximal entropy for billiard flows satisfying a condition of sparse recurrence to singularities. (We obtain Bernoullicity, but no control of the speed of mixing.)

[1] V. Baladi, J. Carrand, and M. Demers, *Measure of maximal entropy for finite horizon Sinai billiard flows,* `arXiv:2209.00982`, To appear Annales Henri Lebesgue.

Nonlinear dynamics in stochastic systems

Jacob Bedrossian

University of California, Los Angeles

Friday, June 14, 13:45-14:45, Aula Magna

In this overview talk we discuss several results regarding the dynamics of stochastically-forced systems arising in or motivated by fluid mechanics. We discuss several advances in estimating Lyapunov exponents and other dynamical properties, such as strong mixing, quantitative spectral gaps of Markov semigroups, and nonlinear energy transfer in degenerately damped systems. The methods are robust and general, designed to yield some information on high dimensional systems which are far too complicated to "completely" analyze and for which sharp estimates seem out of reach for now. Many of the methods are a mixture of ideas coming from PDEs, stochastic analysis, and dynamical systems. Some applications in passive scalar mixing/turbulence, chaos for Galerkin truncations of the 2d Navier-Stokes equations, and the instability of almost-surely invariant manifolds will be discussed. Includes joint work with Alex Blumenthal (Georgia Tech, USA), Kyle Liss (Duke University, USA), Sam Punshon-Smith (Tulane University, USA), and Chi-Hao Wu (University of California, Los Angeles USA). Work funded by the NSF (USA) and the Simons Foundation.

Nonlinear forward-backward equations

Anne-Laure Dalibard

Sorbonne université/ENS Paris

Wednesday, June 12, 13:45-14:45, Aula Magna

This is a joint work with Frédéric Marbach (CNRS & ENS Paris, France) and Jean Rax (Sorbonne université, France).

The purpose of this talk is to review recent results on equations exhibiting a "forward-backward" structure. Our epitome in the linear case will be the so-called stationary Kolmogorov equation $yu_x - u_{yy} = f$ in the domain $(x_0, x_1) \times (-1, 1)$, which is forward parabolic in the upper half of the domain, and backward parabolic in the lower half. We exhibit explicit singular solutions for this equation (with infinitely smooth data). Hence, the solutions to the equation are regular if and only if the source term and lateral boundary data satisfy a finite number of orthogonality conditions. This is similar to well-known phenomena in elliptic problems in nonsmooth domains. We then step on this linear analysis to address nonlinear equations such as $uu_x - u_{yy} = f$ in the vicinity of the linear shear flow, subject to perturbations of the source term and lateral boundary conditions. We construct smooth solutions thanks to an iterative scheme, taking care to satisfy the orthogonality conditions at every step of the process. We will also review related results by Sameer Iyer and Nader Masmoudi in the framework of the Prandtl system.

Covariance-modulated optimal transport and gradient flows

Franca Hoffmann

California Institute of Technology

Thursday, June 13, 13:45-14:45, Aula Magna

This is a joint work with Prof. Martin Burger (Computational Imaging Group and Helmholtz Imaging, Deutsches Elektronen-Synchrotron DESY, 22607 Hamburg, Germany and Fachbereich Mathematik, Universität Hamburg, 20146 Hamburg, Germany), Prof. Matthias Erbar (Universität Bielefeld, Bielefeld, Germany), Daniel Matthes (Technical University of Munich, Munich, Germany) and André Schlichting (University of Münster, Münster, Germany).

We present a variant of the dynamical optimal transport problem in which the energy to be minimised is modulated by the covariance matrix of the current distribution. Such transport metrics arise naturally in mean-field limits of certain ensemble Kalman methods for solving inverse problems. We show that the transport problem splits into two coupled minimization problems up to degrees of freedom given by rotations: one for the evolution of mean and covariance of the interpolating curve, and one for its shape. Similarly, on the level of the gradient flows a similar splitting into the evolution of moments and shapes of the distribution can be observed. Those show better convergence properties in comparison to the classical Wasserstein metric in terms of exponential convergence rates independent of the Gaussian target.

Compactness of Solutions to Stochastic Transport Equations

Kenneth Karlsen

University of Oslo

Monday, June 10, 11:15-12:15, Aula Magna

We explore the dynamics of sequences of linear and nonlinear transport equations with stochastic influences, including stochastically forced kinetic equations in heterogenous environments and stochastic conservation laws with spatially irregular flux. We present new results on the strong compactness of the velocity averages of solutions under general integrability conditions. Additionally, we study nonlinear and nonlocal transport equations related to shallow water wave models perturbed by noise, like the gradient-noise perturbed Camassa-Holm equation. For this equation we demonstrate global existence of weak solutions for finite-energy data, primarily using a propagation of compactness method. Other models exhibiting analogous structural features, for which similar global existence results can be established, include the Hunter-Saxton and variational wave equations, each subjected to perturbations from gradient noise.

The talk draws upon a collaborative series of papers authored with

M. Erceg, M. Kunzinger, D. Mitrovic

and

L. Galimberti, H. Holden, P. Pang.

[1] M. Erceg, K. H. Karlsen, and D. Mitrović. Velocity averaging under minimal conditions for deterministic and stochastic kinetic equations with irregular drift. Submitted, 2023.

[2] H. Holden, K. H. Karlsen, and P. H. Pang. Global well-posedness of the viscous Camassa–Holm equation with gradient noise. *Discrete Contin. Dyn. Syst.*, 43(2):568–618, 2023.

[3] L. Galimberti, H. Holden, K. H. Karlsen, and P. H. Pang. Global existence of dissipative solutions to the Camassa–Holm equation with transport noise. *J. Differential Equations*, 387:1–103, 2024.

[4] K. H. Karlsen, M. Kunzinger, and D. Mitrovic. A dynamic capillarity equation with stochastic forcing on manifolds: a singular limit problem. *Trans. Amer. Math. Soc.*, 377(1):85–166, 2024.

Stochastic Bifurcations and Early-Warning Signs

Christian Kühn

TUM

Monday, June 10, 9:45-10:45, Aula Magna

I am going to provide an overview of recent progress in the analysis of bifurcations for dynamical systems with a focus on early-warnings signs. The need for a more detailed understanding of stochastic multiscale bifurcations has emerged over the last decades in the context of tipping points (or critical transitions) in many complex systems. As a benchmark application, I am going to motivate the development of the mathematical analysis via problems in climate systems, and in particular the possibility of the tipping of the Atlantic Meridional Overturning Circulation. We shall discuss the ideas, how to prove and utilize scaling laws as early-warning signs for finite-dimensional fast-slow stochastic ODEs and then proceed to try to carry over this theory for stochastic PDEs. The work presented is based upon the series of papers [1–10].

[1] S. Baars, J.P. Viebahn, T.E. Mulder, C. Kuehn, F.W. Wubs, and H.A. Dijkstra. Continuation of probability density functions using a generalized Lyapunov approach. *J. Comput. Phys.*, 336(1):627–643, 2017.

[2] P. Bernuzzi and C. Kuehn. Bifurcations and early-warning signs for SPDEs with spatial heterogeneity. *J. Dyn. Diff. Equat.*, pages 1–45, 2023.

[3] P. Bernuzzi and C. Kuehn. Early-warning signs for SPDEs with continuous spectrum. *arXiv:2307.14080*, 2023.

[4] K. Gowda and C. Kuehn. Warning signs for pattern-formation in SPDEs. *Comm. Nonl. Sci. & Numer. Simul.*, 22(1):55–69, 2015.

[5] C. Kuehn. A mathematical framework for critical transitions: bifurcations, fast-slow systems and stochastic dynamics. *Physica D*, 240(12):1020–1035, 2011.

[6] C. Kuehn. A mathematical framework for critical transitions: normal forms, variance and applications. *J. Nonlinear Sci.*, 23(3):457–510, 2013.

[7] C. Kuehn, K. Lux, and A. Neamtu. Warning signs for non-Markovian bifurcations: color blindness and scaling laws. *Proc. R. Soc. A*, 487(2259):20210740, 2022.

[8] C. Kuehn and F. Romano. Scaling laws and bifurcations for SPDEs. *Eur. J. Appl. Math.*, 30(5):853–868, 2019.

[9] F. Romano and C. Kuehn. Analysis and predictability for tipping points with leading-order nonlinear terms. *Int. J. Bifurc. Chaos*, 28(8):1850103, 2018.

[10] X. Zhang, S. Hallerberg, and C. Kuehn. Predictability of critical transitions. *Phys. Rev. E*, 92:052905, 2015.

Analysis of (fast-slow) reaction-diffusion systems using gradient structures

Alexander Mielke

Weierstraß-Institut für Angewandte Analysis und Stochastik

Friday, June 14, 11:15-12:15, Aula Magna

We consider parabolic reaction-diffusion systems where the reactions are given by the so-called mass-action law from chemistry. Assuming the so-called detailed balance condition and no-flux boundary conditions the system has a gradient structure, where the Boltzmann entropy acts as the driving functional and reaction and diffusion generate the dissipation geometry. We show that the gradient structure can also be extended to energy-electro-reaction-diffusion equations, where an additional heat equation for the temperature and an elliptic Poisson equation for the electrostatic potential are added, cf. [Mie11].

The aim is to exploit such gradient structures for the qualitative analysis of the system. Hence, we will discuss results concerning the existence of equilibria and the exponential convergence of solutions to equilibrium, cf. [HHMM18, MiM18, LaM23]. Finally, we will discuss how convergence in the sense of the energy-dissipation principle can be used to study the limit in fast-slow reaction diffusion systems, cf. [LMPR17, FrL21, MPS21]. The so-called EDP-convergence focuses on the convergence of thermodynamic functionals but, as a by-product, also yields convergence of solutions to solutions of the limiting system.

[FrL21] T. Frenzel and M. Liero: *Effective diffusion in thin structures via generalized gradient systems and EDP-convergence*. Discr. Cont. Dynam. Systems Ser. S **14**:1 (2021) 395–425.

[HHMM18] J. Haskovec, S. Hittmeir, P. A. Markowich, and A. Mielke: *Decay to equilibrium for energy-reaction-diffusion systems*. SIAM J. Math. Analysis **50**:1 (2018) 1037–1075.

[LaM23] V. Laschos and A. Mielke: *Evolutionary Variational Inequalites on the Hellinger-Kantorovich and the spherical Hellinger-Kantorovich spaces*. Submitted (2023) , arXiv:2207.09815v3.

[LMPR17] M. Liero, A. Mielke, M. A. Peletier, and D. R. M. Renger: *On microscopic origins of generalized gradient structures*. Discr. Cont. Dynam. Systems Ser. S **10**:1 (2017) 1–35.

[Mie11] A. Mielke: *A gradient structure for reaction-diffusion systems and for energy-drift-diffusion systems*. Nonlinearity **24** (2011) 1329–1346.

[MiM18] A. Mielke and M. Mittnenzweig: *Convergence to equilibrium in energy-reaction-diffusion systems using vector-valued functional inequalities*. J. Nonlinear Sci. **28**:2 (2018) 765–806.

[MPS21] A. Mielke, M. A. Peletier, and A. Stephan: *EDP-convergence for nonlinear fast-slow reaction systems with detailed balance*. Nonlinearity **34**:8 (2021) 5762–5798.

One-dimensional hydrodynamic PDE model of turbulent flow with an anomalous enstrophy cascade

Takashi Sakajo

Kyoto University

Thursday, June 13, 11:15-12:15, Aula Magna

Turbulence is one of the complex fluid phenomena ubiquitously observed in nature, but its comprehensive definition is unclear as yet. Since turbulence is a highly random phenomenon, its property is often described in terms of ensemble averages of physical quantities. One of the well-known statistical properties of turbulent flows is the cascade phenomenon of inviscid conserved quantities. For instance, the energy is a conserved quantity in non-viscous flow, but when a flow with an extremely small viscosity is in a turbulent state in a three-dimensional domain, the ensemble average of the energy spectrum cascades subject to the scaling law, $\langle E(k,t)\rangle \sim C_E k^{-\frac{5}{3}}$, where $k = |\mathbf{k}|$ is the scale of wave number and C_E is a certain constant. In other words, it indicates that the energy dissipates anomalously in the zero viscous limit. A goal of the mathematical theory of turbulence is to explain how the cascade phenomenon occurs through anomalous dissipation using solutions to 3D Navier-Stokes equations. However, mathematical attempts to answer this question are under development and it is an open problem, see e.g. [1]. To get some insights into the possible mathematical description of turbulence, we propose a one-dimensional analogue of the Navier-Stokes equations generating turbulent flows with the cascade phenomenon of the enstrophy (the L^2 norm of the vorticity), which is an inviscid conserved quantity [2]. We construct the model equation from the generalized Constantin-Lax-Majda-DeGregorio equation [3] by adding a viscous dissipation and a large-scale forcing. In the talk, we show recent mathematical and numerical studies of this hydrodynamic model under several types of forcing functions, which are stochastic/deterministic [2] and random [4]. The key to understanding the anomalous enstrophy cascade is a steady singular solution of the equation having sharp peaks. The equation is also utilized for further investigations of the statistics of turbulent flows such as higher-order moments with intermittency.

[1] T. Sakajo, *Singular solutions of nonlinear hydrodynamic equations arising in turbulence theory*, Sugaku

Expositions **36** No. 1 (2023), 93–117.

[2] T. Matsumoto and T. Sakajo, *One-dimensional hydrodynamic model generating turbulent cascade*, Phys. Rev. E **93** No. 5 (2016), 053101.

[3] H. Okamoto, T. Sakajo and M. Wunsch, *On a generalization of the Constantin-Lax-Majda equation*, Nonlinearity **21** (2008), 2447-2461.

[4] Y. Tsuji and T. Sakajo, *Statistical laws of a one-dimensional model of turbulent flows subject to an external random force*, Nonlinearity **36** No. 8 (2023), 4283–4302.

Relative entropy method for measure-valued solutions

Agnieszka Świerczewska-Gwiazda

University of Warsaw

Tuesday, June 11, 13:45-14:45, Aula Magna

Solving nonlinear partial differential equations within any reasonable class of generalised solutions is undoubtedly one of the foremost tasks of mathematical analysis, notably when the problems one considers are motivated by physically observable phenomena. This task is considered extremely difficult, and rigorous mathematical treatment of many systems of enormous practical importance (e.g. the fundamental equations of fluid mechanics or conservation laws) remains vastly incomplete. To make any progress, one repeatedly needs to deal with somewhat feeble notions of solutions formulated as rather abstract objects. Among these notions, the concept of measure-valued solutions stands out as particularly useful and convenient.

The idea behind measure-valued solutions is a relatively simple one: by embedding the given PDE problem into the space of parameterised measures, one gains linearity and shifts somehow the difficulties that need to be faced in qualitative analysis.

The main concern of the talk will lie in the exposition of the applications of the relative entropy method, which has recently seen very pronounced success. The results using this technique for establishing the weak-strong uniqueness property promoted the notion of measure-valued solutions considerably. With further applications to the convergence of numerical schemes and singular limits, the scope of the relative entropy method for measure-valued solutions seems far from fully discovered.

Firstly, the application to scalar conservation laws will be mentioned, and then the results for compressible models of hydrodynamics will be recalled. The talk is based on several works with a large group of co-authors, that are listed below. The presentation of the topic closely follows the monograph (in preparation, to be published in Oxford University Press) *Measure-Valued Solutions for Time-Dependent Partial Differential Equations* by Tomasz Debiec (University of Warsaw), Piotr Gwiazda (Polish Academy of Sciences), Emil Wiedemann (University of Erlangen-Nuremberg) and myself.

[1] J. A. Carillo, T. Debiec, P. Gwiazda, A. Świerczewska-Gwiazda, E. Wiedemann, *Dissipative measure-valued solutions to the Euler-Poisson equation* SIAM J. Math. Anal. **56** (2024), 304–335.

[2] D. Gallenmüller, P. Gwiazda, A. Świerczewska-Gwiazda, J. Woźnicki, *Cahn-Hillard and Keller-Segel systems as high-friction limits of Euler-Korteweg and Euler-Poisson equations* Calc. Var. Partial Differential Equations **63** (2024), Paper No. 47

[3] P. Gwiazda, O. Kreml, A. Świerczewska-Gwiazda, *Dissipative measure-valued solutions for general conservation laws* Ann. Inst. H. Poincaré C Anal. Non Linéaire **37** (2020), 683–707

[4] P. Gwiazda, A. Świerczewska-Gwiazda, E. Wiedemann, *Weak-strong uniqueness for measure-valued solutions of some compressible fluid models* Nonlinearity **28** (2015), 3873–3890

[5] E. Feireisl, P. Gwiazda, A. Świerczewska-Gwiazda, E. Wiedemann, *Dissipative measure-valued solutions to the compressible Navier-Stokes system* Calc. Var. Partial Differential Equations **55** (2016), Art. 141.

[6] P. Gwiazda, A. Świerczewska-Gwiazda, E. Wiedemann, *Weak-strong uniqueness for measure-valued solutions of some compressible fluid models* Nonlinearity **28** (2015), 3873–3890.

Shock formation and maximal hyperbolic development in multi-D gas dynamics

Vlad Vicol
Courant Institute of Mathematical Sciences, New York University
Tuesday, June 11, 11:15-12:15, Aula Magna

This is a joint work with Prof. Steve Shkoller (University of California at Davis, USA).

We consider the Cauchy problem for the multi-dimensional compressible Euler equations evolving from an open set of compressive and generic smooth initial data. We construct unique solutions to the Euler equations which are as smooth as the initial data, in the maximal spacetime region characterized by: at any point in this spacetime, the solution can be smoothly and uniquely computed by tracing both the fast and slow acoustic characteristic surfaces backward-in-time, until reaching the Cauchy data prescribed along the initial time-slice. The future temporal boundary of this spacetime region is a singular hypersurface, consisting of the union of three sets: first, a co-dimension-2 surface of "first singularities" called the pre-shock; second, a downstream co-dimension-1 surface emanating from the pre-shock, on which the Euler solution experiences a continuum of gradient catastrophes; third, an upstream co-dimension-1 surface consisting of a Cauchy horizon emanating from the pre-shock, which the Euler solution cannot reach. To establish this result, we develop a new geometric framework for the description of the acoustic characteristic surfaces, and combine this with a new type of differentiated Riemann-type variables which are linear combinations of gradients of velocity/sound speed and the curvature of the fast acoustic characteristic surfaces. This is the first result on the maximal hyperbolic development of compressive Cauchy data.

Minisymposia

[MS1] Boundary behavior of solutions to fractional elliptic problems

Serena Dipierro
University of Western Australia
Tuesday, 15:15, 1A305

This is a joint work with Nicola Soave (Università degli Studi di Torino) and Enrico Valdinoci (University of Western Australia).

Solutions of nonlocal equations typically depend rather significantly on their values outside of a given region of interest and, in this sense, it is often convenient to assume "global" conditions to deduce "local" results.

In this talk, we present instead a Hopf Lemma for solutions to some integro-differential equations that does not assume any global condition on the sign of the solutions. We also show that non-trivial radial solutions cannot have infinitely many zeros accumulating at the boundary.

[MS1] Harnack estimates for parabolic-type problems with time nonlocalities

Katarzyna Ryszewska
Warsaw University of Technology
Tuesday, 15:45, 1A305

This is a joint work with Prof. Rico Zacher and Adam Kubica.

The theory of Harnack inequalities is a wide and important topic in the analysis of elliptic and parabolic equations. It provides several properties of weak solutions to respective problems, the most significant of which is the Hölder continuity. Thus, developing this theory for nonlocal problems is very desirable. In the talk I will describe the main ideas regarding the application of de Giorgi-Nash -Moser theory for nonlocal problems. Firstly, to give an intuition, I will present the results concerning the Harnack estimates for parabolic-type problem with time fractional derivative of order $\alpha \in (0,1)$. Then, I will comment on recent results for problems with more general time nonlocality.

[MS1] A continuity equation with space time nonlocal Darcy law and fractional time derivative

Nicola Zamponi

Ulm University

Tuesday, 16:15, 1A305

This is a joint work with Esther Daus (formerly Vienna University of Technology, Vienna, Austria), Maria Gualdani (The University of Texas at Austin, Austin TX, USA), Jingjing Xu (formerly George Washington University, Washington DC, USA), Xinyu Zhang (formerly George Washington University, Washington DC, USA).

We investigate existence of solutions for a continuity equation coupled with a space time nonlocal Darcy law. In both equations the classical time derivative is replaced by the Caputo derivative, which models memory effects in time, while the spatial diffusion in the Darcy law is given by the fractional Laplacian, representing Lévy diffusion. We prove global existence of nonnegative weak (super)solutions that satisfy a variational inequality. The proof uses several approximations steps, including an implicit Euler time discretization. We show that the proposed discrete Caputo derivative satisfies several important properties, including positivity preserving, convexity and rigorous convergence towards the continuous Caputo derivative. Most importantly, we give strong compactness criteria for piecewise constant functions, in the spirit of Aubin–Lions theorem, based on bounds of the discrete Caputo derivative.

[1] E. S. Daus, M. P. Gualdani, J. Xu, N. Zamponi, X. Zhang, *Non-Local Porous Media Equations with Fractional Time Derivative.* Nonlinear Analysis 211 (2021): 112486.

[MS1] A doubling-splitting method for space-fractional PDEs

Paul Andries Zegeling

Utrecht University

Tuesday, 16:45, 1A305

In this presentation a doubling-splitting method will be discussed for space-fractional PDE models.

First, the doubling part of the approach yields a higher-order non-fractional equation. The next (splitting) part re-writes the model as a system of lower order non-fractional PDEs. Via the method-of-lines a system of ordinary differential equations is obtained, which can be integrated in time with an appropriate time-integration method. However, traditional time-integrators, such as explicit (or even implicit) linear multistep methods or Runge-Kutta methods do not possess the correct stability properties for such doubling-splitting systems. This is due to the fact that the eigenvalues of the matrices in the semi-discrete ODEs could be located in all four quadrants of the complex plane $\mathbb{C}$. This opens the possibility for the application of so-called boundary-value methods with non-standard stability regions compared to the traditional integrators. Examples will be given for the half-Laplacian operator $-(-\Delta)^{1/2}$ (related to the Hilbert transform $\mathcal{H}$ and the backward wave equation $u_{tt} = -u_{xx}$) and the left-fractional heat equation with operator $\mathcal{D}_L^{3/2}$, yielding the dispersive wave equation $u_{tt} = u_{xxx}$. The results are based on the article [2]. It is also interesting and worthwhile to mention that we recently found a relatively unknown letter by Liouville in 1835 [1], where he briefly discusses a similar doubling-splitting approach for a specific fractional differential equation.

[1] J. Liouville, Manuscript MS3615, 1835.

[2] P.A. Zegeling and M.W.F. van Spengler, *A generalized midpoint-based BV-method for unstable partial differential equations* J. of Comp. and Appl. Maths. **424** (2023).

[MS1] Hadamard variation of eigenvalues with respect to general domain perturbations

Takashi Suzuki

Osaka University

Wednesday, 8:45, 1A305

This is a joint work with T. Tsuchiya.

We study Hadamard variation of eigenvalues of the Laplacian with respect to general general domain perturbations. We show the existence and continuity of their unilateral derivatives up to the second order and their characterizations using finite dimensional eigenvalue problems. Then a smooth rearrangement for multiple eigenvalues is explicitly given. This result has been known for a long while, but we make the proof simpler, rigorous, and comprehensive. Remarkably, a non-local equation commits in the second derivatives.

[MS1] Concentration phenomenon in a parabolic Lotka-Volterra model for a structured population dynamics

Havva Yoldaş

KTH Sweden

Wednesday, 9:15, 1A305

This is a joint work with Vincent Calvez (University of Western Brittany, Brest, France) and Hé1ène Hivert (University of Rennes, Rennes, France).

We consider a Lotka-Volterra type model describing the evolution of a population structured with a phenotypic trait, that influences the adaptation of individuals to their environment. The model is a nonlocal parabolic PDE given by

$$\begin{cases} \varepsilon\partial_t n_\varepsilon(t,x) - \varepsilon^2\Delta_x n_\varepsilon(t,x) = n_\varepsilon(t,x)R(x, I_\varepsilon(t)), & x \in \mathbb{R}^d,\ t \geq 0, \\ I_\varepsilon(t) = \int_{\mathbb{R}^d} \psi(x) n_\varepsilon(t,x)\,\mathrm{d}x, & t \geq 0, \end{cases} \tag{1}$$

where $n_\varepsilon(t,x)$ is the population density depending on time $t \geq 0$ and the phenotypic trait $x \in \mathbb{R}^d$, R is the net growth rate and the nonlocal term I_ε represents the weigted total population. The population is subjected to small mutations, modeled by a small parameter $\varepsilon > 0$. Equation (1) is complemented with an initial datum $n_\varepsilon(0,x) = n_\varepsilon^0(x) \in L^1(\mathbb{R}^d)$.

The problem involves two time scales: the one in which the population evolves and the one where the small mutations occur. In the long-time, the population concentrates around dominant traits and the dominant traits evolve in time with the effect of mutations. This concentration phenomenon is studied mathematically by means of a Hopf-Cole transform and in the asymptotic limit, a constrained Hamilton-Jacobi equation is obtained in [1]. The uniqueness of the constrained Hamilton-Jacobi equation is proven in [2]. The main challenge in the analysis arises from the nonlinearity of the constraint leading to jumps in the solution.

In this talk, we will revise the existence and uniquness results for model (1) and present a numerical treatment of the model based on [3]. More precisely, we propose an asymptotic-preserving scheme for the problem, show that the approximations converge to the solution of the PDE and the scheme is asymptotically stable.

[1] G. Barles, B. Perthame, Concentrations and constrained Hamilton–Jacobi equations arising in adaptive dynamics, *Contemp. Math.* **439** (2007), 57–68.

[2] V. Calvez, K.-Y. Lam, Uniqueness of the viscosity solution of a constrained Hamilton–Jacobi equation, *Calc. Var. Partial. Differ. Equ.* **59(5)** (2020) 163.

[3] V. Calvez, H. Hivert, H. Yoldaş, Concentration in Lotka–Volterra parabolic equations: An asymptotic-preserving scheme, *Numer. Math.* **154** (2023)103–153.

[MS1] Barenblatt profiles and rarefaction waves in Euler alignment system

Grzegorz Karch

Uniwersytet Wroclawski

Wednesday, 9:45, 1A305

I shall present very recent results, obtained jointly with Szymon Cygan, on an existence and large time behavior of solutions to the following Euler alignment system

$$\begin{aligned} &\rho_t + (\rho u)_x = 0, \\ &u_t + uu_x = \int_{\mathbf{R}} \frac{u(y,t) - u(x,t)}{|x-y|^{1+\alpha}} \rho(y,t)\, dy, \end{aligned} \qquad x \in \mathbf{R}, \quad t > 0, \tag{2}$$

with $\alpha \in (0, 1)$. This model arises as the macroscopic realization of the Cucker and Smale agent model dynamics which describes the collective motion of N individuals in particular alignment and flocking.

We proved that, for a large class of initial conditions including bounded and compactly supported $\rho(x, 0)$ and suitably chosen $u(x, 0)$, the corresponding solutions of the initial value problem are global-in-time and behave for large values of time either as the Barenblatt profiles (*i.e* explicit self-similar solution) of the nonlocal porous medium equation or as a rarefaction wave (*i.e* the explicit self-similar solution of the inviscid Burgers equation.)

[MS1] Fractional Impedance Model for Drug Distribution in Obese Patients Under General Anesthesia: BMI-Dependent Analysis

Dana Copot

University of Gent

Wednesday, 10:15, 1A305

General anesthesia involves carefully balanced multi-drug cocktails to induce hypnosis, analgesia, and neuromuscular blockade, ensuring patients remain unaware during surgery. Population-based pharmacological models typically assume drug dispersal follows a Gaussian distribution within tissues like muscle and fat. Notably, drugs are absorbed and eliminated faster in muscle tissue than in fat tissue due to its higher blood flow density. Comorbidities require significant adjustments in patient pharmacokinetic (PK) and pharmacodynamic (PD) models for computerized closed-loop control of general anesthesia. Prolonged surgical procedures, lasting 6 to 18 hours, challenge traditional compartmental models, requiring consideration of non-normal drug diffusion patterns, potentially linked to tissue structure fractals and fractal Brownian motion models [1]. This work introduces the concept of anomalous diffusion in slow-acting soft tissue metabolite binding, influenced by body fat percentage [2,3]. Experimental data has been collected using the Modulab XM device to measure its complex impedance (Z) over a wide range of frequency in high resolution steps, as depicted in figure below.

The augmented model's simulation aligns well with extended drug release tails, attributed to long-term post-anesthesia effects in major surgery patients. Experimental data suggest anomalous diffusion dynamics in fat tissue, well-captured by Cole-Cole models. Future advancements target sensing technology for localized fat tissue data, enhancing patient-specific adipose tissue models.

[1] Munoz-Gil, G., Volpe, G., Garcia-March, M., and et al, *Objective comparison of methods to decode anomalous diffusion,* Nature Communications, **12**, (2021).

[2] Ionescu, C.M., Copot, D., Yumuk, E., De Keyser, R., Muresan, C., Birs, I.R., Ben Othman, G., Farbakhsh, H., Ynineb, A.R., and Neckebroek, M, *Development, validation, and comparison of a novel nociception/anti-nociception monitor against two commercial monitors in general anesthesia,* Sensors **24(7)** (2024), 20-31.

[3] Copot, D. and Ionescu, C., *Tailored pharmacokinetic model to predict drug trapping in long-term anesthesia,* Journal of Advanced Research **32** (2021), 27-36.

[MS1] Optimal control of a class of semilinear fractional elliptic equations

Mahamadi Warma

George Mason University

Thursday, 8:45, 21A342

This is a joint work with Prof Cyrille Kenne (University of Antilles, Martinique, French West Indies) and Prof Gisèle Mophou (University of Antilles, Guadeloupe, French West Indies)

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In this talk, a class of semilinear fractional elliptic equations associated to the spectral fractional Dirichlet Laplace operator is considered. The control appears nonlinearly in the state equation and the objective function. After showing several existence and regularity results of the state equation, we establish the existence of optimal solutions as well as a minimum principle of Pontryagin type and the first order necessary optimality conditions of the associated optimal control problem. Second order conditions for optimality are also obtained for $L^\infty-$ and L^2- local solutions under some structural assumptions and sharp assumptions on the nonlinear functions involved.

[MS1] Neural Networks and Fractional Differential Equations

Luís L. Ferrás

University of Porto

Thursday, 9:15, 21A342

This is a joint work with Dr. Cecília Coelho and Professor M. Fernanda P. costa (Centre of Mathematics (CMAT) - University of Minho, Braga, Portugal).

Fractional Differential Equations (FDEs) are powerful tools in science and engineering for modeling complex systems. Unlike traditional differentiation and integration, FDEs accommodate non-integer orders, providing a more precise representation of processes with non-local and memory-dependent behaviors. Concurrently, Neural Networks (NNs) and Machine Learning (ML) have seen significant success in solving complex problems.

This study merges Fractional Differential Equations with Neural Networks to model complex dynamical systems. Drawing inspiration from concepts like Neural Ordinary Differential Equations (Neural ODE) [1] and fractional calculus in Neural systems [2], we introduce the Neural FDE [3]. This NN architecture, parameterized by $\boldsymbol{\theta}$ (weights and biases), adjusts the function $\boldsymbol{f_\theta}$ of a FDE of order α (see (3)). The objective is to accurately fit the curve of solutions of this FDE to the provided data by adapting the weights and biases in $\boldsymbol{f_\theta}$ [3],

$$ {}_0^C D_t^\alpha \boldsymbol{h}(t) = \boldsymbol{f_\theta}(\boldsymbol{h}(t), t) \text{ with } \boldsymbol{h}(t_0) = \boldsymbol{h}_0, \ \ \alpha = \alpha_{\boldsymbol{\phi}}. \tag{3} $$

Here, ${}_0^C D_t^\alpha \boldsymbol{h}(t)$ denotes the Caputo fractional derivative [4], $\boldsymbol{h}(t)$ represents the state of the dynamical system and instant t and $\boldsymbol{h}(t_0) = \boldsymbol{h}_0$ is the initial condition. As this model introduces an additional parameter (α) compared to the Neural ODE, the order of the FDE, α, is also learned by another NN, $\alpha_{\boldsymbol{\phi}}$, with parameters $\boldsymbol{\phi}$. We restrict our consideration to $\alpha \in (0, 1)$, as this is applicable in many scenarios and simplifies the formulation.

[1] R.T. Chen, Y. Rubanova, J. Bettencourt, D.K. Duvenaud, *Neural ordinary differential equations.* Adv Neural Inf Process Syst **31** (2018), 1–13.

[2] B.N. Lundstrom, M.H. Higgs, W.J. Spain, A.L. Fairhall, *Fractional differentiation by neocortical pyramidal neurons.* Nat. Neurosci. **11** (2008), 1335–1342.

[3] C. Coelho, M.F.P. Costa, L.L. Ferrás, *Neural Fractional Differential Equations.* https://doi.org/10.48550/arXiv.2403.02737.

[4] K. Diethelm, The Analysis of Fractional Differential Equations: An Application - Oriented Exposition Using Differential Operators of Caputo Type. Springer-Verlag, Berlin Heidelberg, 2010.

[MS1] Non-unique weak solutions for active scalar equations

Mimi Dai

University of Illinois at Chicago

Thursday, 9:45, 21A342

This is a joint work with S. Friedlander (University of Southern California, Los Angeles, USA).

We will discuss recent development in construction of weak solutions for active scalar equations which belong to a class of nonlocal PDEs. In particular, weak solutions are constructed using the convex integration method such that the uniqueness and certain conservation law are violated. The purpose is to verify the sharp regularity threshold that separates the rigidity and flexibility regimes.

[MS1] Continuous and discrete fractional differential equations applied in constitutive models of real materials

Leif Kari

KTH Royal Institute of Technology

Thursday, 10:15, 21A342

Stress–strain relationships and other constitutive models for real materials, such as elastomers, often involve multiple material parameters to accurately match experimental data. However, the utilization of fractional differential equations significantly decreases the number of material parameters required. From a physical perspective, real materials exhibit a wide range of relaxation times in their relaxation processes, making fractional differential equations well-suited for modeling their constitutive behavior. This presentation employs various continuous and discrete fractional differential equations in constitutive models, encompassing stress–strain, as well as physical and chemical aging relations, for real materials like elastomers and hydrogels [1–10].

[1] L. Kari, M. Vizcaíno-Vergara, J.J.C. Busfield, Energy flow in a filled rubber isolator at physical room temperature ageing. Constitutive Models for Rubber XIII. Boca Raton, CRC Press, 2024.

[2] L. Kari, *Dynamic stiffness of chemically and physically ageing rubber vibration isolators in the audible frequency range Part 1: Constitutive equations.* Contin. Mech. Thermodyn. **29** (2017), 1027–1046.

[3] L. Kari, *Dynamic stiffness of chemically and physically ageing rubber vibration isolators in the audible frequency range Part 2—Waveguide solution.* Contin. Mech. Thermodyn. **29** (2017), 1047–1059.

[4] L. Kari, *Effective visco-elastic models of tough, doubly cross-linked, single-network polyvinyl alcohol (PVA) hydrogels. Additively separable fractional-derivative based models for chemical and physical cross-links.* Contin. Mech. Thermodyn. **33** (2021), 2315–2329.

[5] L. Kari, *Are single polymer network hydrogels with chemical and physical cross-links a promising*

dynamic vibration absorber material? Materials **13** (2020), 5127.

[6] L. Kari, *Numerically exploring the potential of abating the energy flow peaks through tough, single network hydrogel vibration isolators with chemical and physical cross-links.* Materials **14** (2021), 886.

[7] B. Wang, L. Kari, *A nonlinear constitutive model by spring, fractional derivative and modified bounding surface model to represent the amplitude, frequency and the magnetic dependency for magneto-sensitive rubber.* J. Sound Vib. **438** (2019), 344–352.

[8] L. Kari, *On the dynamic stiffness of preloaded vibration isolators in the audible frequency range: Modeling and experiments.* J. Acoust. Soc. Am. **113** (2003), 1909–1921.

[9] L. Kari, *On the waveguide modelling of dynamic stiffness of cylindrical vibration isolators. Part I: The model, solution and experimental comparison.* J. Sound Vib. **244** (2001), 211–233.

[10] L. Kari, *On the waveguide modelling of dynamic stiffness of cylindrical vibration isolators. Part II: The dispersion relation solution, convergence analysis and comparison with simple models.* J. Sound Vib. **244** (2001), 235–257.

[MS1] Local well-posedness for a novel nonlocal model for cell-cell adhesion

Anna Zhigun

Queen's University Belfast

Thursday, 15:15, 21A342

This is a joint work with Mabel Lizzy Rajendran (University of Birmingham, Birmingham, UK). A result on local well-posedness for a highly nonlocal nonlinear diffusion-adhesion system will be presented. Macroscopic systems of this kind were previously obtained through upscaling in [1] and can account for the effect of microscopic receptor binding dynamics in cell-cell adhesion. The system couples an integro-PDE featuring degenerate diffusion of the porous media type and nonlocal adhesion with a novel nonlinear integral equation. The approach is based on decoupling the system and using Banach's fixed point theorem to solve each of the two equations individually and subsequently the entire system. The main challenge of the implementation lies in selecting a suitable framework. One of the key results is the local well-posedness for the integral equation with a Radon measure as a parameter. The analysis of this equation utilizes the Kantorovich-Rubinstein norm, marking what appears to be the first application of this norm in handling a nonlinear integral equation.

[1] A. Zhigun and M.L. Rajendran, *Modelling non-local cell-cell adhesion: a multiscale approach*, J. Math. Biol. **88**, 55 (2024)

[2] A. Zhigun and M.L. Rajendran, *Local well-posedness for a novel nonlocal model for cell-cell adhesion via receptor binding*, in preparation.

[MS1] A PRIORI ERROR ESTIMATES OF RUNGE-KUTTA DISCONTINUOUS GALERKIN SCHEMES TO SMOOTH SOLUTIONS OF FRACTIONAL CONSERVATION LAW

Fabio Leotta

TU Darmstadt

Thursday, 15:45, 21A342

This is joint work with Prof. Jan Giesselmann (TU Darmstadt, Germany).

We consider the nonlinear scalar fractional conservation law (FCL)

$$\partial_t u + \partial_x f(u) = -(-\partial_x^2)^{\lambda/2}(u) \qquad \text{in} \quad \mathbb{R} \times (0,T), \tag{4}$$

where $-(-\partial_x^2)^{\lambda/2}$ is the nonlocal fractional Laplace operator with $\lambda \in (0,1)$. The fractional Laplacian can be used to model diffusion processes that are governed by Lèvy processes. Physically motivated applications may range from molecular biology and radiation to hydrodynamics as well as mathematical finance.

It is well known that solutions to FCLs may develop shocks in finite time if the diffusion fails to counterbalance the convection. Accordingly, in the context of numerical methods, low convergence rates have to be expected and have indeed already been proven by e.g. Cifani et al. [1]. Contrary to these worst-case estimates, one may be interested in achievable convergence rates when dealing with sufficiently smooth solutions to obtain a more differentiated picture of the numerical performance. This line of reasoning has also been followed in the setting of pure conservation laws $\partial_t u + \partial_x f(u) = 0$, which share some properties with FCLs.

Following a paper from Zhang and Shu [2], we introduce a novel upwind projection operator whose time dependency can be controlled by the regularity assumptions on the flux f and the exact solution u. This ultimately leaves us with improved bounds in an anticipated Gronwall argument and consequently in the a priori estimate:

Main result. Let $\alpha \geq 3$, $f \in C^{\alpha+1}(\mathbb{R})$ and $u \in C^{\alpha}([0,T]; H^{k+1}(\mathbb{R}))$ be the exact solution to the FCL (4). Let $u_h^n \in \mathbb{P}^k$, $k \geq 1$, be the numerical solution at time level t^n of our second order in time upwind RKDG scheme with CFL condition $\tau \leq ch$ for a certain $c > 0$ in the case $k = 1$ and $\tau \lesssim h^{4/3}$ for $k \geq 2$. Then we have

$$\|u^n - u_h^n\|_{L^2(\mathbb{R})} + \left(\sum_{m=0}^{n-1} \tau |u^m - u_h^m|^2_{H^{\lambda/2}(\mathbb{R})}\right)^{1/2} \lesssim h^{k+1-\frac{1}{\alpha}} + h^{k+1-\frac{\lambda}{2}} + \tau^2, \tag{5}$$

for all $n \in \{1, \ldots, N\}$.

[1] Simone Cifani, Espen R. Jakobsen and Kenneth H. Karlsen: The discontinuous Galerkin method for fractal conservation laws, 2011, IMA Journal of Numerical Analysis

[2] Qhiang Zhang and Chi-Wang Shu: Error estimates to smooth solutions of Runge-Kutta discontinuous Galerkin methods for scalar conservation laws, 2006, ESAIM Mathematical Modelling and Numerical Analysis

[MS1] Analytical treatment of time-fractional gradient flows

Marvin Fritz

Austrian Academy of Sciences

Thursday, 16:15, 21A342

This is a joint work with Prof Endre Süli (Oxford University) and Prof Barbara Wohlmuth (Technical University of Munich).

This talk is on analytical and numerical methods for treating time-fractional gradient flows i.e. equations of the form

$$\partial_t^\alpha u = -\nabla E(u),$$

where ∂_t^α is the fractional derivative (in the sense of Caputo) of order $\alpha \in (0,1)$ and E is the underlying energy. Typical examples are the heat equation and the Cahn–Hilliard equation.

It is still unknown whether the energy of time-fractional gradient flows is dissipating in time. This characteristic is critical for integer-order gradient flows, and many numerical systems utilize it. In [2], we suggest a new energy functional that incorporates the solution's history, which is reasonable given that time-fractional partial differential equations are nonlocal-in-time and feature a natural memory effect. On the basis of this new energy, we demonstrate that a time-fractional gradient flow is equivalent to an integer-order flow (with respect to the new energy). In addition, this connection guarantees the dissipative nature of the new energy and permits the development of appropriate numerical schemes.

In [2], we proved a theorem on the well-posedness of general time-fractional gradient flows with certain restrictive assumptions. We further investigated the time-fractional Cahn–Hilliard equation with degenerating mobility in [3], the time-fractional Fokker–Planck equation with general forcing in [1], and a time-fractional dilute polymer model in [4]. We discuss the analytical challenges such as low regularity of weak solutions. Further, we examine the modeling of such systems and their applications to real-world behavior.

Lastly, we discuss several numerical algorithms to discretize the time-fractional derivative such as (nonuniform) L1 schemes and kernel compressing schemes.

[1] M. Fritz, *Well-posedness and simulation of weak solutions to the time-fractional Fokker-Planck equation with general forcing.* Discrete Contin. Dyn. Syst. Ser. B (2024), DOI: 10.3934/dcdsb.2024036.

[2] M. Fritz, U. Khristenko and B. Wohlmuth, *Equivalence between a time-fractional and an integer-order gradient flow: The memory effect reflected in the energy.* Adv. Nonlinear Anal. **1** (2023), 20220262, DOI: 10.1515/anona-2022-0262.

[3] M. Fritz, M. L. Rajendran and B. Wohlmuth, *Time-fractional Cahn–Hilliard equation: Well-posedness, degeneracy, and numerical solutions.* Comput. Math. Appl. **108** (2023), 66–87, DOI: 10.1016/j.camwa.2022.01.002.

[4] M. Fritz, E. Süli and B. Wohlmuth, *Analysis of a dilute polymer model with a time-fractional derivative.* SIAM J. Math. Anal. **56** (2023), 2063–2089, DOI: 10.1137/23M1590767.

[MS1] Nonlocality-induced instabilities in reaction diffusion systems

Cordula Reisch

TU Braunschweig

Thursday, 16:45, 21A342

Modeling inflammatory processes motivates reaction-diffusion models with nonlocal and heterogeneous reaction terms [1]. The parameters of the nonlocal terms influence the stability of stationary states from stable behavior to instability. Starting from stable reaction diffusion equations without nonlocality, I present results on instabilities induced by a heterogeneous nonlocal term in the reaction functions in systems of the form

$$\begin{aligned} \partial_t u &= \vartheta \Delta u + au + bv - u^3 \\ \partial_t v &= \Delta v + cu + dv + \delta \chi_{\Omega_1}(x) \frac{1}{|\Omega_1|} \int_{\Omega_2} u \, \mathrm{d}x \end{aligned} \tag{6}$$

with $\Omega_1, \Omega_2 \subset \Omega \subset \mathbb{R}$, and χ_{Ω_1} being the characteristic function on Ω_1.

The introduction of nonlocality in the reaction function like in (6) poses new challenges: Linearizing the nonlocal system leads to a strongly coupled infinite-dimensional system of ordinary differential equations for the eigenmodes. We show that the infinite dimensional linear system can be approximated by a truncated system. A combination of analytical and numerical results for the nonlinear, the linearized, and the truncated system shows the existence of instability depending on the nonlocality parameter.

This is a joint work with Bao Quoc Tang (University of Graz, Austria) and Cinzia Soresina (University of Trento, Italy).

[1] C. Reisch, D.Langemann, *Chemotactic effects in reaction-diffusion equations for inflammation* J. Biol. Phys. **45**(3), (2019), 253–273.

[MS2] Improved bounds on turbulent convection with Navier-slip boundary conditions

Camilla Nobili

University of Surrey

Tuesday, 8:45, 1B309

This is a joint work with Fabian Bleitner (University of Hamburg).

In this seminar we are interested in rigorously proving power law scalings for the heat transport, as measured by the nondimensional Nusselt number N u for the Rayleigh-Bénard convection problem. In this specific case the scaling laws have the functional form Nu $\sim$ Ra$^{\alpha}$, where Ra is the thermal driving of the system. We will present the latest rigorous upper bounds on the Nusselt number for flows subject to Navier-slip boundary conditions in flat domains. In particular we will show how to substantially simplify arguments used in the seminal works of Doering and Constantin in the 90's and improve bounds.

[MS2] Transition phenomena in vortex layer flows

Francesco Gargano

University of Palermo

Tuesday, 9:15, 1B309

We present a numerical investigation on the evolution of 2D vortex layers at high Reynolds numbers, governed by the incompressible Navier-Stokes equations. Vortex layer flows are characterized by intense vorticity concentrated around a planar curve, where the vorticity rapidly decays to zero away from this curve. In this study, we consider vortex layers whose initial thickness is proportional to the square root of the inverse of the Reynolds number. Consequently, as Re $\to \infty$, the layer simplifies to a vortex sheet. Our investigation reveals the roll-up process as critical in the initial stages of flow evolution, driven by the growth of palinstrophy and the interplay between the stretching of vorticity gradients and dissipation. We identify two distinct Reynolds regimes: low and moderate- high. For moderate to high Reynolds numbers, the flow evolution exhibits transition phenomena, leading to the development of small-scale structures and the formation of self-similar patterns within the flows. Furthermore, we compare our findings with the vortex-sheet solution derived from the governing Birkhoff-Rott equation for vortex sheet flows. Through analyzing complex singularities, we demonstrate that in the limit of Re $\to \infty$ the Navier-Stokes solution behaves differently from the Birkhoff-Rott solution.

[1] PR.E. Caflisch, F. Gargano, M. Sammartino, and V. Sciacca, Complex singularity analysis for vortex layer flows, J. Fluid. Mech., 932, A21 (2022)

[MS2] Layer separation and energy dissipation for 3D NSE at high Reynolds number

Jincheng Yang
University of Chicago
Tuesday, 9:45, 1B309

This is a joint work with Prof. Alexis F. Vasseur (The University of Texas at Austin, Austin, USA).

In this talk, we consider the 3D incompressible Navier-Stokes equation in a bounded domain, with a canonical example of Poiseuille flow in mind. We provide an unconditional upper bound for the boundary layer separation and energy dissipation of Leray–Hopf weak solutions, uniformly in high Reynolds numbers. We estimate layer separation by measuring the energy norm of the discrepancy between a (turbulent) low-viscosity Leray–Hopf solution and a fixed (laminar) regular Euler solution with similar initial conditions and body forces. This is accomplished by a new nonlinear boundary vorticity estimate, achieved along with several new trace estimates and higher derivative estimates using blow-up methods.

[MS2] Hydrostatic limit for inviscid stratified fluids

Lucas Ertzbischoff
Imperial College London
Tuesday, 10:15, 1B309

This is a joint work with Prof R. Bianchini (IAC-CNR, Italy) and Prof M. Coti Zelati (Imperial College London, United-Kingdom)

I will talk about the hydrostatic approximation of the 2d Euler-Boussinesq system, describing the evolution of an inviscid stratified fluid where the vertical length scale is much smaller than the horizontal one. Even though of importance in oceanography, the justification of the hydrostatic limit in this context has remained an open problem. I will discuss some recent results showing that some instability mechanisms may prevent this limit to hold. I will also try to provide some key features of the associated equations and highlight several related challenges.

[MS2] On energy balance in 2D incompressible ideal fluid flow

Helena Nussenzveig Lopes

Universidade Federal do Rio de Janeiro

Thursday, 8:45, 1A305

Based on joint papers with A. Cheskidov (Westlake University, China), M.C. Lopes Filho (Universidade Federal do Rio de Janeiro, Brazil), R. Shvydkoy (University of Illinois, Chicago, USA), and with S. Lanthaler (CalTech University, USA), M.C. Lopes Filho (Universidade Federal do Rio de Janeiro, Brazil) and F. Jin (ETH-Zurich, Switzerland).
Consider the incompressible 2D Euler equations with an external force. We are interested in special properties of weak solutions obtained by the vanishing viscosity method; these are known as physically realizable weak solutions. Energy conservation at supercritical regularity has been studied for unforced flows. In this talk I will discuss conditions on the regularity of external force terms which lead to necessary and sufficient conditions for energy balance to hold for physically realizable weak solutions of the 2D incompressible Euler equations. This is motivated by turbulence modeling and is meant to be understood as a contrast with the behavior of wild solutions.

[MS2] Onsager's conjecture and dissipation anomaly for long time averages

Alexey Cheskidov

Institute for Theoretical Sciences, Westlake University

Thursday, 9:15, 1A305

In turbulent flows, the energy injected at forced low modes (large scales) cascades to small scales through the inertial range where viscous effects are negligible, and only dissipates above Kolmogorov's dissipation wavenumber. The persistence of the energy flux through the inertial range is what constitutes dissipation anomaly for viscous fluid flows as well as anomalous dissipation for the limiting inviscid flows. We show the existence of dissipation anomaly for long time averages, expected in turbulent flows, proving that the Doering-Foias upper bound is sharp. Also, we analyze whether anomalous dissipation can occur in Onsager critical spaces.

[MS2] A one-dimensional model for suspension flows

Charlotte Perrin

CNRS, I2M

Thursday, 9:45, 1A305

We will present in this talk a mathematical model for a mixture composed by solid particles immersed in a viscous liquid. In a dense regime (high concentration of solid particles), the lubrication effects are predominant in the dynamics. Our goal is to study mathematically a minimal effective model, based on compressible Navier-Stokes equations, which take into account lubrication effects via a singular dissipation term. We will also consider the regime where the viscosity of the interstitial fluid tends to 0.

[1] Chaudhuri, N., Mehmood, M. A., Perrin, C., & Zatorska, E. (2024). Duality solutions to the hard-congestion model for the dissipative Aw-Rascle system. arXiv preprint arXiv:2402.08295.

[2] Chaudhuri, N., Navoret, L., Perrin, C., & Zatorska, E. (2022). Hard congestion limit of the dissipative Aw-Rascle system. arXiv preprint arXiv:2209.12449.

[MS2] A proof of Onsager conjecture for SQG

Mimi Dai

University of Illinois at Chicago

Thursday, 10:15, 1A305

This is a joint work with V. Giri (ETH Zurich, Zurich, Switzerland) and R. Radu (Princeton University, Princeton, NJ, USA).

We construct weak solutions for the surface quasi-geostrophic (SQG) equation which do not conserve the Hamiltonian. The weak solutions are in C^{0-} space. Thus our result gives a proof of the Onsager type of conjecture for the SQG, since it is known that the Hamiltonian is conserved in space with higher regularity.

[MS3] Mean-field limits of non-exchangeable multi-agent systems on hypergraphs

David Poyato

University of Granada

Wednesday, 8:45, 1B309

This is a joint work with N. Ayi and N. Pouradier-Duteil (Sorbonne Université, Paris, France).

Multi-agent systems are ubiquitous in Science, and they can be regarded as large systems of interacting particles with the ability to generate large-scale self-organized structures from simple local interactions rules between each agent and its neighbors. It is well known that the precise architecture of the network plays a crucial role in the resulting emergent behavior of the full group. In most real-life applications, connections are not based on uniform all-to-all couplings, but on highly heterogeneous couplings, sometime dense, but often sparse.

I will start the talk by reviewing some recent literature about the rigorous derivation of macroscopic PDE-based models approximating the microscopic ODE-based multi-agent system as the number of individuals tends to infinity. The strategy to tackle this problem is often based on a careful combination of methods ranging from analysis of PDEs and stochastic analysis, to kinetic equations and graph theory. In many cases, interactions among agents are considered binary and they are modulated by the underlying weighted graph of connections. In this direction, I will briefly discuss a recent work [1] in collaboration with Pierre-Emmanuel Jabin (Penn State University) and Juan Soler (University of Granada) where the underlying graphs can be taken asymmetric, sparse, and the resulting graph limit is encoded by a so-called extended graphon (a measure-valued extension of classical graphons).

Then, I will focus on the main part of the talk, where I will present a recent work [2] for the case of higher-order interactions, which has largely captured the attention of the applied community in the last years. In these multi-agent systems, individuals interact by groups so that a full group jointly generates a non-linear force on any given individual, and the underlying graph of connections is then replaced by a hypergraph. We show that when the interaction kernels are regular enough and the underlying hypergraphs are described by dense simplicial complexes (possibly of unbounded order), then the mean-field limit is determined by a limiting Vlasov-type equation, where the resulting mean-field force admits infinitely-many orders of interactions for the first time in the literature.

[1] P.-E. Jabin, D. Poyato, J. Soler, *Mean-field limit of non-exchangeable systems*, preprint.

[2] N. Ayi, D. Poyato, N. Pouradier-Duteil, *Mean-field limit of multi-agent systems over dense simplicial complexes*, preprint.

[MS3] Exploring Nonlinear Velocity Alignment in the Compressible Euler System: Derivation and Asymptotic Behavior

McKenzie Black

University of South Carolina

Wednesday, 9:15, 1B309

This is a joint work with Dr. Changhui Tan (University of South Carolina, Columbia, South Carolina, USA).

In this talk, our focus is on the pressureless compressible Euler system, where we expand it to include a family of nonlinear velocity alignment. This expansion signifies a nonlinear iteration of the Euler-alignment system within collective dynamics, uncovering asymptotic emergent phenomena like alignment and flocking. Our exploration encompasses various nonlinearity types and nonlocal communication protocols, revealing a diverse spectrum of asymptotic behaviors within the system. A particular emphasis is placed on the derivation, where we delve into a specific class of Vlasov-type kinetic flocking models characterized by nonlinear velocity alignment. Our primary goal is to systematically derive the hydrodynamic limit, leading to the compressible Euler system with nonlinear alignment. Building upon Figalli and Kang [1], which focused on linear velocity alignment using the relative entropy method, our study introduces nonlinearity, resulting in an additional alignment term discrepancy during the limiting process. To effectively address this challenge, we employ the monokinetic ansatz along with the relative entropy approach.

[1] A. Figalli and M.-J. Kang, *A rigorous derivation from the kinetic Cucker– Smale model to the pressureless Euler system with nonlocal alignment.* Anal. PDE. **12.3** (2018), 843–866.

[MS3] Graph limit for interacting particle systems on weighted deterministic and random graphs

Nathalie Ayi

Sorbonne Université

Wednesday, 9:45, 1B309

This is a joint work with Nastassia Pouradier Duteil (INRIA, Sorbonne Université, Paris, France).

In this talk, we start by studying a particular model for opinion dynamics where the influence weights of agents evolve in time via an equation which is coupled with the opinions' evolution. We explore the natural question of the large population limit with two approaches: the now classical mean-field limit and the more recent graph limit. After establishing the existence and uniqueness of solutions to the models that we will consider, we provide a rigorous mathematical justification for taking the graph limit in a general context. Then, establishing the key notion of indistinguishability, which is a necessary framework to consider the classical mean-field limit, we prove the subordination of the mean-field limit to the graph one in that context. We finish with the study of interacting particle systems posed on weighted random graphs. In that aim, we introduce a general framework for the construction of weighted random graphs. We prove that as the number of particles tends to infinity, the finite-dimensional particle system converges in probability to the solution of a deterministic graph-limit equation in which the graphon prescribing the interaction is given by the first moment of the weighted random graph law.

[MS3] Coupled Stuart-Landau Oscillators: Amplitude Death and Synchronization

David Nicholas Reynolds

University of Granada

Wednesday, 10:15, 1B309

This is a joint work with Ana P Millán (UGR), David Poyato (UGR), and Francesco Tudisco (University of Edinburgh). The Stuart-Landau equation is one of the simplest descriptions of a limit cycle oscillator,

$$\dot{z} = (\sigma - |z|^2)z \tag{7}$$

where $\sigma = \alpha + i\omega \in \mathbb{C}$. Kuramoto's famous model of phase synchronization was derived from coupling oscillators of this form and applying certain reductions to remove the dependency on the amplitudes of the oscillators. Despite the vast literature that has developed around the Kuramoto model, relatively little mathematical work has been done to understand coupled Stuart-Landau oscillators which still retain dependency on their amplitudes. In this talk we will discuss amplitude death and the synchronization of diffusively coupled Stuart-Landau oscillators with full heterogeneity of parameters,

$$\dot{z}_j = (\sigma_j - |z_j|^2)z_j + \frac{\gamma}{N}\sum_{l=1}^{N}(z_l - z_j), \tag{8}$$

where each $\sigma_j = \alpha_j + i\omega_j$ can be any complex number. We investigate the discrete model with N coupled oscillators. As in the Kuramoto model, the relationship between the coupling strength γ and the natural frequencies ω_j play a role in whether or not synchronization occurs. However, with the amplitude dependence, unlike Kuramoto, the values α_j relative to the coupling strength and natural frequencies also play an integral role in the potential asymptotic outcomes. We begin the discussion with the special case of all natural frequencies $\omega_j \equiv 0$ for which the model can be seen as a model of opinion dynamics that has been studied in [1–3]. We will then investigate the various asymptotic states of the model with the full range of parameters, highlighting particular regimes of interest.

[1] Lear D, Reynolds DN, Shvydkoy R, Grassmannian reduction of cucker-smale systems and dynamical opinion games. Discrete Contin. Dyn. Syst. 41 (12) : 5765-5787, 2021.

[2] Reynolds DN, Tudisco F, Unique Nash equilibrium of a nonlinear model of opinion dynamics on networks with friction-inspired stubbornness, arXiv:2304.07556 2023.

[3] Nguyen V, Shvydkoy R. Continuous model of opinion dynamics with convictions. Discrete Contin. Dyn. Syst., 2023, 43(11): 4010-4026.

[MS3] Application of the nonlocal Bresh-Desjardins estimate for a hydrodynamic model of interacting agents

Ewelina Zatorska

University of Warwick

Friday, 8:45, 1A305

This is a joint work with Nilasis Chaudhuri, Young-Pil Choi, and Oliver Tse.

In this talk I will explore a two-velocity formulation of the one-dimensional hydrodynamic model of interacting agents. The studied system encompasses compressible Navier-Stokes equations with nonlocal attraction-repulsion and alignment terms. I will show that under certain conditions between the nonlocal terms, one can obtain additional global-in-time energy-type estimate, the so-called Bresch-Desjardins estimate, that can further be used to study the long-time asymptotics of solutions.

[MS3] Computational modeling of cancer as a collective of cells: investigating morphology and malignancy

Erik Blom

Uppsala University

Friday, 9:15, 1A305

Biological systems are remarkably interesting and naturally ubiquitous, but a proper understanding of the phenomena that arise from a collective of cooperating biological units can be difficult to attain. Computational modeling provides us with the tools for interpreting current biological understanding about these systems into precise and testable mathematical frameworks, with the potential to refine how we think about the underlying mechanisms. Without 'Maxwell's' or 'Navier Stokes' equations of biology, the computational modeler must consider important trade-offs between different models in addition to the various simulation techniques used to test them.

In this talk, we will investigate how these ideas apply to cancer biology, from the modeling of tumor growth curves to tumor cell population modeling. Specifically, we shall investigate the collective phenomena of a stochastic, cell-based model of avascular tumor growth and how it compares to its spatially continuous counterpart described by partial differential equations [1]. We find insightful morphological stability conditions and experience the utility of leaning against two similar models of different character in the analysis. We will also see how the model can be used formulate the hallmarks of cancer to study the emergence of malignancy from a population of cells in homeostasis [2]; and we discuss how these endeavours connect back to the aforementioned potential of computational modeling.

[1] E. Blom, S. Engblom, *Morphological stability for in silico models of avascular tumors*, arXiv preprint (2023): https://doi.org/10.48550/arXiv.2309.07889

[2] E. Blom, S. Engblom, G. Menz, *Modeling the hallmarks of cancer*, ENUMATH23 proceedings (submitted, 2024)

[MS3] Conditional regularity for a chemotaxis-Navier-Stokes system

Mimi Dai

University of Illinois at Chicago

Friday, 9:45, 1A305

This is a joint work with H. Liu (University of Memphis, Memphis, USA).

We will discuss the regularity problem of a three dimensional chemotaxis-Navier-Stokes (CNS) system on a periodic domain. The CNS system describes a scenario in which both the bacteria and oxygen are transported by the fluid and diffuse randomly. We show that the model permits global regular solution under certain condition. Such condition is in terms of only low modes of the oxygen concentration and the fluid velocity, and weaker than many existing regularity conditions in the literature.

[MS3] Mean-field limit for 2D stationary systems with Coulombian interactions

Jan Peszek

University of Warsaw

Friday, 10:15, 1A305

This is a joint work with Rémy Rodiac from the University of Warsaw. I will present a recent study of the mean-field limit of solutions to the system

$$\frac{1}{M_N}\sum_{j\neq i} d_j[\nabla g(x_i - x_j) + \nabla F(x_i - x_j)] + \nabla V(x_i) = 0, \quad x_i \neq x_j, \ \forall i \neq j \in \{1, \ldots, N\}.$$

Here d_j are real charge coefficients, $M_N := \sum_{i=1}^{N} |d_i|$, and the functions $g + F$ and V represent interaction and potential energy, respectively. Key features above are the Coulombian singularity of g and the admissibility of negative charges d_i. Such a setting includes the stationary attraction-repulsion problem with Coulombian singularity and the stationary system of point-vortices.

In 2D, we prove that the associated empirical measures converge to a limiting measure μ satisfying a two-fold criticality condition: in velocity form or in vorticity form. The first approach follows ideas of Sandier-Serfaty in the context of Ginzburg-Landau vortices [1]. The second criticality condition, in the vorticity form, is obtained by arguments closer to Schochet [2].

[1] E. Sandier and S. Serfaty. *Limiting vorticities for the Ginzburg-Landau equations.* Duke Math. J., **117**(3), (2003), 403–446.

[2] S. Schochet, *The weak vorticity formulation of the 2-D Euler equations and concentration-cancellation.* Comm. Partial Differential Equations, **20**(5-6), (1995), 1077–1104.

[MS4] On the high Mach number limit for capillary fluids

Donatella Donatelli

University of L'Aquila

Tuesday, 8:45, 1D222

This is a joint work with Matteo Caggio (Institute of Mathematics of the Academy of Sciences of the Czech Republic, Prague, Czech Republic).

It is well known that in fluid dynamics a variety of models is given as a result of a scaling analysis. The Mach number, physically, is given by the ratio of the reference fluid velocity and the sound speed. In general the value of the Mach number depends on the conditions of the physical phenomenon under consideration and it is related to the compressibility of the fluid according to its low or high values. In the case of a low Mach number regime the speed of sound tends to infinity while the pressure becomes almost constant and doesn't generates density variations, as a consequence compressibility can be ignored and the final asymptotical physical state is the incompressible one. Conversely if the fluid speed increases beyond the sound speed then the Mach number is high and the compressibility effects have to be taken into consideration and the asymptotical physical state is that of a pressureless fluid. In this talk we will perform rigorously the high Mach number limit for capillary fluids and discuss the main properties of the related pressureless capillary fluid model.

[MS4] Cahn–Hillard and Keller–Segel systems as high-friction limits of Euler–Korteweg and Euler–Poisson equations

Jakub Woźnicki

University of Warsaw

Tuesday, 9:15, 1D222

This is a joint work with Dennis Gallenmüller, Piotr Gwiazda (Polish Academy of Sciences, Warsaw, Poland), Agnieszka Świerczewska-Gwiazda (University of Warsaw, Warsaw, Poland). We consider a combined system of Euler–Korteweg and Euler–Poisson equations with friction and exponential pressure with exponent $\gamma > 1$. We show the existence of dissipative measure-valued solutions in the cases of repulsive and attractive potential in the Euler–Poisson system. The latter case requires additional restriction on γ. Furthermore in case of $\gamma \geq 2$, we show that the strong solutions to the Cahn–Hillard–Keller–Segel system are a high-friction limit of the dissipative measure-valued solutions to Euler–Korteweg–Poisson equations. We will discuss the ideas of the solutions for both systems, and how one can compare both of them in spite of a different structure. We will also look at different technical issues appearing in the proof, which reduce the possibilities of the regularity of the pressure, and compare those with similar results in the field.

[MS4] Viscous perturbations to discontinuous solutions of the

Alexis Vasseur
University of Texas at Austin
Tuesday, 9:45, 1D222

This is a joint work with Prof Geng Chen (University of Kansas) and Prof. Moon-Jin Kang (KAIST, South Korea).

The compressible Euler equation can lead to the emergence of shocks-discontinuities in finite time, notably observed behind supersonic planes. A very natural way to justify these singularities involves studying solutions as inviscid limits of Navier-Stokes solutions with evanescent viscosities. The mathematical study of this problem is however very difficult because of the destabilization effect of the viscosities.

Bianchini and Bressan proved the inviscid limit to small BV solutions using the so-called artificial viscosities in 2005. However, until very recently, achieving this limit with physical viscosities remained an open question.

In this presentation, we will present the basic ideas of classical mathematical theories to compressible fluid mechanics and introduce the recent a-contraction with shifts method. This method is employed to describe the physical inviscid limit in the context of the barotropic Euler equation.

[MS4] Heat-conducting fluids on time-dependent domains

Aneta Wróblewska-Kamińska
Institute of Mathematics, Polish Academy of Sciences
Tuesday, 10:15, 1D222

We consider a general compressible viscous, heat conducting fluid flow described by the Naiver-Stokes-Fourier system on domain which shape may change in time.

We concentrate on the existence of a variational solution to the system supplemented with slip boundary conditions for velocity and conservative boundary condition for the heat flux. The no-slip boundary condition for the velocity will be also discussed. The proof is based on a proper

choice of the penalization. I will mentioned also the result in low Mach number regime.
This result is based on the joint work with O. Kreml, V. Machá, Š. Nečasová and is a part of the very recent book *Mathematical theory of compressible fluids on moving domains* (O. Kreml, V. Mácha, S. Nečasová, T. Piasecki).

[MS4] On a compressible fluid-structure interaction problem with slip boundary conditions

Sarka Necasova

Academy of Sciences

Thursday, 8:45, 1B309

This is a joint work with Yadong Liu (Nanjing Normal University, Nanjing, China) and Sourav Mitra (Indian Institute of Technology Indore,Indore, India).
We study a system describing the compressible barotropic fluids interacting with (visco) elastic solid shell/plate. In particular, the elastic structure is part of the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, via a modified relative entropy method in time-dependent domains, we prove the weak-strong uniqueness property of weak solutions. Finally, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data.

[1] Y. Liu, S. Mitra, Š. Nečasová, *On a compressible fluid-structure interaction problem with slip boundary conditions,* Preprint 2024.

[MS4] Global Solutions of the one-dimensional Compressible Euler Equations with Non-local Interactions

Ewelina Zatorska

University of Warwick

Thursday, 9:15, 1B309

This is a joint work with Jose A. Carrillo, Gui-Qiang G. Chen, and Difan Yuan

I will talk about the global existence of finite-energy solutions of the one-dimensional compressible Euler equations with damping and non-local interactions: Newtonian repulsion and quadratic confinement. The result holds for both: the polytropic gases and a general pressure law, but I will present only the first simple case. The existence result is achieved by constructing a sequence of solutions of the one-dimensional compressible Navier-Stokes type equations with density-dependent viscosity under the stress-free boundary condition and then taking vanishing viscosity limit. I will focus the talk on the main difficulties which arise from the appearance of the nonlocal terms. In particular, I will derive uniform higher moments estimates and explain the construction of approximate solutions.

[MS4] From nonlocal Euler-Korteweg to local Cahn-Hilliard via the high-friction limit

Piotr Gwiazda

University of Warsaw

Thursday, 9:45, 1B309

Several recent papers considered the high-friction limit for systems arising in fluid mechanics. Following this approach, we rigorously derive the nonlocal Cahn-Hilliard equation as a limit of the nonlocal Euler-Korteweg equation using the relative entropy method. Applying the recent result by the first and third author, we also derive rigorously the local degenerate Cahn-Hilliard equation. The proof is formulated for dissipative measure-valued solutions of the nonlocal Euler-Korteweg equation which are known to exist on arbitrary intervals of time. Our work provides a new method to derive equations not enjoying classical solutions via relative entropy method by introducing the nonlocal effect in the fluid equation.

[MS4] Homogenization of compressible Navier–Stokes–Fourier system in domains with tiny holes

Milan Pokorny

Charles University, Prague

Thursday, 10:15, 1B309

This is a joint work with Yong Lu (University of Nanjing, Nanjing, China), Emil Skříšovský (Charles University, Prague, Czech Republic) and Florian Oschmann (Czech Accademy of Sciences, Prgue, Czech Reublic).

We consider the compressible Navier–Stokes–Fourier system in a domain with large number of holes. Under the assumption that the holes are sufficiently small, together with certain standard assumptions on the adiabatic exponent and the behaviour of the heat conductivity, we show that if passing simultaneously with the number of holes to infinity and their size to zero, in the limit we obtain again a solution to the compressible Navier–Stokes–Fourier system in the domain without holes. The talk includes both the steady and evolutionary situations. The talk is based on the results from the below listed papers.

[1] F. Oschmann, M. Pokorný, *Homogenization of the unsteady compressible Navier–Stokes equations for adiabatic exponent* $\gamma > 3$. J. Differential Equations **377** (2023), 271–296.

[2] M. Pokorný, E. Skříšovský, *Homogenization of the evolutionary compressible Navier–Stokes–Fourier system in domains with tiny holes.* J. Elliptic Parabol. Equ. **7** (2021), 361–391.

[3] Y. Lu, M. Pokorný: *Homogenization of stationary Navier–Stokes–Fourier system in domains with tiny holes.* J. Differential Equations **278** (2021), 463–492.

[MS5] Computation of elliptic invariant tori via parametrization method

Chiara Caracciolo

University of Padua

Wednesday, 15:15, 1D222

This is a joint work with J.-L. Figueras (Uppsala University, Uppsala, Sweden) and A. Haro (University of Barcelona, Barcelona, Spain).

In KAM theory, constructive techniques are essential to produce realistic results in physical problems, as there is tipically a big gap between purely analytical results and numerical ones. The parametrization method, also in view of its natural adaptation to Computer-Assisted procedures, is among the techniques that have been proven successful in bridging this gap.

We present a parametrization algorithm for computing lower dimensional elliptic tori in Hamiltonian systems. In this extension of KAM theorem, the focus is on tori that are characterized by quasi-periodic motions with fewer frequencies than the degrees of freedom of the system. Moreover, the linear dynamics in the transverse dimensions is elliptic. As a main difference with respect to previous works, we fix both the frequency vectors involved in the dynamics: the one describing the quasi-periodic motion on the torus and the one related to the harmonic oscillations in the transverse dimensions. This is done by considering a family of Hamiltonian functions, that in particular are not required to be nearly-integrable nor expressed in action-angle variables.

As an example, we apply the algorithm to compute elliptic tori in a model of four coupled pendula.

[MS5] Construction of the monodromy action via rigorous forward integration of differential equations

Akitoshi Takayasu

University of Tsukuba

Wednesday, 15:45, 1D222

This talk presents a constructive approach to finding the monodromy action of n-dimensional linear differential equations (DEs) in a complex domain D. The monodromy action characterizes the multivaluedness of the solution of DEs, which reflects the topological property of the domain. Our approach is based on rigorous forward integration of DEs from the fundamental solution at a base point b in D, which is equivalent to analytic continuation of solutions to the DEs into the complex domain. In general, if the region of analytic connection is simply connected, the monodromy action is an identity. That is, this is a trivial identity element of the fundamental group $\pi_1(D, b)$. On the other hand, if there are singular points inside the domain, rigorous integration along a loop γ enclosing the singular points yields a nonsingular matrix called the monodromy matrix M_γ. Constructing the monodromy matrix based on interval arithmetic, we provide a nontrivial monodromy action that represents a group homomorphism

$$\rho : \pi_1(D, b) \to \mathrm{GL}_n(\mathbb{C}), \quad [\gamma] \mapsto M_\gamma,$$

where $[\gamma]$ denotes the homotopy class of the loop γ. This is called monodromy representation of the fundamental group and the image of ρ is called the monodromy group.

In the talk, we start with an example of a toy problem for the hypergeometric differential equation, and then present the results of computer-assisted proofs for constructing the monodromy action for the Picard-Fuchs differential equation. We also give an example of a computer-assisted proofs for nonintegrability of dynamical systems via the rigorous construction of the monodromy action.

[MS5] Existence of non convex V-states

Gerard Castro López

Universitat Politècnica de Catalunya and Brown University

Thursday, 8:45, 1B364

This is a joint work with Javier Gómez-Serrano (Brown University, Providence, US).

V-states are uniformly rotating vortex patches of the 2D Euler equation. The only known explicit examples are circles and ellipses, the rest of positive existence results use local or global bifurcation arguments and don't give any quantitative information of the solutions. I will talk about the existence of solutions far from the perturbative regime, being able to extract nontrivial features of them and a precise quantitative description. We will use a combination of analysis and computer assisted proofs techniques.

[MS5] Proof of a conjecture of Marcha

Jason Mireles James

Florida Atlantic University

Thursday, 9:15, 1B364

In a series of papers in the early 2000s, Chenciner, Montgomery, Fejoz, and Marchal proved that the gravitational three body with equal masses admits a one parameter family of periodic orbits bifurcating from the equilateral triangle solution of Lagrange which have all the symmetries of the figure eight choreography. Marchal conjectured that this family actually contains the eight. I will discuss a recently completed computer assisted proof which settles this conjecture in the affirmative. The idea is to recast the problem as a delay differential equation describing the motion of only one of the bodies.

This is joint work with Carlos Garcia-Azpeitia, Renato Calleja, Olivier Henot, and Jean-Philippe Lessard.

[MS5] Exploring Differential Equations with Non-Polynomial Nonlinearities: Leveraging the FFT for Computer-Assisted Proofs

Jean-Philippe Lessard

McGill University

Thursday, 9:45, 1B364

This presentation introduces a methodology for generating computer-assisted proofs (CAPs) aimed at establishing the existence of solutions for nonlinear differential equations featuring non-polynomial analytic nonlinearities. Our approach combines the Fast Fourier Transform (FFT) algorithm with interval arithmetic and a Newton-Kantorovich argument to effectively construct CAPs. A key highlight is the rigorous management of Fourier coefficients of the nonlinear term Fourier series, achieved through insights from complex analysis and the Discrete Poisson Summation Formula. We demonstrate the effectiveness of our method through two illustrative examples: firstly, proving the existence of periodic orbits in the Mackey-Glass (delay) equation, and secondly, establishing the existence of periodic localized traveling waves in the two-dimensional suspension bridge equation.

[MS5] Self-similar singular solutions for the complex Ginzburg-Landau equation

Joel Dahne

Uppsala University

Thursday, 10:15, 1B364

This is a joint work with Jordi-Lluís Figueras (Uppsala University).

In 2001, Plecháč and Šverák [1] gave strong numerical evidence for the existence branches of backwards self-similar singular solutions to the complex Ginzburg-Landau equation. We now present a rigorous proof of these branches existence. Our proof follows the same strategy as Plecháč and Šverák, which reduces the problem to proving the existence of a solution to a certain ODE with prescribed behaviour at zero and infinity. Near zero the solution is constructed using a rigorous numerical ODE solver and near infinity by carefully analysing the asymptotic expansion. These two solutions are then glued together to form the full solution.

[1] P. Plecháč and V. Šverák. *On self-similar singular solutions of the complex Ginzburg-Landau equation.* Comm. Pure Appl. Math. **54.10** (Apr. 2001), 1215–1242.

[MS5] Spectral Stability via the Maslov Index and Computer Assisted Proofs

Jonathan Jaquette

New Jersey Institute of Technology

Thursday, 15:15, 1B364

In this talk we present current work on developing a computer assisted proof for the stability of pulse solutions to the Swift-Hohenberg equation, generalizing the framework based on counting conjugate points / the Maslov index developed in Beck & Jaquette (2022). We have made significant theoretical advancements to this end, needed to account for essential degeneracies, however more work remains on the CAP front.

The general set up is to first compute a homoclinic orbit of a 4D Hamiltonian ODE. Next one evolves unstable vector bundles of the linearized system from one end of the homoclinic to the other. These vector bundles define a heteroclinic orbit in the space of Lagrangian Grassmanians (limiting from a point to a periodic orbit), and the number of conjugate points counts the number of times the heteroclinic orbit passes through a fixed reference frame. To get a validated count of the conjugate points, one must show that they only occur in a compact region, which requires a careful analysis of the limiting behavior of this path of Lagrangian subspaces, and the taming of external resonances in an application of the parameterization method.

This talk is based on joint work with Hannah Pieper and Margaret Beck.

[MS5] A tight KAM theorem for Lagrangian tori in the Sun-Jupiter-Saturn system

Jordi-Lluís Figueras

Uppsala Universitet

Thursday, 15:45, 1B364

In this talk we will explain the abstract KAM theorem we developed that suits for proving quasiperiodic solutions for the Sun-Jupiter-Saturn system. This talk is based on the papers [1] and [2]. ¡This work has been developed with Alex Haro.

[1] Jordi-Lluís Figueras, Alex Haro. *A modified parameterization method for invariant Lagrangian tori for partially integrable Hamiltonian systems.* Physica D 462 (2024), Paper No. 134127.

[2] Jordi-Lluís Figueras, Alex Haro. *Sun-Jupiter-Saturn System may exist: A verified computation of quasiperiodic solutions for the planar three body problem.* (Submitted) `arXiv:2403.10152`.

[MS5] Computer-Assisted Proofs of existence of invariant tori in quasi-periodic systems via Fourier Methods

Eric Sandin Vidal

Vrije Universiteit Amsterdam

Thursday, 16:15, 1B364

The goal of this talk is to provide a methodology to prove existence of (fiberwise hyperbolic) real-analytic invariant tori in real-analytic quasi-periodic skew-product dynamical systems, close to nearly-invariant tori (obtained, for instance, through numerical computations). The methodology is based on the application of a Newton-Kantorovich theorem [1] (similar to the radii polynomial approach seen in [2]) whose hypotheses are tested using the Fourier analysis methods in [3], for a numerical approximation of the parameterization of an invariant torus.

This is a joint work with Alex Haro (Universitat de Barcelona and Centre de Recerca Matemàtica).

[1] Jordi-Lluís Figueras and Alex Haro. *Reliable computation of robust response tori on the verge of breakdown.* SIAM Journal on Applied Dynamical Systems, 11(2):597–628, 2012.

[2] Jan Bouwe van den Berg, Maxime Breden, Jean-Philippe Lessard, and Maxime Murray. *Continuation of homoclinic orbits in the suspension bridge equation: a computer-assisted proof.* Journal of Differential Equations, 264(5):3086–3130, 2018.

[3] Jordi-Lluís Figueras, Alex Haro and Alejandro Luque. *Rigorous computer-assisted application of KAM theory: a modern approach.* Foundations of computational mathematics 17, 1123-1193.

[MS5] Arnold diffusion in the elliptic Hill four-body problem: geometric method and numerical verification

Marian Gidea

Yeshiva University

Thursday, 16:45, 1B364

This is a joint work with Jaime Burgos–García (Universidad Autónoma de Coahuila, Saltillo, México) and Claudio Sierpe (Universidad del Bío-Bío, Concepción, Chile).

We consider the planar elliptic restricted four-body problem (ER4BP), describing the dynamics of a massless body under the gravitational influence of three massive bodies (primaries) of masses $m_1 > m_2 > m_3$ forming an equilateral central configuration, where each primary moves on an elliptic orbit about the common center of mass. We derive the elliptic Hill four-body problem (EH4BP), which is an approximation of the ER4BP describing the dynamics of the infinitesimal body in a neighborhood of the smaller body, in the limit case when $m_3 \to 0$ and m_1, m_2 are sent to infinity. The EH4BP can be written as a perturbation of the circular Hill four-body problem (CH4BP), with the eccentricity ε of the elliptic orbits being the small parameter. While for some orbits the effect of the perturbation may average out, for other orbits it may accumulate in the long run. We show that the EH4BP exhibits Arnold diffusion, in the sense that there exist orbits of the infinitesimal body that undergo significant changes over time. In particular, the energy along these orbits increases by $O(1)$ with respect to the perturbation parameter ε.

Our mechanism of diffusion relies on the existence of two normally hyperbolic invariant manifolds (NHIM's), and on the corresponding homoclinic and heteroclinic connections. The dynamics along homoclinic/heteroclinic orbits is encoded via scattering maps. Having several scattering maps, we select the scattering map that gives the largest gain in energy on the domains where it grows, and we select the scattering map that gives the smallest loss in energy on the domains where it decays. Using Birkhoff's Ergodic Theorem we show that there are pseudo-orbits generated by the selected scattering maps along which, on average, the energy grows by an amount independent of the small parameter. A shadowing lemma yields the existence of diffusing orbits.

A motivation for this work is the system consisting of Sun, Jupiter, the Trojan asteroid (624) Hektor, and its moonlet Skamandrios, which can be modeled by the EH4BP. In this context, the effect of the Arnold diffusion is that it can push the moonlet Skamandrios towards collision with Hektor, or, on the contrary, to escape from Hektor's capture.

In this work, the normally hyperbolic invariant manifolds, the homoclinic and heteroclinic connections, and the unperturbed and perturbed scattering maps are computed numerically [1]. A rigorous computer assisted proof can be done using, e.g., the CAPD library.

[1] J. Burgos Garcia, M. Gidea, C. Sierpe. Arnold diffusion in the elliptic Hill four-body problem: geometric method and numerical verification *Preprint*, 2024.

[MS5] KAM tori in a particular class of extrasolar planetary models

Ugo Locatelli

Università di Roma Tor Vergata

Friday, 8:45, 1A306

This is a joint work with PhD. C. Caracciolo (University of Uppsala – Univ. of Padua).

Since the birth of KAM theory, planetary three-body problems were indicated as a natural benchmark to study its applicability to realistic systems of physical interest. It is well known that KAM statements proved in a purely analytical way completely fail such a challenging purpose. This the reason way, since the last two decades of the previous century Computer-Assisted Proofs (hereafter CAPs) are commonly used in this context.

We revisit this general problem in the case of a few planar models of extrasolar systems hosting one star and two exoplanets; we consider them with values of the parameters that are in agreement with the observations. The existence of invariant KAM tori in correspondence with orbital motions is investigated by using a publicly available software code, that is specially designed to perform CAPs. Such an approach can be successful if and only if the problem is described by a Hamiltonian (in action-angle coordinates) that is close enough to a Kolmogorov normal form.

We describe the bare minimum of preliminar canonical transformations, which are needed to bring the Hamiltonian in a suitable form to start a CAP. This strategy is implemented in such a way to rigorously prove the existence of KAM tori for three exoplanetary systems (namely HD11964, HD142 and HD4732). A rather simple argument allows us to give a characterization of the planetary systems for which our proof scheme has good chance of success.

[MS5] Lower bounds on the Hausdorff dimensions of Julia sets

Warwick Tucker

Monash University

Friday, 9:15, 1A306

This is a joint work with Artem Dudko (Institute of Mathematics of Polish Academy of Sciences, Warsaw, Poland) and Igors Gorbovickis (Constructor University, Bremen, Germany).

We present an algorithm for a rigorous computation of lower bounds on the Hausdorff dimensions of Julia sets for a wide class of holomorphic maps [1]. We apply this algorithm to obtain lower bounds on the Hausdorff dimension of the Julia sets of some infinitely renormalizable real quadratic polynomials, including the Feigenbaum polynomial $p_{\mathrm{Feig}}(z) = z^2 + c_{\mathrm{Feig}}$. In addition to that, we construct a piecewise constant function on $[-2, 2]$ that provides rigorous lower bounds for the Hausdorff dimension of the Julia sets of all quadratic polynomials $p_c(z) = z^2 + c$ with $c \in [-2, 2]$. Finally, we verify the conjecture of Ludwik Jaksztas and Michel Zinsmeister that the Hausdorff dimension of the Julia set of a quadratic polynomial $p_c(z) = z^2 + c$, is a C^1-smooth function of the real parameter c on the interval $c \in (c_{\mathrm{Feig}}, -3/4)$.

[1] Dudko A., Gorbovickis I., and Tucker W., *Lower bounds on the Hausdorff dimension of some Julia sets* Nonlinearity. **36:5** (2023), 2865.

[MS5] Existence of partly hyperbolic invariant tori in Hamiltonian systems: an *a-posteriori approach*

Álvaro Fernandez-Mora

Universitat de Barcelona

Friday, 9:45, 1A306

In this talk, we consider the existence of partially hyperbolic invariant tori with Diophantine frequencies and their invariant bundles in periodic and quasi-periodic analytic Hamiltonian systems. We present an *a-posteriori* KAM theorem stating that if we have embeddings for the torus and its stable bundle satisfying that the error in their functional invariance equations is small enough, then there is an invariant torus with stable and unstable invariant bundles nearby. The method of proof is based on the parameterization method and consists on constructing an iterative procedure that defines sequences of embeddings that converge to solutions of their invariance equations in a complex strip of the torus. In particular, we obtain both the stable and unstable bundles simultaneously and the application of the theorem to autonomous Hamiltonian systems is straightforward. The results rely on the geometrical properties of the system and of invariant tori and do neither assume action-angle coordinates nor a perturbative setting. Additionally, the theorem is constructed to be suitable for numerical implementations and computer assisted proofs. Non-rigorous implementations have been applied to compute 3-dimensional invariant tori in the Elliptic Restricted Three Body Problem in [1].

[1] F-M, Á., Haro, A., and Mondelo, J.M.: Flow Map Parameterization Methods for Invariant Tori in Quasi-Periodic Hamiltonian Systems *In SIAM Journal on Applied Dynamical Systems* (2024).

[MS5] Computer-assisted proofs for some self-similar profiles on $\mathbb{R}^d$

Maxime Breden

Ecole polytechnique

Friday, 10:15, 1A306

This is a joint work with Hugo Chu (Imperial College London, UK).

We develop computer-assisted tools to study semilinear equations of the form

$$-\Delta u - \frac{x}{2} \cdot \nabla u = f(x,u), \quad x \in \mathbb{R}^d, \quad d \in \{1,2,3\}. \tag{9}$$

Equations of this form appear naturally in several contexts, and in particular when looking for self-similar solutions of the nonlinear heat equation or of the nonlinear Schrödinger equation.
We introduce a general computer-assisted methodology for equations of the form (9), allowing not only to prove the existence of solutions, but also to describe them very precisely. Our argument revolves around a fixed-point/Newton-Kantorovich theorem which is standard for computer-assisted proofs, but many of the required ingredients then differ. We use a spectral approach based on an eigenbasis of $\mathcal{L} := -\Delta - \frac{x}{2} \cdot \nabla$, together with appropriate weighted Sobolev spaces, and a quadrature rule allowing to deal with nonlinearities.
We obtain radially symmetric self-similar profiles for the nonliear heat equation and the nonlinear Schrödinger equation, which had already been extensively studied, as well as radially asymmetric profiles about which less is known.

[MS6] On solution manifolds of some differential equations with general state-dependent delay

Hans-Otto Walther

Justus-Liebig-Universitaet Giessen

Monday, 15:15, 1B364

Differential equations with state-dependent delays define a semiflow of continuously differentiable solution operators in general only on the associated *solution manifold* in the Banach space $C_n^1 = C^1([-h,0],\mathbb{R}^n)$. For a prototypic example we develop a new proof [3] that its solution manifold is diffeomorphic to an open subset of the subspace given by $\phi'(0) = 0$, without recourse to a restrictive hypothesis about the form of delays which is instrumental in earlier work [1,2] on the nature of solution manifolds. The new proof uses the framework of algebraic-delay systems.

[1] T. Krisztin and H.O. Walther, *Solution manifolds of differential systems with discrete state-dependent delays are almost graphs.* Discrete Contin. Dyn. Syst. **43** (2023), 2973–2984.

[2] H.O. Walther, *On solution manifolds for algebraic-delay systems.* Ukrainian Mathematical Journal **75**

(2023), 1591–1607.

[3] H.O. Walther, *On solution manifolds of some differential equations with more general state-dependent delay.* https://arxiv.org/abs/2402.07636, 16 pp, 2024.

[MS6] On regularity of mild solutions for linear delay differential equations

Junya Nishiguchi

Tohoku University

Monday, 15:45, 1B364

The notion of mild solutions for linear delay differential equations (DDEs) has been introduced in [1] for the purpose of defining fundamental matrix solutions and obtaining a variation of constants formula for linear DDEs. This notion gives a definition of solutions to linear DDEs under discontinuous history functions. For a given linear DDE, the fundamental matrix solutions are locally absolutely continuous on the interval $[0, \infty)$, however, it is not apparent that the same property is true for the mild solutions. Here we obtain a result which shows the regularity of mild solutions by proving the local absolute continuity of a function obtained by the history segment of a locally absolutely continuous function. We further obtain a result which reveals the regularity of forcing terms determined by initial history functions by developing a density argument used in a previous study. This talk will be based on [2].

[1] J. Nishiguchi, *Mild solutions, variation of constants formula, and linearized stability for delay differential equations*, Electron. J. Qual. Theory Differ. Equ. **2023**, No. 32, 1–77. https://doi.org/10.14232/ejqtde.2023.1.32.

[2] J. Nishiguchi, *On regularity of mild solutions for linear delay differential equations*, submitted. http://dx.doi.org/10.2139/ssrn.4699292.

[MS6] Morse decomposition of state-dependent delay differential equations

Ábel Garab

University of Szeged, Hungary

Monday, 16:15, 1B364

We consider state-dependent delay differential equations of the form

$$\dot{x}(t) = f(x(t), x(t - r(x_t))),$$

where f is continuously differentiable and fulfills a negative feedback condition in the delayed term. Under suitable conditions on r and f, we construct a Morse decomposition of the global attractor, giving some insight into the structure of it. The so-called Morse sets in the decomposition are closely related to the level sets of an integer valued Lyapunov functional that counts the number of sign changes along solutions on intervals of length of the delay [1]. This generalizes former results for constant delay by J. Mallet-Paret [2].

Joint work with Ferenc A. Bartha and Tibor Krisztin (University of Szeged).

[1] T. Krisztin, O. Arino, *The Two-Dimensional attractor of a differential equation with state-dependent delay*, J. Dyn. Differerential Equations, **Vol. 13, No. 3** (2001), 453–522.

[2] J. Mallet-Paret, *Morse decomposition for differential delay equations*, J. Differential Equations, **72** (1988), 270–315.

[MS6] Existence and exponential stability of low-order and high-order Cohen-Grossberg neural network models with infinite delays

José J. Oliveira

University of Minho

Monday, 16:45, 1B364

This is a joint work with A. Elmwafy (PhD student) and César M. Silva (University of Beira Interior, Covilhã, Portugal).

In this presentation, we present sufficient conditions for the existence and global exponential stability of a periodic solution of the following family of functional differential equations with infinite delays,

$$x_i'(t) = -a_i(t, x_i(t))\Big[- b_i(t, x_i(t)) + f_i(t, x_t)\Big], \;\; t \geq 0, \, i \in \{1, \ldots, n\}.$$

The stability criterion depends on the dominance of nondelay terms over the delay terms.

The criterion for the existence of a periodic solution is obtained with application of the coincidence degree theorem.
The general results are applied to obtain new criteria for the existence and global exponential stability of a periodic solution for both low-order and high-order Cohen-grossberg neural network models.
A numerical example is presented to illustrate the effectiveness of the new results.

[MS6] Periodic solutions in a delayed dendritic cell therapy model

Yang Kuang

Arizona State University

Tuesday, 8:45, 1B364

We formulate a tumor-immune interaction model with a constant delay to capture the behavior following application of a dendritic cell therapy. The model is validated using experimental data from melanoma-induced mice. Through theoretical and numerical analyses, the model is shown to produce rich dynamics, such as a Hopf bifurcation and bistability. We provide thresholds for tumor existence and, in a special case, tumor elimination. Our work indicates a sensitivity in model outcomes to the immune response time. We provide a stability analysis for the high tumor equilibrium. For small delays in response, the tumor and immune system coexist at a low level. Large delays give rise to fatally high tumor levels. Our computational and theoretical work reveals that there exists an intermediate region of delay that generates complex oscillatory, even chaotic, behavior. The model then reflects uncertainty in treatment outcomes for varying initial tumor burdens, as well as tumor dormancy followed by uncontrolled growth to a lethal size, a phenomenon seen in vivo. Theoretical and computational analyses suggest efficacious treatments to use in conjunction with the dendritic cell vaccine. Analysis of a highly aggressive tumor confirms the importance of representation with a time delay, as periodic solutions are generated when a delay is present.

[MS6] Stability analysis of a viral infection model with distributed delay

Jacques Bélair

Université de Montréal

Tuesday, 9:15, 1B364

Mathematical modeling of the immune system has provided significant insight into its regulation and possible control measures to limit the effects of viral infections. We consider in this context the system

$$\begin{aligned} C'(t) &= \lambda(\Lambda - C(t)) - \beta C(t)V(t) \\ I'(t) &= \beta C(t)V(t) - aI(t) - \rho F(t)I(t) \\ F'(t) &= F(t)V(t)(\omega + \delta C(t)) - bF(t) \\ V'(t) &= aNI(t-\tau)e^{-a\tau - \rho\int_{t-\tau}^{t} F(u)du} - \gamma F(t)V(t) - kV(t) \end{aligned}$$

which has been suggested [1] to reflect the dynamics between non-infected (C) and infected (I) $CD4^+T$ cells, virions (V) and the ensuing immune response (F).

We present a stability analysis of the stationary solutions, which amounts to investigating a number of characteristic equations with delay-dependent coefficients, e.g. (in the simplest case)

$$x^2 + (a+k)x + ak - \beta\Lambda aNe^{-a\tau}e^{-x\tau} = 0$$

The associated bifurcations leading to change in their stability are completely determined as well. Numerical simulations are performed to complement the analytical results.

This is a joint work with Marc-Antoine Trahan.

[1] M.-A. Trahan, Sur un modèle d'infection virale avec délai distribué. Mémoire de maitrise, Université de Montréal (2024).

[MS6] The influence of seasonality and diapausing stages in patchy models for tick populations: A global attraction analysis

Alfonso Ruiz-Herrera

University of Oviedo

Tuesday, 9:45, 1B364

This is a joint work with Prof. Juan Quevedo (University of Oviedo, Spain).

Lyme borreliosis, tick-borne encephalitis, and human granulocytic anaplasmosis are some important tick-borne diseases. Motivated by their notable epidemiological role, this paper explores the interplay among seasonality, diapause, and diffusion on the population dynamics of ticks. To reach this goal, we analyze a metapopulation model in a fragmented ecosystem of m-patches, with ticks as a motivating example. Specifically, we offer a new methodology to derive criteria of global attraction in nonautonomous metapopulation models with delay without assuming monotonicity requirements. Roughly speaking, we extend the results of [1] introducing tick-diffusion among different geographical locations. A strength of our approach is that the results apply to metapopulations with any number of patches and topology. From a practical point of view, our theoretical analysis allows us to corroborate some previous experimental work. Another important conclusion of this paper is that the presence of sinks can benefit the tick population, simplifying the dynamical behavior of the whole metapopulation and increasing the total population size. To assess the real repercussions of the results, we analyze our model with parameters derived from real observations.

[1] HA El-Morshedy, A Ruiz-Herrera, *Global attractivity in tick population models incorporating seasonality and diapausing stages.* Proceedings of the Royal Society A **480** (2024), 20230235.

[MS6] Blow-up to infinity or convergence to zero for a delay differential equation

Yukihiko Nakata

Aoyama Gakuin University

Tuesday, 10:15, 1B364

Recently, there have been studies concerning the existence of finite-time blow-up solutions for delay differential equations, see [1–5]. There are issues related to the blow-up phenomena, which we would like to shed light on: when and which solution blows up in finite time? In this talk, we review the previous studies and then we study the dynamics of the following delay differential equation:

$$x'(t) = x(t)^2 - x(t - \tau), \tag{10}$$

with $\tau > 0$. We discuss the blow-up phenomena of (10), comparing it to (10) with $\tau = 0$. We also investigate the convergence of the solution to the zero solution. This study is based on the collaboration work with T. Ishiwata (Shibaura Institute of Technology).

[1] J.A.D. Appleby, D.D. Patterson, Blow-up and super exponential growth in superlinear Volterra equations. Disc. Cont. Dyn. Syst. A 38(8), pp. 3993–4017 (2018)

[2] A. Eremin, E. Ishiwata, T. Ishiwata, Y. Nakata, Delay-induced blow-up in a planar oscillation model. Jpn. J. Indust. Appl. Math. 38, pp. 1037–1061 (2021)

[3] K. Ezzinbi, M. Jazar, Blow-up results for some nonlinear delay differential equations. Positivity 10(2), pp. 329–341 (2006)

[4] I. Győri, Y. Nakata, G. Röst, Unbounded and blow-up solutions for a delay logistic equation with positive feedback. Comm. Pure Appl. Anal. 17(6), pp. 2845–2854 (2018)

[5] T. Ishiwata, Y. Nakata, A note on blow-up solutions for a scalar differential equation with a discrete delay, Jpn. J. Indust. Appl. Math. 39, pp. 959–971 (2022) (Correction: 39, p. 1109 (2022))

[MS6] Periodic and connecting orbits for Mackey–Glass type equations

Tibor Krisztin

University of Szeged

Tuesday, 15:15, 1B364

This is a joint work with G. Benedek (University of Szeged, Szeged, Hungary) and R. Szczelina (Jagiellonian University, Kraków, Poland).

We consider delay differential equations of the form

$$y'(t) = -ay(t) + bf(y(t-1)) \tag{11}$$

with positive parameters a, b and a hump-shaped nonlinear $f : [0,\infty) \to [0,\infty)$. The prototype nonlinearity is $f(\xi) = \frac{\xi^k}{1+\xi^n}$ with $k > 0$ and a large n. The case $k = 1$ is the famous Mackey–Glass equation, the case $k > 1$ appears in population models with the Allee effect, and $k \in (0,1)$ arises in some economic growth models.

It is assumed that the nonlinear f is close to a $g : [0,\infty) \to [0,1]$ with $g(\xi) = 0$ for all $\xi > 1$. For the prototype nonlinearity $g(\xi^k) = \xi^k$ for $\xi \in [0,1]$. The fact $g(\xi) = 0$, $\xi > 1$, allows to construct a stable periodic orbit of the equation $x'(t) = -cx(t) + dg(x(t-1))$ with some $d > c > 0$. Then it is shown that Equation (11) also has a stable periodic orbit, provided a, b, f are close to c, d, g. In addition, for some nonlinearities, heteroclinic connections are proven between equilibrium points, between equilibria and periodic orbits, and between periodic orbits.

[MS6] Homoclinic orbits for the limiting case of the Mackey–Glass equation

Ferenc A. Bartha

University of Szeged, Bolyai Institute

Tuesday, 15:45, 1B364

This is a joint work with Gabriella Vas and Tibor Krisztin (Bolyai Institute, University of Szeged). We consider the Mackey-Glass equation and let $n \to \infty$ to obtain the limiting case, namely,

$$x'(t) = -ax(t) + bf(x(t-1)), \tag{12}$$

where $f(\xi) = \xi$ for $\xi \in [0,1)$, $f(1) = 1/2$, and $f(\xi) = 0$ for $\xi > 1$. In a previous work [1], we established the existence of *complicated* looking, orbitally asymptotically stable periodic orbits for (12) utilizing rigorous numerics. Now, we present how those techniques can be extended to localize an unstable periodic orbit p and, then, show the existence of a homoclinic orbit to p.

[1] F.A. Bartha, T. Krisztin, A. Vígh, *Stable periodic orbits for the Mackey–Glass equation* J. Differ. Equ. **296** (2021), 15–49.

[MS6] Periodic, nonperiodic, and chaotic solutions for a class of difference equations with negative feedback

Benjamin Kennedy

Gettysburg College

Tuesday, 16:15, 1B364

We discuss the scalar difference equation

$$x(k+1) = x(k) + \frac{f(x(k-N))}{N},$$

where f is nonincreasing with negative feedback. This equation is a discretization of the well-studied differential delay equation

$$x'(t) = f(x(t-1)).$$

We examine explicit families of such equations for which we can find, for large N and appropriate parameter values, various dynamical behaviors including chaotic solutions and periodic solutions with many sign changes per minimal period. We contrast these behaviors with the dynamics of the limiting differential equation. Our primary tool is the analysis of return maps for the difference equations that are conjugate to continuous self-maps of the circle.

[MS6] Computer assisted proofs of symbolic dynamics in some Delay Differential Equations

Robert Szczelina

Jagiellonian University

Tuesday, 16:45, 1B364

This is a joint work with Anna Gierzkiewicz (Jagiellonian University, Poland).

We will briefly describe the algorithm from [1] for obtaining rigorous bound on solutions to Delay Differential Equations (DDEs) using the interval arithmetic. Those bounds can be combined with the topological tools such as covering relations in infinite dimensional spaces to check the necessary conditions for the existence of various invariant objects in the system, in particular periodic orbits or symbolic dynamics. As an application, we combine the rigorous estimates with theorems presented in [2] to give a computer-assisted proof of symbolic dynamics in a chaotic ODE perturbed with delayed term of the form $z'(t) = f(z(t)) + \varepsilon \cdot f(z(t-\tau))$ for explicit values of parameters τ and ε.

Acknowledgments: This research was funded in whole or in part by National Science Centre, Poland, 2023/49/B/ST6/02801.

[1] R. S. and P. Zgliczyński, *High-order Lohner-type algorithm for rigorous computation of Poincaré maps in systems of Delay Differential Equations with several delays* J. Found. Comp. Math. **In Press** (2023), doi:10.1007/s10208-023-09614-x.

[2] A. Gierzkiewicz, P. Zgliczyński, *From the Sharkovskii theorem to periodic orbits for the Rössler system.* J. Differential Equations **314** (2022), 733–751.

[MS6] Hyers–Ulam stability and hyperbolicity for nonautonomous linear delay differential equations

Mihály Pituk

University of Pannonia

Wednesday, 8:45, 1B364

This is a joint work with Professors Lucas Backes (Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil) and Davor Dragičević (University of Rijeka, Rijeka, Croatia).

It is known that hyperbolic nonautonomous linear delay differential equations in finite dimensional spaces are Hyers–Ulam stable and hence shadowable. The converse result is available only in the special case of autonomous and periodic linear delay differential equations with a simple spectrum. In this talk, we show the converse and hence the equivalence of all three notions in the title for a general class of nonautonomous linear delay differential equations with uniformly bounded coefficients. The importance of the boundedness assumption will be shown by an example.

[1] L. Backes, D. Dragičević, and M. Pituk, *Shadowing, Hyers–Ulam stability and hyperbolicity for nonautonomous linear delay differential equations,* Commun. Contemp. Math. (2024), Online Ready,

https://doi.org/10.1142/S0219199724500123

[MS6] Effect of delays on the asymptotic stability of linear differential systems

Hideaki Matsunaga

Osaka Metropolitan University

Wednesday, 9:15, 1B364

This is a joint work with Yuki Hata (Osaka Prefecture University).

We summarize some recent results on stability properties of a linear delay differential system

$$\begin{cases} x'(t) = -ax(t-\tau_{11}) - by(t-\tau_{12}), \\ y'(t) = -cx(t-\tau_{21}) - dy(t-\tau_{22}), \end{cases} \tag{1}$$

where a, b, c, $d \in \mathbb{R}$ and $\tau_{ij} \geq 0$ for $i, j = 1, 2$. Clarifying stability dependence on parameters of delay differential systems is important. In particular, the derivation of delay-dependent stability results has received considerable attention. We provide new stability criteria and some examples to illustrate the delay-dependent stability switches for (1) with delay in the diagonal terms. The proof technique is based on careful analysis of the existence and the transversality of characteristic roots on the imaginary axis.

[1] Y. Hata, H. Matsunaga, *Delay-dependent stability switches in a delay differential system.* Discrete Contin. Dyn. Syst. Ser. B **28** (2023), 4910–4936.

[2] A. Matsumoto, F. Szidarovszky, *Time delays and chaos in two competing species revisited.* Appl. Math. Comput. **395** (2021), Paper No. 125862, 14 pp.

[MS6] The role of symmetry in delay effects on stability

JOHN STAVROULAKIS

GEORGIA INSTITUTE OF TECHNOLOGY

Wednesday, 10:15, 1B364

A conjecture of Buchanan and Lillo states that all nontrivial oscillatory solutions of

$$x'(t) = p(t)x(t - \tau(t)),$$

with $0 \leq p(t) \leq 1, 0 \leq \tau(t) \leq 2.75 + \ln 2 \approx 3.44$ tend to a known antiperiodic function. We discuss recent developments on this question, focusing on the periodic solutions characterizing the threshold case, for both positive feedback with $\sup \tau(t) = 2.75 + \ln 2$ and negative feedback $0 \leq -p(t) \leq 1, \sup \tau(t) = 1.5$ (the 3/2-criterion), as well as higher order equations.

We investigate the behavior of the threshold periodic solutions under perturbation together with the symmetry (antiperiodicity) which characterizes them. This problem is set within the broader background of delay effects on stability for autonomous and nonautonomous equations, taking into account the fundamental relation between oscillation speed and dynamics of delay equations. We highlight the crucial role of symmetry in both the intuitions behind this vein of research, as well as the relevant combinatorial-variational problems.

[MS7] Identification and existence of Boltzmann processes

Barbara Rüdiger

University Wuppertal

Tuesday, 8:45, 1D226

The stochastic differential equation (SDE) of McKean-Vlasov type is identified such that the Fokker-Planck equation associated to it is the Boltzmann equation. Hence, we call its solutions as Boltzmann processes. They describe the dynamics (in position and velocity) of tagged particles expanding in vacuum in accordance with the Boltzmann equation. Given a solution $f := \{f(t, x, v\}_{0 \leq t \leq T}$ of the Boltzmann equation, the existence of solutions to the McKean-Vlasov SDE is established for the non-cutoff hard sphere case.

This is a joint work with Padmanabhan Sundar (Louisiana State University)

[MS7] On the stochastic engine of contagious diseases in exponentially growing populations

Torsten Lindström
Linnaeus University

Tuesday, 9:15, 1D226

The purpose of this paper is to analyze the mechanism for the interplay of deterministic and stochastic models for contagious diseases. Deterministic models for contagious diseases are prone to predict global stability. Small natural birth and death rates in comparison to disease parameters like the contact rate and the removal rate ensures that the globally stable endemic equilibrium corresponds to a tiny average proportion of infected individuals. Asymptotic equilibrium levels corresponding to low numbers of individuals invalidate the deterministic results.

Diffusion effects force probability mass functions of the stochastic model to possess similar stability properties as the deterministic model. Particular simulations of the stochastic model predict, however, oscillatory patterns. Small and isolated populations show longer periods, more violent oscillations, and larger probabilities of extinction.

We prove that evolution maximizes the infectiousness of the disease as measured by the ability to increase the proportion of infected individuals. This holds provided the stochastic oscillations are moderate enough to keep the proportion of susceptible individuals near a deterministic equilibrium.

We close our paper with a discussion of the herd-immunity concept and stress its close relation to vaccination-programs.

[1] T. Lindström, *On the stochastic engine of contagious diseases in exponentially growing populations* Nonlinear Analysis, Real World Applications **77** (2024), article id 104045.

[MS7] Age dependent population modeling with Poissonian births and deaths

Ihsan Arharas

Linnéuniversitetet

Tuesday, 9:45, 1D226

This is a joint work with Prof. Roger Pettersson, Department of Mathematics, Linnaeus University, Vaxjo, Sweden. and Prof. Omar Kebiri, Department of Stochastics and its Applications, BTU Cottbus-Senftenberg, Brandenburg, Germany.

The McKendrick-Von Forster age-structured population model is extended to a stochastic differential equation age-structured model. Various approaches to stochastic modeling of age-dependent populations have been explored in the literature. Typically, authors convert the McKendrick-Von Forster model into a stochastic one by incorporating a noise term into the dynamics. However, it is not clear whether the stochastic model is "more accurate" than the deterministic one. In this paper, our aim is to derive a stochastic model naturally by integrating uncertainty into the model itself rather than introducing uncertainty into the ordinary differential equation (ODE), aiming to develop a more comprehensive stochastic model. To this end, we formulate a model where births and deaths occur according to an age-dependent Poisson process, with each life and death being counted individually. Therefore, our goal is to develop a stochastic age-dependent model where randomness in births and deaths is originated with this Poisson process, and which, with appropriate scaling, can be approximated by the McKendrick-Von Forster model.

[1] S. N. Ethier and T. G. Kurtz, *Markov Processes: Characterization and Convergence.* Wiley Series in Probability and Statistics. Wiley, March 1986. ISBN 0471081868.

[2] M. Chowdhury, *A Stochastic Age-Structured Model.* Master Thesis, Texas Tech University, 1998. ersson, R., Taki, R. (2019).

[MS7] Mathematical modeling of learning process

Yuriy Rogovchenko

University of Agder, Kristiansand, Norway

Tuesday, 10:15, 1D226

We report preliminary findings from the ongoing joint work with Jorge Duarte (Instituto Superior de Engenharia de Lisboa - ISEL and Universidade de Lisboa, Lisbon, Portugal), Cristina Januário (Instituto Superior de Engenharia de Lisboa - ISEL and University of Aveiro, Portugal), Nuno Martins (Universidade de Lisboa, Lisbon, Portugal), and Svitlana Rogovchenko (University of Agder, Grimstad, Norway) regarding mathematical modeling of student learning.

Learning is a complex long-life process by which humans acquire new knowledge and skills; it underpins all aspects of education and constitutes an important part of our daily life. It is widely acknowledged that learning success is determined by a combination of many factors including personal learning preferences and attitude towards learning, lecturer's content knowledge and skills, teaching methods (student- or lecturer centered). Several mathematical models based on variations of SEIR (susceptible-exposed-infected-recovered) epidemiological models have been suggested recently for describing the process of knowledge acquisition, see, for instance, [1]– [3]. Reflecting on the cited research, own teaching experience, and our research in mathematics education, we suggest a model for learning which accounts for individual's aptitude for learning and learning facilitated by social interaction with peer students and the lecturer. We also take into consideration possibility for learning from several sources and possible loss of information with time (retention of knowledge).

[1] S.-G. Liao and S.-P. Yi, *Modeling and analyzing knowledge transmission process considering free-riding behavior of knowledge acquisition: A waterborn disease approach* Physica A **569** (2021), 125769.

[2] G. Makanda and R. Sypkens, *Investigating the dynamics of knowledge acqusition in learning using differential equations* Int. J. Math. Comp. Sci. **11** (2017), 491–497.

[3] Mutiawati, R. Johar, M. Ramli, and Malizar *Mathematical model of student learning behavior with the effect of learning motivation and student social interaction* J. Math. Educ. **13** (2022), 415–436.

[MS7] Wavefronts in Biased Population Dynamics

Diego Berti

University of Turin, Italy

Thursday, 15:15, 1A305

This is a joint work with Andrea Corli (University of Ferrara, Italy) and Luisa Malaguti (University of Modena and Reggio Emilia, Italy).

We deal with wavefront solutions for the reaction-diffusion equation

$$u_t + f(u)_x = \{D(u)u\}_x + g(u) \quad \text{in} \quad \mathbb{R} \times (0, \infty). \tag{13}$$

Here, the unknown $u = u(x,t)$ stands for population density; f is a (non-linear) convection, D is a sign-changing diffusivity, and g is the source term.

For the parabolic equation (13), we brielfy present recent theoretical results on the existence and fine properties of so-called *wavefronts*, namely solutions of the type $u(x,t) = \varphi(x - ct)$, for some *wave profile* $\varphi(\xi)$ and *wave speed* $c \in \mathbb{R}$. The convection component notably influences population spread, potentially leading to the emergence of various wavefront types.

Also, we discuss applications of the theoretical findings to models for biological populations moving in a one-spatial dimension, with directional biases in movement. In particular, we focus on a lattice-based stochastic model, where agents (splitted in two non-interacting classes) can reproduce, die, and move, at varying rates. In this context, the presence of biases is related to convection on the continuum equation.

[MS7] Well Posedness of Biological Models

Rinaldo M. Colombo

University of Brescia

Thursday, 15:45, 1A305

This is a joint work with M. Garavello (University of Milano-Bicocca), F. Marcellini (University of Brescia) and E. Rossi (University of Modena - Reggio Emilia)

Conservation (or balance) Laws enter a variety of biological models. In population dynamics, balance laws describe age (or space) structured dynamics. In epidemiology, balance laws enter several *variazioni* on the SIR model. In predatory-prey dynamics, non local balance laws describe species moving with finite speed towards a well defined target. Other examples of the use of these equations are provided by cancer or cell growth.

Strictly speaking, the usual definition of hyperbolicity hardly applies to non local balance laws. Nevertheless, analytical techniques originally developed in the framework of hyperbolic conservation laws are effective.

An effective approach to general well posedness proofs is based on classical hyperbolic techniques but tackles boundary conditions without resorting to the traces of the solution. A different approach relies on differential equations in metric spaces and allows to show the well posedness of a variety of coupled problems.

Once well posedness is obtained, control problems can be tackled and often have a precise biological relevance, as in the case of devising optimal vaccination strategies.

[MS7] Coupled reaction-diffusion equations with degenerate diffusivity: wavefront analysis

Valentina Taddei

University of Modena and Reggio Emilia

Thursday, 16:15, 1A305

This is a joint work with Prof. Eduardo Muñoz-Hernández (Complutense University of Madrid, Spain) and Prof. Elisa Sovrano (University of Modena and Reggio Emilia, Italy).

We investigate wavefront solutions for the nonlinear system of two coupled reaction-diffusion equations with degenerate diffusivity

$$\begin{cases} n_t = -f(n,b) \\ b_t = [g(n)h(b)b_x]_x + f(n,b). \end{cases} \tag{14}$$

These systems mainly appear in modelling spatial-temporal patterns during bacterial growth. We prove, for sufficiently large speeds, the existence and uniqueness, up to translations, of a traveling wave whose profile is component-wise monotone. On this purpose we combine the fixed-point theory, the central manifold theorem and a comparison technique. Finally, by employing a first-order reduction method, we prove that the set of admissible speeds is connected and closed and that the traveling wave is classical for speeds strictly bigger than the threshold, for which a sharp profile appears.

[1] E.Muñoz-Hernández, E.Sovrano and V.Taddei, *Coupled reaction-diffusion equations with degenerate diffusivity: wavefront analysis*, Submitted.

[MS7] Growth of subsolutions to fully nonlinear equations in halfspaces

Niklas Lundström

Umeå University

Thursday, 16:45, 1A305

We characterize lower growth estimates for subsolutions of fully nonlinear partial differential equations on the form

$$F(x, u, Du, D^2u) = 0$$

in terms of solutions to ordinary differential equations built upon assumptions on F. Using this characterization we derive several sharp Phragmén–Lindelöf-type theorems in halfspaces for certain classes of well known PDEs.

Moreover, we discuss consequences of our theorems in the setting of nonlinear diffusion, relating to p-Laplace-type equations with sources, sinks, and spatially dependent exponent $p = p(x)$.

[MS8] Existence and spectral stability of pattern-forming fronts in the FitzHugh–Nagumo system

Paul Carter

University of California, Irvine

Thursday, 15:15, 1B309

This is joint work with Dr. Montie Avery (Boston University, Boston, USA), Dr. Björn de Rijk (Karlsruhe Institute of Technology, Karlsruhe, Germany), and Prof. Arnd Scheel (University of Minnesota, Twin Cities, USA).

We consider the FitzHugh–Nagumo PDE in the so-called oscillatory regime in which one observes spatially oscillatory patterns that invade an unstable steady state. The pattern is selected from a family of periodic traveling wave train solutions by an invasion front. Using geometric singular perturbation techniques, we construct “pushed" and “pulled" pattern-forming fronts as heteroclinic orbits between the unstable steady state and a periodic orbit representing the wave train in the wake. In the case of pushed fronts, the wave train necessarily passes near a pair of nonhyperbolic fold points on the associated critical manifold. We also discuss implications for the stability of the pattern-forming fronts and the challenges introduced by the fold points in the corresponding spectral stability problem.

[MS8] Pattern formation in a mechano-chemical model

Daphne Nesenberend

Leiden University

Thursday, 15:45, 1B309

During the development of a wide class of organisms, the cues that signal or mitigate a specific developmental stage can be chemical as well as mechanical. To describe and study these two types of cues in the developmental process, we present and analyse a so-called mechano-chemical model, which describes a biological surface (e.g. a cell membrane or a tissue surface) that is evolving over time. This time evolution is determined both by a morphogen chemical acting on (and diffusing over) the surface, and by the local curvature of the surface itself [1]. To analyse the possibility of pattern formation in this model, we reduce the surface to a one-dimensional graph, which reduces the model to a system of reaction-diffusion equations in one spatial dimension. The curve length is fixed by a Lagrange multiplier, inducing an inhibiting effect on the dynamics. We choose specific expressions for the preferred local curvature and the curvature induced morphogen production. Numerical simulations indicate the existence of spatially periodic (and quasi-periodic) steady states. We further analyse the existence of these pattern states using geometric singular perturbation theory, highlighting the importance of pattern selection by the geometric constraints on the curve.

[1] M. Mercker, A. Marciniak-Czochra, T. Richter and D. Hartmann, *Modeling and computing of*

deformation dynamics of inhomogeneous biological surfaces. SIAM J. Appl. Math. **73** (2013), 1768–1792.

[MS8] Slow localized patterns in singularly perturbed reaction-diffusion equations

Arjen Doelman

Leiden University

Thursday, 16:15, 1B309

Localized patterns in singularly perturbed reaction-diffusion equations typically consist of slow parts – in which the associated solution follows an orbit on a slow manifold in a reduced spatial dynamical system – alternated by fast excursions – in which the solution jumps from one slow manifold to another, or back to the original slow manifold. In this talk we consider the existence and stability of stationary and traveling localized patterns in singularly perturbed 2-component reaction-diffusion equations that do not exhibit such jumps, *i.e.*, that are completely embedded in a slow manifold of the singularly perturbed spatial dynamical system. These patterns have rarely been considered before, for two reasons: *(i)* in many models considered in the literature, the flow on the slow manifold is typically linear and thus cannot exhibit homoclinic pulse or heteroclinic front solutions; *(ii)* the slow manifolds occurring in the literature are also often 'vertical' – i.e. given by $u \equiv u_0$, where u is the fast variable – which implies that the associated stability problem is 'trivial' since it follows in that case that the stability is determined by a scalar (reaction-diffusion) equation. This project is motivated by several explicit ecosystem models that do give rise to non-vertical normally hyperbolic slow manifolds on which the flow may exhibit both homoclinic and heteroclinic orbits. In this talk, we consider a general system of singularly perturbed 2-component reaction-diffusion equations for which the associated spatial dynamical system for traveling waves typically has a non-flat, non-vertical 2-dimensional normally hyperbolic slow manifold on which the (slow) flow may exhibit homoclinic and heteroclinic orbits. We find that the spectral stability problems associated to such ('fully slow') pulses and fronts are at leading order given by nonlinear, but scalar, eigenvalue problems with Sturm-Liouville characteristics of which the precise structure is strongly determined by the geometric nature of the underlying slow manifold. We show that homoclinic pulse patterns are typically unstable – as is the case for scalar problems. However, heteroclinic fronts may either be stable or unstable.

Moreover, unlike in scalar problems, homoclinic pulse patterns that are asymptotically close to a heteroclinic cycle may be stable. Based on a detailed analysis of the singularly perturbed problem, we are able to obtain full control over the nature of the bifurcations – saddle-node, Hopf, global, etc. – that determine the existence and stability of the (stationary and/or traveling) heteroclinic fronts and/or homoclinic pulses. Finally, we show that heteroclinic orbits may correspond to stable (slow) interfaces in 2-dimensional space, while the homoclinic pulses must be unstable as localized stripes – even when they are stable in 1 space dimension.

[MS8] Multidimensional Stability of Planar Travelling Waves for Stochastically Perturbed Reaction-Diffusion Systems

Mark van den Bosch

Leiden University

Thursday, 16:45, 1B309

This is a joint work with Prof. dr. Hermen Jan Hupkes (Leiden University, the Netherlands).

Travelling pulses and waves are a rich subset of feasible patterns in reaction-diffusion equations. Many authors have investigated their existence, stability, and other properties, but what happens if the deterministic dynamics is affected by random occurrences? In particular, how does the interplay between diffusion and noise influence the velocity, curvature and stability of the waves?

We consider reaction-diffusion systems with multiplicative noise on a spatial domain of dimension two or higher ($d \geq 2$). The noise process is white in time, coloured in space, and chosen to be invariant under translations|based on applications. In the deterministic setting, multidimensional stability of planar waves on the whole space $\mathbb{R}^d$ has been studied by many, see for instance [1]. Inspired by the works [2] and [3], we show multidimensional stability of planar waves for exponentially long time scales on a cylindrical domain $\mathbb{R} \times \mathbb{T}^{d-1}$, where $\mathbb{T}$ denotes a one-dimensional torus. This is achieved by means of a long-time stochastic phase tracking mechanism. In addition, we demonstrate how asymptotic expansions provide some intuition on how to tackle the problem on $\mathbb{R}^d$.

Our approach is similar to the one on the real line ($d = 1$), but many additional obstacles arise. Indeed, the mild formulation of our problem that we end up with is ill-posed in the sense of the well-understood Itô-integrals. To make sense of the mild formulation where integrands are anticipating, there was a need to exploit recent theory [4] concerning forward integrals and to develop it further.

[1] T. Kapitula (1997). Multidimensional stability of planar travelling waves. *Transactions of the American Mathematical Society* **349**(1), 257–269.

[2] C. H. S. Hamster and H. J. Hupkes (2020). Travelling waves for reaction–diffusion equations forced by translation invariant noise. *Physica D: Nonlinear Phenomena* **401**, 132233.

[3] C. H. S. Hamster and H. J. Hupkes (2020). Stability of traveling waves on exponentially long timescales in stochastic reaction-diffusion equations. *SIAM Journal on Applied Dynamical Systems* **19**(4), 2469-2499.

[4] J. M. A. M. van Neerven, J. and M. C. Veraar (2021). Maximal inequalities for stochastic convolutions and pathwise uniform convergence of time discretisation schemes. *Stochastics and Partial Differential Equations: Analysis and computations*, 1-66.

[MS8] Turing pattern propagates on curved surface

Shuji Ishihara

University of Tokyo

Friday, 8:45, 1B364

This is joint work with Dr. Ryosuke Nishide (University of Tokyo, Tokyo, Japan). We study the Turing pattern on curved surfaces. Since the seminal work by A. Turing [1] in which he already studied a pattern on a sphere, many researchers have investigated the pattern formation on curved surfaces such as spheres and tori. Notably, all of these studies presumed that the Turing pattern remains static on curved surfaces irrespective of the surface geometry. Here, we present that the Turing pattern on curved surfaces moves on curved surfaces [2]. For the sake of analytical tractability, we mainly study reaction-diffusion systems on an axisymmetric surface with periodic boundary conditions, with parameters set so that the Turing pattern appears on a flat plane. Numerical and theoretical analyses reveal that there exist propagating solutions along the azimuth direction, and both the symmetries of the surface and pattern are associated with the initiation of the pattern propagation. We also conduct weakly non-linear analysis and derive the amplitude equations and show that the intricate interaction between modes results in the initiation of pattern propagation and even more complex behaviors such as oscillatory and chaotic pattern dynamics. This study provides a novel and generic mechanism of pattern propagation that is caused by surface curvature (which is not possible in 1D systems), as well as new insights into the potential role of surface geometry in pattern dynamics.

[1] A. M. Turing, The chemical basis of morphogenesis, Phil. Trans. R. Soc. Lond. B **237**, 37-72 (1952).

[2] R. Nishide, and S. Ishihara, Pattern Propagation Driven by Surface Curvature, Phys. Rev. Lett. **128**, 224101 (2022).

[MS8] Periodic and localized waves in the actin cortex of motile cells

Arik Yochelis

Ben-Gurion University

Friday, 9:15, 1B364

This is a joint work with Prof Carsten Beta (Institute of Physics and Astronomy, University of Potsdam, Potsdam, 14476, Germany).

Self-organized patterns in the actin cytoskeleton are essential for eukaryotic cellular life. They are the building blocks of many functional structures that often operate simultaneously to facilitate, for example, nutrient uptake and movement of cells. However, identifying how qualitatively distinct actin patterns can coexist remains a challenge, requiring theoretical investigations [1]. Bifurcation analysis of a mass (actin) conserved reaction–diffusion model, uncovers a generic mechanism about the co-dimension-two T-point, of how different actin waves — traveling waves and excitable pulses — organize and simultaneously emerge. Surprisingly, live-cell imaging experiments indeed reveal that narrow, planar, and fast-moving excitable pulses may coexist with ring-shaped macropinocytic actin waves in the cortex of motile amoeboid cells [2].

[1] C. Beta, L. Edelstein-Keshet, N. Gov, A. Yochelis, *From actin waves to mechanism and back: How theory aids biological understanding*, eLife **12** (2023), e87181.

[2] A. Yochelis, S. Flemming, and C. Beta, *Versatile patterns in the actin cortex of motile cells: Self-organized pulses can coexist with macropinocytic ring-shaped waves*, Phys. Rev. Lett. **129** (2022), 088101.

[MS8] A billiard problem in nonlinear dissipative systems

Tomoyuki Miyaji

Kyoto University

Friday, 9:45, 1B364

We study planar motion of a self-propelled particle that interacts repulsively with a boundary. For instance, a camphor disk floating on the surface of water spontaneously moves by making surface tension inhomogeneous. It is modeled by a system consisting of a planar reaction-diffusion equation and an equation of motion [1]. A traveling spot solution of the system characterizes the motion of the camphor disk. The spot dynamics can be reduced to a system of four-dimensional ODEs under the assumption that the system is sufficiently close to a bifurcation point of a stationary spot solution [1,3]. A reduced system on the half plane has been studied in [2] and [4]. Under certain conditions, it is shown that the particle reflects from the boundary wall without collision, and the angle of reflection is greater than that of incidence. Suppose such a particle is confined in a bounded domain. As it exhibits motion in a straight line and non-specular reflection, it behaves quite differently from classical mathematical billiards, which assume specular reflection. Assuming that the domain is quite large, we may model the motion by a discrete-time dynamical system: the particle moves in a straight line and changes its direction according to a given angle relation when it hits a boundary. We will present an overview of these studies. This talk is based on a work with Prof. Shin-Ichiro Ei (Hokkaido University, Sapporo, Japan) and the late Professor Masayasu Mimura.

[1] X. Chen, S.-I. Ei, and M. Mimura Self-motion of camphor discs. model and analysis, *Netw. Heterog. Media* **4** (2009) 1–18.

[2] S.-I. Ei, M. Mimura and T. Miyaji, Reflection of a self-propelling rigid disk from a boundary. *Discrete Contin. Dyn. Syst. S* **14**:3 (2021) 803–817.

[3] S.-I. Ei, M. Mimura and M. Nagayama, Interacting spots in reaction diffusion systems, *Discrete*

Contin. Dyn. Syst. **14** (2006) 31–62.

[4] T. Miyaji and R. Sinclair, Asymptotic reflection of a self-propelled particle from a boundary wall, *Jpn. J. Ind. Appl. Math.* **41** (2024) 269–295.

[MS8] Annihilation tongue

Yasumasa Nishiura

Hokkaido University

Friday, 10:15, 1B364

In this talk, we focus on annihilation dynamics for the head-on collision of traveling patterns. A representative and well-known example of annihilation is the one observed for one-dimensional traveling pulses of the FitzHugh-Nagumo equations. In this talk, we present a new and completely different type of annihilation arising in a "weak" interaction regime. It is even counterintuitive in the sense that the spots/pulses come together very slowly but do not merge, and then they start to repel each other for a certain time. Finally, up and down oscillatory instability emerges and its amplitude grows enough for patterns to become extinct eventually [1]. There is a kind of hidden instability embedded in the traveling patterns, which causes the above annihilation dynamics, which turns out to be a codimension 2 point consisting of drift and Hopf (DH) instabilities. Annihilation regime emanating from the codimension 2 point forms a tongue shape in an appropriate parameter space. The above scenario can be proved analytically up to the onset of annihilation by reducing it to a finite-dimensional system. We also discuss about how such annihilation dynamics changes into a merging type of annihilation similar to the FitzHugh-Nagumo pulses [2]. When the parameter does not belong the annihilation tongue, the patterns do not annihilate and repel each other even though it is very close to the codimension 2 point. This is a joint work with Kei-Ichi Ueda (Toyama university) and Takashi Teramoto (Kyoto Women's University).

[1] Y. Nishiura, T. Teramoto and K. Ueda, Arbitrarily weak head-on collision can induce annihilation: the role of hidden instabilities. *Japan Journal of Industrial and Applied Mathematics*, **40** (2023) 1695-1743, https://doi.org/10.1007/s13160-023-00607-5

[2] Y. Nishiura, T. Teramoto and K. Ueda, Scattering of traveling spots in dissipative systems, *Chaos*, **15**

(2005) 47509-0475190, DOI: 10.1063/1.1207127.

[MS9] Dynamics of vortex pairs at large Reynolds number in 2D fluids

Michele Dolce

École Polytechnique Fédérale de Lausanne

Monday, 15:15, 1B306

This is a joint work with Thierry Gallay (Université Grenoble Alpes, Grenoble, France).

The evolution of two point vortices in a 2D inviscid fluid in the whole plane, whether counter-rotating or co-rotating, is explicitly determined by the Helmholtz-Kirchhoff system. One translates on a straight line with a constant speed, while the latter rigidly rotates around each other. At a large but finite Reynolds number, vortex core sizes grow due to diffusion, revealing immensely rich phenomena stemming from this simple initial configuration. The goal of the talk is to validate an asymptotic expansion describing the dynamics of the viscous vortex pair. This will be done in terms of small adimensional parameters, particularly useful to isolate inviscid and viscous effects. The expansion is performed at a sufficiently high order where corrections to the Helmholtz-Kirchhoff motion are revealed, in agreement with observations in the applied literature. We then show that the exact solution remains close to our approximation over a time interval that increases boundlessly as the Reynolds number goes to infinity. The proof relies on stability estimates derived from Arnold's variational characterization of the steady states of the 2D Euler equation, as recently revised by Gallay and Šverák and applied to viscous fluids.

[MS9] Global bifurcation for corotating vortex pairs

Claudia García

Universidad de Granada

Monday, 15:45, 1B306

This is a collaboration with Susanna V. Haziot (Brown university, Providence, RI, USA).

The existence of a local curve of corotating vortex pairs was proven by Hmidi and Mateu via a desingularization of a pair of point vortices. In this talk, we will present a global continuation of these local curves. That is, we consider solutions which are more than a mere perturbation of trivial solutions. Indeed, while the local analysis relies on the study of the linear equation at the trivial solution, the global analysis requires on a deeper understanding of topological properties of the nonlinear problem. For our proof, we adapt the powerful analytic global bifurcation theorem due to Buffoni and Toland, to allow for the singularity at the bifurcation point.

[MS9] Vortex dynamics for the Gross-Pitaevskii equation

Rowan Juneman

University of Bath

Monday, 16:15, 1B306

This is joint work with Manuel del Pino and Monica Musso (University of Bath, Bath, UK).

The Gross-Pitaevskii equation in the plane arises as a physical model for an idealized, two-dimensional superfluid. We construct solutions to this equation with multiple vortices of degree ± 1, corresponding to concentration points of the associated fluid vorticity. The vortex dynamics is described on any finite time interval, and at leading order is governed by the classical Kirchhoff law. Compared to previous rigorous results of Bethuel-Jerrard-Smets and Jerrard-Spirn, we use a different method based on linearization around an approximate solution. This approach provides a very precise description of the solutions near the vortex set and information on lower order corrections to the vortex dynamics. Moreover, our analysis of the linearized problem is potentially of independent interest in the study of long-time dynamics.

[MS9] Steady vortex rings with surface tension

Christian Seis

Universität Münster

Monday, 16:45, 1B306

This is a joint work with David Meyer (University of Münster, Münster, Germany).

The existence of steady vortex rings for the two-phase Euler equations with surface tension is studied, describing the evolution of a perfect bubble air ring in water. Such objects are created in nature by cetaceans such as dolphins or beluga whales, and they appear to be surprisingly stable configurations. The mathematical model features a vortex sheet on the surface of the air bubble. We construct such vortex rings with small cross sections with the help of an implicit function theorem and derive the asymptotics of various quantities for small cross sections.

[MS9] A plausible link between low-dimensional point-vortex dynamics and long-time states in the 2-D Euler equations

Klas Modin

Chalmers University of Technology and University of Gothenburg

Thursday, 8:45, 1D222

This is a joint work with Milo Viviani (Scuola Normale Superiore, Pisa, Italy).

If you stir up an incompressible, low-viscosity fluid confined to a thin (almost 2-D) domain, what you see is spectacular, completely different from the situation in full 3-D. The fluid self-organizes into large, coherently swirling vortex condensates. Some regions swirl counterclockwise (positive vorticity) while others swirl clockwise (negative vorticity), with occasional merger between equal-signed regions, and repulsion between opposite-signed regions. What is the mechanism of vortex condensation and under which conditions does it occur?

Lars Onsager, the Norwegian Nobel Laureate in chemistry, made significant headway in 1949 by applying Boltzmann's theory of statistical mechanics to a large but finite number of interacting point vortices (PVs). Onsager realized that, contrary to a Boltzmann gas, the energy can take all values from $-\infty$ to ∞ (just put two PVs of equal of different sign arbitrarily close). From statistical mechanics, it then follows that the analog of thermodynamical temperature can be negative, and in such situations PVs of equal sign tend to cluster. This way, Onsager's theory of statistical hydrodynamics predicts specific configurations of the long-time states.

However, low viscosity, thin film lab experiments, as well as numerical simulations of 2-D Euler equations, are often in conflict with Onsager's statistical predictions: vortex condensation occurs, but the vorticity configuration does not reach the predicted long-time states. Instead, the motion is trapped in persistent unsteadiness via interacting "vortex blob condensates".

Here, I present numerical simulations with *Zeitlin's model* – the only discretization of the 2-D Euler equations that preserves all the underlying geometric structure – which suggests a mechanism that could hinder the statistical predictions. Namely, that Liouville integrability of low dimensional PV dynamics acts as a barrier in phase space for further (large-scale) vortex mixing.

[MS9] Vortex filament reconnection in the 3d critical Abelian Higgs model

Robert Jerrard
University of Toronto
Thursday, 9:15, 1D222

This is a joint work with Amirmasoud Geevechi (University of Toronto, Toronto, Canada).

The critical Abelian Higgs model (AHM) is a nonlinear system of wave equations arising in particle physics. We construct solutions of this system in 3+1 dimensions that exhibit an integer number N of slowly-moving nearly parallel vortex filaments, with the leading-order dynamics of this ensemble of filaments described by a wave map into the moduli space, a manifold carrying a natural Riemannian structure that parametrizes stationary 2d solutions of the AHM. These results allow us to study the poorly-understood phenomenon of vortex reconnection in this setting. In particular, we show that in the regime we study, reconnection is the generic outcome of collisions of vortex filaments.

[MS9] On the binormal curvature flow for unbounded curves

Didier Smets
Sorbonne Université
Thursday, 9:45, 1D222

This is a joint work with Valeria Banica (Sorbonne Université, Paris, France) and Robert L. Jerrard (University of Toronto, Toronto, Canada).

The talk will be devoted to the binormal curvature flow equation

$$\partial_t\Gamma = \partial_s\Gamma \wedge \partial_{ss}\Gamma,$$

sometimes also called the filament flow or the local induction approximation (LIA) flow, since it first appeared as a reduced model for the evolution of filaments in fluids. We will propose an extension of the notion of weak solutions, as developed in [2] in the context of closed curves (more precisely of finite mass integral 1-currents with no boundary), to the case of curves extending towards infinity. One motivation behind this extension is the ability to encompass in our theory the important examples of self-similar solutions which were first described in [1], and which at some time during their evolution correspond to an infinite broken line with a corner singularity. It turns out that we are eventually able to tackle the case of arbitrary Lipschitz curves as initial data, provided they have a finite renormalized length, for a suitable definition of the latter.

[1] S. Gutiérrez, J. Rivas and L. Vega, *Formation of singularities and self-similar vortex motion under the localized induction approximation.* Comm. Part. Diff. Eqs. **28** (2003), 927–968.

[2] R. L. Jerrard and D. Smets, *On the motion of a curve by its binormal curvature.* J. Eur. Math. Soc. **17** (2015), 1148–1515.

[MS9] The binormal flow and the desingularization of the Biot-Savart integral

Luis Vega

UPV/EHU-BCAM

Thursday, 10:15, 1D222

This is a joint work with Prof M.A. Fontelos(ICMAT,Madrid, Spain).

I'll present the so called Localized Induction Approximation that describes the dynamics of a vortex filament according to the Binormal Curvature Flow (BF). I'll give a result about the desingularization of the Biot-Savart integral within the framework of Navier-Stokes equations. Some particular examples regarding BF obtained with Valeria Banica will be also considered.

[MS10] Two recent homogenization results for dielectric elastomer composites

Silvia Jimenez Bolanos

Colgate University

Tuesday, 8:45, 1D236

This is a joint work with Thuyen Dang (Univeristy of Chicago, USA) and Yuliya Gorb (National Science Foundation, USA)

First, we will discuss the periodic homogenization for a weakly coupled electroelastic system of a nonlinear electrostatic equation with an elastic equation enriched with electrostriction.

Such coupling is employed to describe dielectric elastomers or deformable (elastic) dielectrics. We will show that the effective response of the system consists of a homogeneous dielectric elastomer described by a nonlinear weakly coupled system of PDEs whose coefficients depend on the coefficients of the original heterogeneous material, the geometry of the composite and the periodicity of the original microstructure. The approach developed here for this nonlinear problem allows obtaining an explicit corrector result for the homogenization of monotone operators with minimal regularity assumptions.

Next, we will discuss the homogenization of high-contrast dielectric elastomer composite. The considered heterogeneous material consisting of an ambient material with inserted particles is described by a weakly coupled system of an electrostatic equation with an elastic equation enriched with electrostriction. It is assumed that particles gradually become rigid as the fine-scale parameter approaches zero. We will see that the effective response of this system entails a homogeneous dielectric elastomer, described by a weakly coupled system of PDEs. The coefficients of the homogenized equations are dependent on various factors, including the composite's geometry, the original microstructure's periodicity, and the coefficients characterizing the initial heterogeneous material. Particularly, these coefficients are significantly influenced by the high-contrast nature of the fine-scale problem's coefficients. Consequently, as anticipated, the high-contrast coefficients of the original yield non-local effects in the homogenized response.

[MS10] Modeling and simulating the cooling effect during tool grinding

Tom Freudenberg

Universität Bremen

Tuesday, 9:15, 1D236

This is a joint work with Alfred Schmidt (University of Bremen, Germany), Michael Eden (Karlstadt University, Sweden) and the IFW of the Leibniz University Hannover, Germany.

In tool grinding high thermal loads appear, that negatively impact the grinding wheel and product quality. To reduce the friction and cool the system the use of cooling lubricant is important. To optimize the grinding process and the coolant usage a simulation model is needed. Different micro structures play an important role in grinding, like the porosity and granular surface of the grinding wheel and rough surface of the workpiece. Resolving these structures numerically leads to high memory cost and computation times. Therefore, effective multi-scale approaches are needed.

In this talk we present first homogenization results concerning porous structures, small grains at an interface and the grinding gap that were derived with the concept of two-scale convergence [1,2]. Investigated are systems of heat equations coupled with imperfect heat transfer and ε-dependent parameters, where ε describes the size of the underlying micro structure, as well as the Stokes equation for describing the coolant flow. Additionally, we present numerical simulations of the homogenized model and compare them with resolved micro models.

[1] M. Eden and T. Freudenberg, *Effective Heat Transfer Between a Porous Medium and a Fluid Layer: Homogenization and Simulation*. Multiscale Model. Simul., to appear (2024). arxiv.org/abs/2212.09291.

[2] T. Freudenberg and M. Eden, *Homogenization and simulation of heat transfer through a thin grain layer*. Submitted. arxiv.org/abs/2312.02704.

[MS10] Boundary Layer Estimates in Stochastic Homogenization

Claudia Raithel

TU Dresden

Tuesday, 9:45, 1D236

This is a joint work with P. Bella (TU Dortmund, Germany), J. Fischer (ISTA, Austria), and M. Josien (CEA Cadarache, France).

We are interested in quantifying the boundary layer in the stochastic homogenization of linear elliptic PDEs. In particular, we obtain optimal decay of the boundary layer tails. As an application of this decay, we are able to show the optimal fluctuations-scaling of the error in the Representative Volume Element (RVE) method for approximating the homogenized coefficients (without the presence of a screening term).

[MS10] A sequential global programming approach for two-scale optimization of homogenized multiphysics problems with application to Biot porous media

Michael Stingl

Friedrich-Alexander Universität Erlangen - Nürnberg

Tuesday, 10:15, 1D236

This is a joint work with Prof. Eduard Rohan and Dr. Vladimir Lukeš (University of West Bohemia in Pilsen, Czech Republic) as well as Dr. Bich Ngoc Vu (Friedrich-Alexander Universität Erlangen-Nürnberg, Germany).

We present a new approach and an algorithm for solving two-scale material optimization problems to optimize the behaviour of a fluid-saturated porous medium in a given domain. While the state problem is governed by the Biot model describing the fluid-structure interaction in homoge- nized poroelastic structures, the approach is widely applicable to multiphysics problems involving several macroscopic fields in which homogenization provides the relationship between the microcon- figurations and the macroscopic mathematical model. The optimization variables describe the local microstructure design by virtue of the pore shape which determines the effective medium proper- ties, namely the material coefficients, computed by the homogenization method. The numerical optimization strategy involves a) precomputing a database of the material coefficients associated with the geometric parameters, and b) applying the sequential global programming (SGP) method for solving the problem of macroscopically optimized distribution of material coefficients. Due to the flexibility of the SGP approach, different types of microstructures with fully independent parametrizations can easily be handled. The efficiency of the concept is demonstrated by a series of numerical experiments that show that the SGP method can simultaneously handle multiple types of microstructures with nontrivial parametrizations using a considerably low and stable number of state problems to be solved.

[MS10] Multiscale methods with applications in electrophysiology

Irina Pettersson

Chalmers University of Technology and University of Gothenburg

Wednesday, 8:45, 21A342

This is a joint work with V. Rybalko and A. Rybalko (Chalmers University of Technology and University of Gothenburg, Sweden).

Electrophysiology is a branch of physiology studying electric properties of biological cells and tissues. The electrical response of nerves or other biological cells is modelled by coupled systems of partial differential equations. The typical size of cells or thickness of axons in a nerve bundle is small compared with the size of the domain, and so such problems become computationally heavy. Deriving a macroscopic problem using the methods of asymptotic analysis and homogenization provides one with a more feasible model. The talk focuses on the derivation of effective models for nerve bundles both in periodic and random setting.

It was suggested in [1], that signal propagation in bundled axons can be, similar to the cardiac tissue, described by an effective bidomain model, and a rigorous derivation of it was carried out in [2] in periodic setting. In the present work we extend the results and the methods of [2] to the case of randomly distributed axons. We adopt the FitzHugh-Nagumo dynamics on the interfacial membrane. Introducing a small parameter $\varepsilon > 0$ describing the microscale of the fascicle, we assume that both the radii of axons and the distances between neighboring ones are of the order ε. Combining the techniques of homogenization in media with random geometry (see, e.g., [3], [4]) and the Minty monotonicity method, we derive an effective problem of bidomain type for signal propagation when ε is sufficiently small. Since the axons are disjoint inside the bundle, the effective conductivity of intracellular part degenerates in directions orthogonal to axons.

[1] E. Mandonnet, and O. Pantz, *The role of electrode direction during axonal bipolar electrical stimulation: a bidomain computational model study* Acta neurochirurgica 153 (2011), 2351-2355.

[2] C. Jerez-Hanckes, I. A. Martínez Ávila, I. Pettersson, and V. Rybalko, *Derivation of a bidomain model for bundles of myelinated axons.* Nonlinear Analysis: Real World Applications 70 (2023): 103789.

[3] S. Wright, A. Mikelic, and A. Bourgeat, *Stochastic two-scale convergence in the mean and applications.* (1994): 19-52.

[4] A. Piatnitski, and M. Ptashnyk, *Homogenization of biomechanical models of plant tissues with randomly distributed cells.* Nonlinearity 33, no. 10 (2020): 5510.

[MS10] Homogenization of a Signorini type problem in a domain with inclusions

Sara Monsurrò

University of Salerno

Wednesday, 9:15, 21A342

This is a joint work with Carmen Perugia (University of Sannio, Benevento, Italy) and Federica Raimondi (University of Salerno, Fisciano (Sa), Italy)

Aim of the talk is to describe some homogenization results for Signorini's type problems in domains with inclusions. The Signorini condition is expressed in terms of two complementary interfacial conditions involving the jump of the solution on the interface and its conormal derivatives via a parameter γ. According to different values of the parameter different limit behaviours are obtained.

[MS10] Homogenization of a reaction-diffusion model involving moving boundaries at the micro scale

Sorin Pop

Hasselt University

Wednesday, 9:45, 21A342

This is a joint work with M. Gahn (University of Heidelberg, Germany), M. Neuss-Radu (University of Erlangen, Germany), M.A. Peter (Augsburg University, Germany), David Wiedemann (University of Dortmund, Germany).

Reactive transport in evolving complex media occur in a variety of real-life applications related to e.g. porous media. We mention here mineral precipitation and dissolution, biofilm growth, colloid deposition, or water diffusion into absorbent particles. Referring to the former, the medium consists of alternating solid grains and voids. The latter is filled by a fluid, wherein solute species are transported by diffusion. These species can precipitate at the fluid-solid interface to form a solid mineral layer (e.g. salt), the reverse process of dissolution being also possible. Around each grain, the mineral layer has a variable, a priori unknown thickness, which means that the solid microstructure is evolving, the fluid-solid interface being a free boundary.

We discuss the rigorous homogenization for a reaction-diffusion model defined in a complex domain with an evolving microstructure. In the initial state, the microscopic domain is periodically perforated, each perforation representing a spherical solid grain. We assume that the distance between two neighboring grains, as well as the radii of the grains, are of the order ϵ (a small, positive parameter). We assume that, around each grain, the evolution of the fluid-solid interface is radially symmetric, but the radius depends on the unknown (the solute concentration) at its surface. Therefore, these radii are model unknowns. By employing a transformation of the evolving microscopic domain to a fixed, periodic domain, we prove the existence of a weak solution and obtain a priori estimates that are uniform with respect to ϵ. Finally, letting $\epsilon \to 0$, we derive a macroscopic model, the solution of which approximates the micro-scale solution. For this, we use the method of two-scale convergence, and obtain strong compactness results enabling us to pass to the limit in the nonlinear terms.

[1] M. Gahn, I.S. Pop, *Homogenization of a mineral dissolution and precipitation model involving free boundaries at the micro scale.* J. Differential Equations **343** (2023), 90–151.

[2] D. Wiedemann, M.A. Peter *Homogenisation of local colloid evolution induced by reaction and diffusion.* Nonlinear Anal. **227** (2023), 113168.

[MS10] Contructible Approximate Cloaks by De-homogenization

YVES CAPDEBOSCQ

Université Paris Cité

Wednesday, 10:15, 21A342

This is a joint work with Eleanor Gemida (Université Paris Cité).

In a smooth bounded domain $\Omega \subset \mathbb{R}^d$, with $B_2 := \{x : \|x\| \leq 2\} \subset \Omega$, the Dirichlet-to-Neumann map associated to a positive definite matrix-valued functions $A \in L^\infty\left(\Omega; \mathbb{R}^{d\times d}\right)$ is

$$\Lambda_A : H^{\frac{1}{2}}(\partial\Omega) \to H^{-\frac{1}{2}}(\partial\Omega)$$

$$\phi \to A\nabla u \cdot n : \begin{cases} -\operatorname{div}(A\nabla u) = 0 & \text{in } \Omega, \\ u = \phi & \partial\Omega. \end{cases}$$

For $\rho \in]0,1[$ a ρ-approximate cloak for the conductivity equation is a matrix-valued functions $a_\rho \in L^\infty\left(\Omega \setminus B_1;;\mathbb{R}^{d\times d}\right)$ such that for any $\gamma \in L^\infty\left(B_1;\mathbb{R}^{d\times d}\right)$, with $A_{\gamma,\rho} = \begin{cases} a_\rho & \text{in } \Omega \setminus B_1 \\ \gamma & \text{in } B_1 \end{cases}$ there holds $\|\Lambda_{A_{\gamma,\rho}} - \Lambda_{I_d}\| \leq C\rho^d$, where C is independent of γ. The celebrated cloaking-by-mapping approach has provided a method for constructing such approximate cloaks mathematically. The conductivity coefficient a_ρ is both anisotropic and space dependent, and can be thought of as a composite (meta)-material. In this presentation we introduce piecewise isotropic ρ-approximate cloaks with some optimality properties.

[MS10] Hypocoercivity for non-autonomous linear evolution equations: an operator-theoretic approach

Artur Stephan

Technische Universität Wien

Thursday, 16:45, 1D222

Motivated by a recent result of *Achleitner et al* [1], we investigate the notion of *hypocoercivity* for linear evolution equations, where the generator of the evolution depends explicitly on time. Here, we consider a family of bounded operators $\{A(t)\}_{t\geq 0}$ on a Hilbert space H that are assumed to be uniformly bounded in time. To track the explicit time-dependence, we utilize the Howland-Evans-Neidhardt approach to formulate the problem time-independently on the space $L^2([0,T],H)$ including time and space. As it turns out the desired solution operator (or propagator) is obtained by a semigroup generated by a sum $K = D_0 + \mathcal{A}$, where D_0 generates the shift-semigroup and $\mathcal{A}$ is the multiplication operator induced by the operator family $\{A(t)\}_{t\geq 0}$. Moreover, estimates for the propagator translate to estimates on the semigroup $e^{-\tau K}$ (and vice versa), and hence, allow to answer questions regarding hypocoercivity now in a time-independent setting.

In my talk, we focus on quantitative result for the short-time behavior of the evolution. For this we rely on an explicit formula for the semigroup $e^{-\tau K}$ by the so-called Dyson-Phillips series, and, moreover, on calculations from [1].

[1] F. Achleitner, A. Arnold, V. Mehrmann, E.A. Nigsch, *Hypocoercivity in Hilbert spaces*,

arXiv:2307.08280v1, July 2023.

[MS10] Homogenization of an eigenvalue problem through rough surfaces

Federica Raimondi

University of Salerno

Thursday, 15:15, 1D222

This is a joint work with Prof Sara Monsurrò (University of Salerno, Salerno, Italy) and PhD Jake Avila (University of the Philippines Diliman, Quenzon City, Philippines).

In Raimondi:article we study the asymptotic behaviour of the spectrum and its associated eigenspaces for a stationary heat propagation problem posed in a bounded cylinder with a rough interface. The main novelty concerns the proof of the uniform a priori estimates for the eigenvalues. In fact, due to the peculiar geometry of the domain, standard techniques do not apply and a suitable new approach is developed.

[1] J. Avila, S. Monsurrò, F. Raimondi, *Homogenization of an eigenvalue problem through rough surfaces* Asymptot. Anal. (2023), 1–25.

[MS10] Asymptotically based optimization of flow-induced deformation of periodic flexural structures made of thin yarns

Julia Orlik

Fraunhofer ITWM, Kaiserslautern

Thursday, 15:45, 1D222

This is a joint work with Dr. Maxime Krier (Fraunhofer ITWM, Germany), Prof. Grigory Panasenko (Univ. St. Etienne, France).

The two-way coupled fluid-structure interaction of Stokes flow through a linear elastic, thin porous structure is considered. The micro-resolved structure is deterministic with an in-plane periodicity of ε and consists of long thin yarns in contact. It can be described by a handful of geometrical and mechanical parameters such as distances between yarns, friction coefficients, linear material properties and yarn's cross-sections. Such problems typically arise e.g. in filtration applications with woven filter media.

Due to drastically differing length scales between fluid domain and the micro-resolved structure, direct numerical approaches for the problem are impractical in most scenarios. An asymptotic model was derived in [1–3] for the limit $\varepsilon \to 0$ by means of two-scale convergence. In the asymptotic limit, the problem is reduced to 3D Stokes flow coupled to a 2D Kirchhoff plate with non-standard interface coupling: the plate's vibration is proportional to the jump of fluid stresses across the plate.

The plate's effective elasticity tensors are determined by auxiliary problems on the smallest periodic unit of the structure, so called cell problems [4, 5]. The model is augmented by an interface flux term obeying Darcy's law to incorporate mass transport through the plate. The corresponding permeability tensor is attained by Stokes flow problems in the micro resolved periodic unit.

An optimization problem of tracking type is considered: The microscopic design of the structure is to be chosen such that a desired flow-induced deformation profile of the structure in steady-state is reached. The deformation profile is directly linked to the structure's filtration efficiency and is therefore of high interest in industrial application. An adjoint based optimization approach is considered. In this presentation, a complete overview over the numerical workflow is given. On the micro-scale, the elasticity cell problems are solved utilizing a dimension reduction approach to 1D beams with additional contact conditions [5], while the permeability tensor is attained by solving three Stokes problems. On the macro-scale, a monolithic FEM formulation for the fluid-structure interaction is employed both for the arising forward and adjoint systems.

[1] J. Orlik, G. Panasenko, R. Stavre: Asymptotic analysis of a viscous fluid layer separated by a thin stiff stratified elastic plate, Applicable Analysis, **100(3)**, 589-629, (2021)

[2] M. Gahn, W. Jäger, M. Neuss-Radu: Derivation of Stokes-plate-equations modeling fluid flow interaction with thin porous elastic layers, Applicable Analysis, **101(12)**, 4319-4348, (2022)

[3] G. Griso, J. Orlik, S. Wackerle: Asymptotic behavior for textiles in von-Kármán regime Journal de Mathématiques Pures et Appliquées, **144**, 164-193, (2020)

[4] G. Griso, L. Khilkova, J. Orlik, O. Sivak: Asymptotic Behavior of Stable Structures Made of Beams. J. Elast (2021). https://doi.org/10.1007/s10659-021-09816-w

[5] J. Orlik, G. Panasenko, V. Shiryaev: Optimization of textile-like materials via homogenization and dimension reduction. SIAM Multiscale Model. Simul., **14(2)**, 637–667 (2016)

[MS10] Multi-continuum homogenization and odd elasticity

Grigor Nika

Karlstad University

Thursday, 16:15, 1D222

We derive an effective system from a periodically heterogeneous Cosserat continuum. The homogenization process encompasses intrinsic lengths related to scale-size effects and leads to an asymmetric effective stress related to concept of *odd elasticity*. The corresponding local problem exhibits asymmetry as well, due to the micropolar couple modulus inherited from the original microscopic Cosserat problem. We validate our results by conducting numerical simulations using the finite element method on circularly perforated square and rectangular unit cells, highlighting the impact of the micropolar couple modulus on effective coefficients. Furthermore, we numerically show that there exists a threshold micropolar couple modulus which affects the behavior of the cell and compare the cell displacement in the presence and absence of a micropolar couple modulus.

[1] G. Nika, *Derivation of effective models from heterogenous Cosserat media via periodic unfolding.* Ricerche mat. **73** (2024) 381–406.

[2] G. Nika, *On a hierarchy of effective models for the biomechanics of human compact bone tissue.* IMA J. App. Math. **88**(2) (2023), 282-307.

[MS12] Stability and Periodicity in Heat-Wave Systems

Justin Webster

University of Maryland, Baltimore County

Tuesday, 15:15, 1B306

Time-periodic behaviors arise in multi-physics systems as (i) responses to periodic forcing (blood flow in human tissues) or (ii) emergent "self-excitations" (aeroelastic flutter). Motivated by fluid-structure interactions [4,5], we consider weak solutions to an idealized heat-wave interaction, with physical coupling conditions across lower dimensional interface, with time-periodic forcing.

For undamped hyperbolic systems, one must avoid resonance when seeking periodic solutions [1,3]. For parabolic systems, the theory of periodic existence and uniqueness well-understood [3, 4]. In the heat-wave case, the system is only partially damped yielding an indeterminate case. Indeed, owing to the work Galdi et al. [1], there is a deep connection between the stability type of the homogeneous solution semigroup and the existence and uniqueness of periodic solutions for finite energy time-periodic forcing. For our heat-wave system, the underlying semigroup for the unforced problem is polynomial stabile, but not exponentially stable [2]. Thus the periodic behaviors of the system are not fully characterized by the existing abstract theory [1].

In this talk, we provide an explicit construction of a unique periodic solution for our partially damped heat-wave system. Without dissipation in the wave component, a priori estimates for the wave must be recovered through the interface via heat dissipation. This is done through boundary control estimates, which introduce geometric constraints. We demonstrate that for certain classes of domains, unique periodic solutions can be obtained from temporally smooth forcing, thereby eliminating the possibility of systemic resonance. The regularity loss between the forcing and solution depends on the wave domain's geometry, and, owing to time-periodicity in the dynamics, no additional spatial regularity is required of the forcing. It remains an open question whether the geometric control constraints are technical (arising from the method), or if such restrictions are necessary to circumvent resonance. This is joint work with B. Muha and S. Schwarzacher.

[1] Galdi, G.P., Mohebbi, M., Zakerzadeh, R. and Zunino, P., 2014. Hyperbolic-Parabolic Coupling and the Occurrence of Resonance in Partially Dissipative Systems. **Fluid-structure Interaction and Biomedical Applications**, pp.197–256. *Springer*, Basel.

[2] Zhang, X. and Zuazua, E., 2007. Long-time behavior of a coupled heat-wave system arising in fluid-structure interaction. *Archive for Rational Mechanics and Analysis*, 184, pp.49–120.

[3] Bostan, M., 2002. *Periodic Solutions for Evolution Equations*. Department of Mathematics, Texas State University-San Marcos.

[4] Casanova, J.J., 2019. Existence of time-periodic strong solutions to a fluid-structure system. *Discrete and Continuous Dynamical Systems*, 39(6), pp.3291–3313.

[5] Kaltenbacher, B., Kukavica, I., Lasiecka, I., Triggiani, R., Tuffaha, A. and Webster, J.T., 2018. **Mathematical Theory of evolutionary Fluid-Flow Structure Interactions**. *Birkhäuser*.

[MS12] Periodic motions of a harmonic oscillator interacting with incompressible fluids

Giusy Mazzone

Queen's University

Tuesday, 15:45, 1B306

This is a joint work with Mahdi Mohebbi (Queen's University, Kingston, Canada).

We consider the interactions between a harmonic oscillator (consisting of a mass-spring system) and a fluid occupying an infinite three-dimensional channel. The fluid flow is governed by the Navier-Stokes equations subject to a prescribed time-periodic flow rate. External time-periodic body forces (with the same period as the flow rate) may also be applied on the fluid and on the mass. We show that, with no restriction on the period of the flow rate (and of the external forces), when the flow rate is "small", there exists a weak time-periodic solution to the coupled fluid-oscillator system. Under some more regularity and "smallness" conditions on the flow rate (and on the external forces) we also show that these solutions are, indeed, strong solutions.

[1] G. Mazzone and M. Mohebbi, *On periodic motions of a harmonic oscillator interacting with incompressible fluids* (submitted) (2024), 22pp. https://arxiv.org/abs/2312.08690

[MS12] The effect of time-periodic boundary flux on the decay of viscous flow past a body

Thomas Eiter

University of Kassel

Tuesday, 16:15, 1B306

This is a joint work with Ana Leonor Silvestre (Universidade de Lisboa, Lisbon, Portugal).

We study the spatial decay of time-periodic flow of an incompressible viscous fluid past an obstacle that moves with constant non-zero velocity. Using the time-periodic fundamental solution to the associated linearized problem in the whole space, we first derive representation formulas for weak solutions under suitable regularity assumptions. These formulas lead to asymptotic expansions of the velocity and the pressure field at spatial infinity, and thus to sharp pointwise decay estimates of these quantities. Comparing the results with the cases of steady flow past a body or of time-periodic flow in the whole space, we observe a slower decay rate of the pressure and of the oscillatory part of the velocity. The asymptotic expansions show that this phenomenon depends on the flux through the boundary, and that a time-independent total flux induces faster decay, which is of the same rate as known for these two special cases.

[MS12] Free compressible inviscid model of flow-structure interaction

Igor Kukavica

University of Southern California

Tuesday, 16:45, 1B306

This is a joint work with Professors Šárka Nečasová and Amjad Tuffaha.

We consider a system that describes the interaction between a compressible inviscid fluid, modeled by the compressible Euler equations, and an elastic plate, represented by a fourth-order hyperbolic PDE. We investigate the local existence of solutions in spaces of minimal regularity. We will also discuss the incompressible case.

[MS12] On the long-time behaviour of solutions to unforced evolution Navier-Stokes equations under Navier boundary conditions

Alessio Falocchi

Politecnico di Milano

Wednesday, 8:45, 1D222

This is a joint work with Prof. Elvise Berchio (Politecnico di Torino, Italy) and Clara Patriarca (Université libre de Bruxelles, Belgium).

We study the asymptotic behaviour of the solutions to Navier-Stokes unforced equations under Navier boundary conditions in a wide class of merely Lipschitz domains of physical interest that we call *sectors*. The main motivations come from the celebrated results by Foias-Saut [1] related to the long time behaviour of the solutions to Navier-Stokes equations under Dirichlet conditions.

Here the choice of the boundary conditions requires carefully considering the geometry of the domain Ω, due to the possible lack of the Poincaré inequality in presence of axial symmetries. In non-axially symmetric domains we show the validity of the Foias-Saut result about the limit at infinity of the Dirichlet quotient, in axially symmetric domains we provide two invariants of the flow which completely characterize the motion and we prove that the Foias-Saut result holds for initial data belonging to one of the invariants.

[1] C. Foias, J.C. Saut, *Asymptotic behavior, as $t \to +\infty$ of solutions of Navier-Stokes equations and nonlinear spectral manifolds*, Indiana Univ. Math. J. 33, 3, 459–477 (1984)

[MS12] Rigorous elastohydrodynamic lubrication approximation

Mario Bukal

University of Zagreb

Wednesday, 9:15, 1D222

Motivated by widespread applications of microfluidic channels and chips, we consider an interaction between a thin layer of an incompressible viscous fluid and an elastic structure. Starting from fluid-structure interaction (FSI) problems, our aim is to rigorously derive corresponding reduced models, which are favorable in engineering applications. We formulate this as a singular limit problem in terms of the vanishing relative fluid thickness, which is carried out based on suitable energy estimates and uniform no-contact results for FSI systems. Reduced models are justified in terms of weak convergence results in the sense that weak limits of solutions to the FSI problem are identified in a relation with solutions of the reduced model. The talk is based on joint work [1–3] with Boris Muha (University of Zagreb) and ongoing collaboration with Igor Kukavica (University of Southern California) and Boris Muha (University of Zagreb).

[1] M. Bukal and B. Muha, *A review on rigorous derivation of reduced models for fluid-structure interaction systems.* Waves in Flows, Advances in Mathematical Fluid Mechanics (Eds. T. Bodnár, G. P. Galdi, Š. Nečasová), 2021, pp. 171–199, Birkhäuser.

[2] M. Bukal and B. Muha, *Rigorous Derivation of a Linear Sixth-Order Thin-Film Equation as a Reduced Model for Thin Fluid-Thin Structure Interaction Problems*, Appl. Math. Optim. 84 (2021), 2245–2288.

[3] M. Bukal and B. Muha, *Justification of a nonlinear sixth-order thin-film equation as the reduced model*

for a fluid-structure interaction problem, Nonlinearity 35 (2022), 4695–4726.

[MS12] Numerical simulations of mass-diffusive compressible and nearly incompressible fluids flows models

Tomas Bodnar

Czech Technical University

Wednesday, 9:45, 1D222

This contribution presents first numerical tests of some recently published alternative models for solution of viscous compressible and nearly incompressible models. All models are solved by high resolution compact finite difference scheme with strong stability preserving Runge-Kutta time stepping. Three simple but challenging computational test cases are presented, based on the double-periodic shear layer, Taylor-Green vortex and the Kelvin-Helmholtz instability. The obtained time-dependent flow fields are showing pronounced shear and vorticity layers being resolved by the standard as well as by the new mass-diffusive modified models. The preliminary results show that in some cases the new models are a viable alternative to the well established classical models.

Recently there have been attempts to revise and possibly improve the traditional mathematical models describing the fluids flows. The works of Brennen [2] and Svärd [5] are example of such possible model updates. In these new models, the basic physical principles (conservation/balance laws) are still being used, but the interpretation of certain physical variables and processes brings other options for into the mathematical formulations of such revised models. These changes are bringing some interesting results from the point of view of mathematical analysis of the corresponding models as well as possible increase in the efficiency of numerical methods [3], [4], [1].

[1] T. Bodnár and Ph. Fraunié, Numerical evaluation of mass-diffusive compressible fluids flows models. In: Topical Problems of Fluid Mechanics 2024, pages 23–30, Prague, 2024. Institute of Thermomechanics CAS.

[2] H. Brenner. Navier–Stokes revisited. *Physica A*, 349:60–132, 2005.

[3] V. Dolejší and M. Svärd. Numerical study of two models for viscous compressible fluid flows. *Journal of Computational Physics*, 427:110068, 2021.

[4] A. Kajzer and J. Pozorski. The mass diffusive model of Svärd simplified to simulate nearly incompressible flows. *Computers and Mathematics with Applications*, 121:18–29, 2022.

[5] M. Svärd. A new Eulerian model for viscous and heat conducting compressible flows. *Physica A*, 506:350–375, 2018.

[MS12] Numerical analysis of a heat–wave system with added mass

Marija Galić

University of Zagreb

Wednesday, 10:15, 1D222

Fluid–structure interaction (FSI) problems are multi–physics problems which arise in many applications, e.g. aeroelasticity and biomedicine. Since they are often too complex to be solved analytically it can be useful to consider corresponding simplified models which, in some sense, share the same properties with the original problem. For this reason, we consider a heat–wave system with added mass which consists of the linear heat equation and the linear wave equation coupled through a common interface with mass. In this talk, we present the comparison between two different approaches in numerical analysis of the problem: a monolithic approach where the entire coupled system is solved as one monolithic system and a partitioned approach where the coupled problem is partitioned into subproblems, following the different physics in the problem, i.e. parabolic and hyperbolic nature of the equations involved.

[1] M. Bukac and B. Muha, *Stability and convergence analysis of the extensions of the kinematically coupled scheme for the fluid–structure interaction.* SIAM J. Numer. Anal. 54(5): 3032–3061, 2016.

[2] B. Muha, *A note on optimal regularity and regularizing effects of point mass coupling for a heat-–wave system.* J. Math. Anal. Appl. 425(2): 1134–1147, 2015.

[3] X. Zhang and E. Zuazua, *Long-time behavior of a coupled heatwave system arising in fluid-structure interaction.* Arch. Ration. Mech. Anal. 184(1): 49—120, 2007.

[MS12] On a toy model for fluid-structure interaction problems with contact

Sridan Trifunovic

Faculty of Sciences, University of Novi Sad, Serbia

Thursday, 15:15, 1D226

Joint work with professor Boris Muha from the Faculty of Sciences of the University of Zagreb.

In this talk, I will present a recent work on a simple model that is used to study contact problem for the interaction between a beam and a fluid. The model consists of a viscoelastic beam described by function η denoting the vertical displacement. The beam is located above an obstacle $y = 0$ and satisfies the equation

$$\partial_{tt}\eta - \partial_{xx}\eta - \partial_{txx}\eta = F_{con} \tag{15}$$

where F_{con} represents the contact force which acts only on the set $\{\eta = 0\}$ and ensures that $\eta \geq 0$. The contact is assumed to be completely inelastic, i.e. the beam loses all kinetic energy upon contact. For this problem, a weak solution is constructed by approximative method – the contact force is approximated with a penalizing term $\frac{1}{\varepsilon}\chi_{\{\eta<0\}}(\partial_t\eta)^-$ which does not depend on the distance from the obstacle. The two main novelties are that the following properties are preserved for the constructed limiting solution as $\varepsilon \to 0$:

1. The dissipation due to contact for the constructed weak solution is a singular measure which only exists when the beam is going down (it is zero on $\text{int}\{\partial_t\eta \geq 0\}$);
2. The velocity after contact is zero in a weak sense.

Some numerical examples are also included which demonstrate certain properties and difficulties considering the contact set $\{\eta = 0\}$ and the contact force F_{con}, and their behavior throughout the convergence $\varepsilon \to 0$.

Acknowledgments: This research was supported by the Science Fund of the Republic of Serbia, GRANT No TF C1389-YF, Project title - FluidVarVisc.

[MS12] Multiscale Interface Coupling of PDEs and ODEs for Tissue Perfusion

Lorena Bociu

NC State University

Thursday, 15:45, 1D226

This is a joint work with Matthew Broussard (NC State University, USA), Giovanna Guidoboni (University of Maine, USA), Daniele Prada (Consiglio Nazionale delle Ricerche, Pavia, Italy), Riccardo Sacco (Politecnico di Milano, Italy) and Sarah Strikwerda (University of Pennsylvania, USA).

In biomechanics, local phenomena, such as tissue perfusion, are strictly related to the global features of the whole blood circulation. We propose a heterogeneous model where a local, accurate, 3D description of tissue perfusion by means of poroelasticity equations is coupled with a systemic 0D lumped model of the remainder of the circulation. This represents a multiscale strategy, which couples an initial boundary value problem to be used in a specific tissue region with an initial value problem in the rest of the circulatory system. We discuss well-posedness analysis for this multiscale interface model, as well as solution methods focused on a detailed comparison between functional iterations and an operator splitting method and how they handle the interface conditions.

[MS12] Homogenization of Leray's flux problem for the steady-state Navier-Stokes equations in a multiply-connected planar domain

Gianmarco Sperone

Politecnico di Milano

Thursday, 16:15, 1D226

The steady motion of a viscous incompressible fluid in a multiply-connected, planar, bounded domain (perforated with a large number of small holes) is modeled through the Navier-Stokes equations with non-homogeneous Dirichlet boundary data satisfying the general outflow condition. Under either a symmetry assumption on the data or under a smallness condition on each of the boundary fluxes (therefore, no constraints on the magnitude of the boundary velocity

are imposed), we apply the classical energy method in homogenization theory and study the asymptotic behavior of the solutions to this system as the size of the perforations goes to zero: it is shown that the effective equations remain unmodified in the limit. The main novelty of the present work lies in the obtainment of the required uniform bounds, which are achieved by a contradiction argument based on Bernoulli's law for solutions of the stationary Euler equations. This is a joint work with Clara Patriarca (Université Libre de Bruxelles, Belgium).

[MS12] Motion of a large number of small rigid bodies in a viscous incompressible fluid

Arnab Roy

TU Darmstadt

Thursday, 16:45, 1D226

This is a joint work with Prof Eduard Feireisl (Institute of Mathematics, Czech Academy of Sciences, Prague, Czech Republic) and Prof. Arghir Zarnescu (Basque Center for Applied Mathematics, Bilbao, Spain).

In this talk, we consider the motion of several small rigid bodies immersed in a viscous incompressible fluid contained in a domain. We show that the fluid flow is not influenced by the presence of the infinitely many bodies in the asymptotic limit. The result depends solely on the geometry of the bodies and is independent of their mass densities.

[MS13] Hardy-Sobolev interpolation inequalities

Charlotte Isabel Dietze

LMU Munich

Tuesday, 15:15, 1D222

We derive a family of interpolation estimates which improve Hardy's inequality and cover the Sobolev critical exponent. We also determine all optimizers among radial functions in the endpoint case and discuss open questions on nonrestricted optimizers.

[MS13] Non-degeneracy, stability and symmetry for the fractional Caffarelli–Kohn–Nirenberg inequality

Tobias König

Goethe University

Tuesday, 15:45, 1D222

The fractional Caffarelli–Kohn–Nirenberg (CKN) inequality states that

$$\int_{\mathbb{R}^n}\int_{\mathbb{R}^n} \frac{(u(x)-u(y))^2}{|x|^\alpha |x-y|^{n+2s}|y|^\alpha}\,\mathrm{d}x\,\mathrm{d}y \geq \Lambda_{n,s,p,\alpha,\beta}\left[u|x|^{-\beta}\right]^2_{L^p},$$

for $0 < s < \min\{1, n/2\}$, $2 < p < 2^*_s$, and $\alpha, \beta \in \mathbb{R}$ so that $\beta - \alpha = s - n(\frac{1}{2} - \frac{1}{p})$ and $-2s < \alpha < \frac{n-2s}{2}$.

Even in the classical ($s = 1$) case, the exact parameter region of symmetry-breaking for minimizers of CKN has only been determined fairly recently in seminal work by Dolbeault et al. (2016).

Continuing the program started in Ao et al. (2022), we establish the non-degeneracy and sharp quantitative stability of minimizers for $\alpha \geq 0$ in the fractional case. Furthermore, we show that minimizers remain symmetric when $\alpha < 0$ for p very close to 2. As a key proof ingredient for the non-degeneracy, we discover a Hardy-type inequality for a general class of radial weights that might be of independent interest.

This is joint work with Nicola De Nitti (EPF Lausanne) and Federico Glaudo (Princeton).

[MS13] Sobolev improvements on sharp Rellich inequalities

Gerassimos Barbatis

National and Kapodistrian University of Athens

Tuesday, 16:15, 1D222

There are two main Rellich inequalities for the bilaplacian, the one involving $|\nabla u|$ and the other involving $|u|$ at the RHS. In this talk we present some new sharp Sobolev-type improvements of these inequalities. More precisely, in our first result we improve the Rellich inequality with $|\nabla u|$ obtained by Beckner in dimensions $n = 3, 4$ by a sharp Sobolev term thus complementing existing results for the case $n \geq 5$. In the second theorem the sharp constant of the Sobolev improvement for the Rellich inequality with $|u|$ is obtained. This is joint work with A. Tertikas.

[MS13] A Schiffer-type problem with applications to stationary Euler flows

Antonio J. Fernández

Universidad Autónoma de Madrid

Tuesday, 16:45, 1D222

If on a smooth bounded domain $\Omega \subset \mathbb{R}^2$ there is a nonconstant Neumann eigenfunction u that is locally constant on the boundary, must Ω be a disk or an annulus? This question can be understood as a weaker analog of the well-known Schiffer conjecture, in that the function u is here allowed to take a different constant value on each connected component of $\partial\Omega$ yet many of the known rigidity properties of the original problem are essentially preserved. In this talk we provide a negative answer by constructing a family of nontrivial doubly connected domains Ω with the above property. Then, we will show how our construction implies the existence of continuous, compactly supported stationary weak solutions to the 2D incompressible Euler equations which are not locally radial. The talk is based on a joint work with Alberto Enciso, David Ruiz and Pieralberto Sicbaldi.

[MS13] Degenerate Boundary Value Problems

Gianmarco Brocchi

Chalmers and University of Gothenburg

Thursday, 8:45, 1D226

In Boundary Value Problems (BVPs) one aims to understand solutions to a differential equation (the problem) under some constraint (value at the boundary).

On smooth domains in Euclidean space, BVPs can be attacked with Fourier methods: the Fourier transform provides us with a representation of the solution. What happens when the boundary of our domain becomes rough and symmetries are lost? Can we still find a way to describe solutions and trace space when Fourier methods break down? Ultimately: how do solutions (and these methods) depend on small perturbations of the boundary?

In this talk I will introduce a “first order approach” for divergence form equations $-\mathrm{div} A\nabla u = 0$, which relates harmonic extensions from the real line and holomorphic functions. This relation works in higher dimensions as well, and allows us to rewrite our problem in a suitable way so that holomorphic functional calculus can be applied.

We will take a closer look at degenerate BVPs: when the coefficient $A(x)$ of our divergence form equation lacks uniform boundedness and accretivity, and can exhibit singularities. Current state-of-the-art results can handle singularities characterised by scalar Muckenhoupt weights. These results have been extended on manifolds satisfying some curvature assumption. But even on flat Euclidean space, anisotropic degenerate coefficients have been out of reach, due to the lack of off-diagonal estimates. Is there another way to handle more general matrix-degenerate coefficients?

In a joint work with Andreas Rosén, we exploit a new metric to prove the first "anisotropic degenerate" Kato square root estimate.

[1] Pascal Auscher, Andreas (Axelsson) Rosén, and Alan McIntosh, *Solvability of elliptic systems with square integrable boundary data*, Ark. Mat., **48** (2010)

[2] P. Auscher, A. Rosén, and D. Rule, *Boundary Value Problems for degenerate Elliptic equations and systems*, Ann. Sci. Éc. Norm. Supér. (4) **48**, No. 4, 951-1000 (2015)

[3] Pascal Auscher, Andrew J. Morris, and Andreas Rosén, *Quadratic Estimates for Degenerate Elliptic Systems on Manifolds with Lower Ricci Curvature Bounds and Boundary Value Problems*, arXiv:2209.11529, (2022)

[4] Brocchi, Gianmarco, and Andreas Rosén. *The metric for matrix degenerate Kato square root operators* arXiv:2404.09580, (2024).

[MS13] Regularity of domain walls in optimal partition problems

Roberto Ognibene

Università di Pisa

Thursday, 9:15, 1D226

Let us consider a bounded domain, divided into a fixed number of disjoint subdomains and, among all the possible configurations, let us consider the one for which the sum of the first

Dirichlet eigenvalues of the subdomains is minimal. In this talk, I will discuss the regularity of the interface which emerges as boundary of such optimal partition and, in particular, I will focus on the regularity up to the fixed boundary. The talk is based on a joint work with B. Velichkov.

[MS13] Ratio convergence rates for infinity harmonic functions

Leon Bungert
University of Würzburg
Thursday, 9:45, 1D226

In this talk I will present two examples how the newly developed concept of ratio convergence can be used to prove explicit convergence rates for approximations of solutions of the infinity-Laplace equation. The first example features the p-Laplace equation, the second one the discrete infinity-Laplace equation on a graph with n vertices. In both cases one can show that solutions satisfy a comparison principle with certain non-Euclidean distance functions. Asymptotically (meaning for $p \to \infty$ or $n \to \infty$, respectively) the quotient of this distance from the center of a ball of radius $r > 0$ to its boundary divided by the same distance for the radius $2r$ converges to $\frac{1}{2}$ with explicit rates. This information can be combined with the "comparison with cones" property of infinity-harmonic functions and suitable strictness transforms to prove convergence rates. In the case of the p-Laplace equation the rate scales like $p^{-\frac{1}{4}}$, for the graph infinity-Laplace equation like $n^{-\frac{1}{9d}}$ up to logarithmic factors, where d is the dimension of the ambient space.

[MS13] Asymptotic behavior of solutions of problems involving power nonlinearities

Francesca De Marchis
Sapienza University of Rome
Thursday, 10:15, 1D226

We discuss, highlighting analogies and differences, the asymptotic behavior both of solutions to the classical Lane Emden equation both to a linear elliptic equation subject to a Neumann boundary condition involving a power nonlinearity. Based on joint papers with M. Grossi, H. Fourti, I. Ianni and F. Pacella.

[MS14] Lorenz-like attractors in systems with central and axial symmetries

Efrosiniia Karatetskaia

National Research University Higher School of Economics

Wednesday, 8:45, 1D236

This is a part of joint work with S. Gonchenko, A. Kazakov, and V. Kruglov.

In this talk we describe bifurcation scenarios leading to the birth of Lorenz-like attractors in 3D systems with central ($x \to -x, \quad y \to -y, \quad z \to -z$) and axial ($x \to -x, \quad y \to -y, \quad z \to z$) symmetries. We start with a stable equilibrium state O. In the first, case after several local and global bifurcation, we obtain a pair of the Lorenz attractors. In the second case, we obtain the so-called conjoined Lorenz twins attractor [1] that is a pair of Lorenz attractor glued together by the unstable separatices of O.

We illustrate both scenarios on the example of specific 3D system of ODE proposed in [1]. Also we extend these scenarios for the 3D maps and illustrate their implementation for the cubic three-dimensional Hénon map. We discuss which coefficients in the normal form are responsible for the formation of the attractors of both types. Finally, we study pseudohyperbolic properties of the observed attractors and found the boundaries of pseudohyperbolic attractor existence.

The work is supported by the RSF grant No. 23-71-30008.

[MS14] Multi-winged Lorenz attractors due to bifurcations of a periodic orbit with multipliers $(-1, i, -i)$

Aleksei Kazakov

HSE University

Wednesday, 9:15, 1D236

his is a part of joint work with E. Karatetskaia, K. Safonov, and D. Turaev.

We show that bifurcations of periodic orbits with multipliers $(-1, i, -i)$ can lead to the birth of pseudohyperbolic Lorenz-like attractors of three different types: one is a discrete analogue of the classical Lorenz attractor, and the other two are new. We call them two- and four-winged "Simó angels". These three attractors exist in an orientation-reversing, three-dimensional, quadratic

Hénon map. This implies the abundance of such attractors in a class of systems with homoclinic tangencies. Our analysis is based on the study of a normal form for this bifurcation, a slow-fast three-dimensional system of differential equations with a Z4-symmetry. The existence of the continuous-time counterparts of the mentioned above pseudohyperbolic attractors is established for the normal form as a part of an extensive numerical analysis of its bifurcations. In particular, we establish that all three types of continuous-time pseudohyperbolic attractors are born out of a certain Z4-symmetric heteroclinic configurations with three saddles.

The work is supported by the RSF grant No. 23-71-30008.

[MS14] High order homoclinic tangencies and universal dynamics for multidimensional diffeomorphisms

Dmitrii Mints

Imperial College London

Wednesday, 9:45, 1D236

Our research is aimed at studying the dynamics of smooth multidimensional diffeomorphisms from Newhouse domain, that is, open regions in the space of maps where systems with homoclinic tangencies are dense. We prove that maps with high order homoclinic tangencies of corank-1 (invariant manifolds forming the tangency have a unique common tangent vector) and maps having universal one-dimensional dynamics are dense in the Newhouse regions in the space of smooth and real-analytic multidimensional maps. We also show that in the space of smooth and real-analytic multidimensional maps in any neighborhood of a map such that it has bi-focus periodic orbit whose invariant manifolds are tangent, there exist open regions (which are subdomain of the Newhouse domain) where maps with high order homoclinic tangencies of corank-2 (invariant manifolds forming the tangency have a plane of common tangent vectors) and maps having universal two-dimensional dynamics are dense.

[MS14] Chaotic dynamics in the Rössler system

Eran Igra

Technion - Israel Institute of Technology

Wednesday, 10:15, 1D236

Originally introduced in 1974 by O.E. Rössler, the Rössler system is one of the most famous examples of chaotic flows, being generated by a stretch-and-fold mechanism. Despite being arguably the least non-linear flows one can think of, the Rs̈sler system is known to be rich in nonlinear phenomena - for example: spiral homoclinic bifurcations, period-doubling routes to chaos to name a few. In this talk we state and prove a topological criterion for the existence of chaotic dynamics for the Rössler system, which include infinitely many periodic trajectories. Time permitting, we will also characterize the topology of these periodic trajectories, and prove their persistence under perturbations.

[MS14] Geometric model for a pseudohyperbolic spiral attractor

Dimitry Turaev

Imperial College

Wednesday, 15:15, 1D226

We describe a class of chain-transitive strange attractors which, robustly with respect to small perturbations, contain a saddle-focus equilibrium with a two-dimensional unstable manifold, along with hyperbolic sets with one and two positive Lyapunov exponents, persistent homoclinic tangencies, and robust heterodimensional cycles.

[MS14] Blenders and robust transitivity for a family of derived from Anosov maps

Marisa Cantarino

Monash University

Wednesday, 15:45, 1D226

This is ongoing work made with the collaboration of Andy Hammerlindl and Warwick Tucker.

Blenders originally emerged as objects in dynamical systems as an example given by C. Bonatti and L. Díaz of a system which is not uniformly hyperbolic but it is robustly transitive. Roughly speaking, on an n-manifold, a blender is a hyperbolic invariant subset of the system that allows for robust intersections of s-dimensional stable manifolds and u-dimensional unstable manifolds, with $s + u < n$. The way to make this intersection robust without "the right dimensions" is to make the stable set of the blender to "fill the space as if it is higher dimensional". This intersection allows the existence of robust heterodimensional cycles, giving conditions for robust transitivity. We present a family of D.A. (derived-from-Anosov) systems on the 3-torus for which we are exploring the presence of blenders, using computer aided strategies.

[MS14] Rigorous enclosure of the discrete spectrum for transfer operators

Isaia Nisoli

UFRJ

Thursday, 15:15, 1D236

In this work, in collaboration with Blumenthal and Taylor-Crush, I present a generalization of a fundamental result, the Gerschgorin circle theorem, to obtain enclosures of the discrete spectrum of a transfer operator preserving a strong Banach space compactly embedded in a weak Banach space. The enclosures are obtained by rigorously bounding the weak resolvent norm of a finite rank approximation of the transfer operator. This result has important consequences, allowing us to understand the finer statistical properties of systems satisfying a Lasota-Yorke inequality, as uniformly expanding maps and systems with additive noise.

[MS14] The Lorenz equations and the modular surface

Tali Pinsky

The Technion

Thursday, 15:45, 1D236

Joint with Christian Bonatti.

In this talk I intend to describe a recent extension of the Lorenz geometric model that is defined on the three dimensional sphere. I'll explain how this new model is strongly related to the geodesic flow on the modular surface. The geodesic flow is defined on the complement of a trefoil knot and a trefoil also exists in the extended geometric model. I'll show that one can find parameters for the Lorenz equations for which there also exists an invariant trefoil knot. For such a parameter, one can prove an analogue of Smale's 14th problem: the equations at these parameters can be continuously deformed to the extended geometric model.

[MS14] Transitions to wild chaos in a 4D Lorenz-like system

Juan Patino-Echeverria
University of Auckland
Thursday, 16:15, 1D236

This is joint work with Hinke Osinga and Bernd Krauskopf (University of Auckland, Auckland, New Zealand).

Wild chaos is a form of higher-dimensional chaotic dynamics that can only arise in vector fields of dimension at least four. This talk explores wild chaos in a four-dimensional system of differential equations, which is an extension of the classic Lorenz equations. Recently, Gonchenko, Kazakov and Turaev (2021) showed, via the computation of Lyapunov exponents, that this system has a wild chaotic attractor at a particular point in parameter space.

To explain how this wild chaotic attractor arises geometrically, we perform a bifurcation analysis of the system in a two-parameter setting. As a starting point, we continue the one-parameter bifurcation structure of the classic Lorenz equations when the relevant new parameter is "switched on". We find that the well-known homoclinic explosion point of the Lorenz system unfolds and gives rise to infinite cascades of curves of Shilnikov-type global connections in the four-dimensional system. These connections are formed by the unstable manifold of the origin, which plays an essential role in the emergence of complicated dynamics in the system. We also compute the kneading diagram that encodes how this one-dimensional manifold repeatedly moves around a pair of equilibria. In combination with the direct computation of curves of global bifurcations, the kneading diagram provides insight that helps identify regions where wild chaos may occur.

[MS14] Lorenz Equations and the Figure Eight Knot

Yara Hatoom
Technion - Israel Institute of Technology
Thursday, 16:45, 1D236

For most parameters the Lorenz equations have three singular points, and for some parameters which are called T-points, there is an invariant curve which joins the three singular points and infinity. At the first T-point the curve is the trefoil knot and in the second T-point it is the figure-eight knot. In this talk, we are going to discuss the system at the second T-point parameter and show that the invariant figure eight knot forces the existence of infinitely many periodic orbits, and we can study the knot types of these orbits.

[MS14] Mostly contracting random maps

Pablo Barrientos

UFF

Friday, 8:45, 1D222

This is a work in collaboration with Dominique Malicet from Université Gustave Eiffel.

We are interested in understanding the long-term behavior of the iteration of a random map f given by the compositions $f^n_\omega = f_{\omega_{n-1}} \circ \cdots \circ f_{\omega_0}$, where each f_{ω_i} is a Lipschitz transformation of compact metric space independently and randomly selected according to the same probability measure. Such a random map is said to be *mostly contracting* if all the Lyapunov exponents associated with the stationary measures are negative. This requires the introduction of the notion of (maximal) Lyapunov exponent in this general context of Lipschitz transformations on compact metric spaces. We will prove:

(1) This class is open with respect to a coarser topology than the C^1-topology.

(2) The global Palis' conjecture holds, i.e., for any mostly contracting random map, there exist finitely many physical measures with disjoint support and whose union of basins covers almost everywhere.

(3) The annealed Koopman operator is quasi-compact, which implies many statistical properties such as central limit theorems, large deviations, and Hölder continuity (of exponents), etc.

Examples of this class of random maps include random products of circle C^1 diffeomorphisms, interval C^1 diffeomorphisms onto their images, and C^1 diffeomorphisms of a Cantor set on a line. All of these are considered under the assumption of no common invariant measure. This class of random maps also include projective actions of locally constant linear cocycle under the assumption of the simplicity of the spectrum. Moreover, as a consequence of a comprehensive general theory, one can extend (1), (2), and (3) above to the setting of Markovian random maps (i.e., random compositions driven by Markov measures).

One of the main tools to prove the above results requires the generalization of *Kingman's subadditive ergodic theorem* and *uniform Kingman's subadditive ergodic theorem* for general *Markov operators*. These two results are of independent interest, since they may have many other applications in other contexts.

[MS14] On the continuation of Lagrangian tori from Kepler to Sun-Jupiter-Saturn system

Jordi-Lluís Figueras

Uppsala Universitet

Friday, 9:15, 1D222

In this talk we will explain how we have computed with high accuracy invariant tori in the Sun-Jupiter-Saturn system using a continuation method from solutions of the Kepler problem, see the paper "Sun-Jupiter-Saturn System may exist: A verified computation of quasiperiodic solutions for the planar three body problem" (submitted) This work has been developed with Alex Haro.

[MS14] Renormalization and mild blow ups in generalized Navier-Stokes equations

Deniz Gaidashev

Uppsala University

Friday, 9:45, 1D222

We will describe how the problem of existence of self-similar blow up solutions in a generalized mild Navier-Stokes system can be stated as a fixed point problem for a "renormalization" operator. We construct renormalization a-priori bounds, that is a renormalization invariant precompact set in an appropriate weighted L_p-space. As a consequence of a-priori bounds, we prove existence of renormalization fixed points, and existence of non-trivial self-similar mild solutions, whose L_p-norm, $p \geq 2$, becomes unbounded in finite time T . These solutions have good regularity properties, but are not strong solutions (which are the subject of the NS existence and uniqueness problem).

[MS14] Birth of discrete Lorenz attractors in bifurcations of homoclinic and heteroclinic cycles

Ivan Ovsyannikov
Constructor University
Friday, 10:15, 1D222

Consider the following three-dimensional Henon map:

$$\bar{x} = y,\ \bar{y} = z,\ \bar{z} = M + Ay + Bx - z^2 \tag{16}$$

It is well-known that in some parameter domain map (1) possesses the discrete Lorenz attractor. This fact helps to prove the existence of such attractors in other problems, where map (1) appears e.g. as a first-return map. In particular, these are homoclinic and heteroclinic cycles with quadratic tangencies of manifolds and having special structures, local and global. The latter include the existence of resonant multipliers in the fixed point or non-simple tangencies of invariant manifolds. These additional conditions are required to avoid the existence of lower-dimensional invariant submanifolds and thus to make the dynamics effectively three-dimensional. Otherwise, the three-dimensional chaotic dynamics e.g. Lorenz-like attractors will be not possible.
Another question arises in this context: is it possible to have Lorenz-like attractors in the same systems, but in the backward time direction? The first return map in this case will be the inverse to (1), that is, the map

$$\bar{x} = y,\ \bar{y} = z,\ \bar{z} = M + Az + Bx - y^2 \tag{17}$$

It is automatically clear that map (2) has a discrete Lorenz repeller near the same bifurcation point, where map (1) has an attractor. The main result of the present work is the numerical proof that map (2) possesses period-6 points with multipliers $(-1, -1, +1)$ near which indeed there exist period-6 discrete Lorenz attractors.

[MS15] The Hypocoercivity Index for the short time behavior of linear time-invariant ODE systems

Franz Achleitner

TU Wien

Wednesday, 8:45, 1D237

This is a joint work with Anton Arnold (TU Wien, Vienna, Austria) and Eric A. Carlen (Rutgers University, Piscataway (NJ), USA).

We consider the class of conservative-dissipative ODE systems, which is a subclass of Lyapunov stable, linear time-invariant ODE systems. We characterize asymptotically stable, conservative-dissipative ODE systems via the hypocoercivity (theory) of their system matrices. Our main result is a concise characterization of the hypocoercivity index (an algebraic structural property of matrices with positive semi-definite Hermitian part introduced in *Achleitner, Arnold, and Carlen (2018)*) in terms of the short time behavior of the norm of the matrix exponential for the associated conservative-dissipative ODE system.

[1] F. Achleitner, A. Arnold, and E.A. Carlen, *On multi-dimensional hypocoercive BGK models.*, Kinet. Relat. Models **11** (2018), 953–1009.

[2] F. Achleitner, A. Arnold, and E.A. Carlen, *The hypocoercivity index for the short time behavior of linear time-invariant ODE systems.* J. Differential Equations **371** (2023), 83–115.

[MS15] Hypocoercivity for non-autonomous linear evolution equations: an operator-theoretic approach

Artur Stephan

Technische Universität Wien

Wednesday, 9:15, 1D237

Motivated by a recent result of *Achleitner et al* [1], we investigate the notion of *hypocoercivity* for linear evolution equations, where the generator of the evolution depends explicitly on time. Here, we consider a family of bounded operators $\{A(t)\}_{t\geq 0}$ on a Hilbert space H that are assumed to be uniformly bounded in time. To track the explicit time-dependence, we utilize the Howland-Evans-Neidhardt approach to formulate the problem time-independently on the space $L^2([0,T],H)$ including time and space. As it turns out the desired solution operator (or propagator) is obtained by a semigroup generated by a sum $K = D_0 + \mathcal{A}$, where D_0 generates the shift-semigroup and $\mathcal{A}$ is the multiplication operator induced by the operator family $\{A(t)\}_{t\geq 0}$. Moreover, estimates for the propagator translate to estimates on the semigroup $e^{-\tau K}$ (and vice versa), and hence, allow to answer questions regarding hypocoercivity now in a time-independent setting.

In my talk, we focus on quantitative result for the short-time behavior of the evolution. For this we rely on an explicit formula for the semigroup $e^{-\tau K}$ by the so-called Dyson-Phillips series, and, moreover, on calculations from [1].

[1] F. Achleitner, A. Arnold, V. Mehrmann, E.A. Nigsch, *Hypocoercivity in Hilbert spaces*, arXiv:2307.08280v1, July 2023.

[MS15] Non-Equilibrium steady states in BGK models for heat transfer in gases

Josephine Evans

University of Warwick

Wednesday, 9:45, 1D237

This is a joint work with Angeliki Menegaki (Imperial College London, United Kingdom).

This talk is about hypocoercivity in the context of non-equilibrium steady states. I will present the general additional challenges in showing convergence to equilibrium for equations with non-equilibrium steady states. I will focus on the results of two papers joint with Angeliki in which we study the BGK model for heat transfer in gasses. In this particular setting we no longer expect the entropy for the fully non-linear function to necessarily be a monotone function. This is because we are in an open system so the second law of thermodynamics no longer holds. In the context of hypocoercivity this potentially breaks the *microscopic coercivity* assumption for the linearized equation around a steady state; the fact that the collision operator pulls the system towards local equilibria. I will present a system where we can show that microscopic coercivity does still hold and another where it doesn't and we are currently unable to show convergence to equlibrium.

[MS15] Stability of bifurcating equilibria by hypocoercivity

Christian Schmeiser

University of Vienna

Wednesday, 10:15, 1D237

This is a joint work with Sara Merino Aceituno (University of Vienna, Austria) and Raphael Winter (Cardiff University, UK).

Several models for particle alignment with stochastic perturbations show a steady state bifurcation behaviour with an exchange of stability between a trivial (non-aligned) state and an ordered (aligned) state. An example is the nondimensionalized Vicsek-BGK equation

$$\partial_t F + \gamma\, \omega \cdot \nabla_x F = \rho_F M_{J_F} - F\,,$$

a kinetic transport model for a distribution function $F(t,x,\omega)$ of particles moving with constant speed and direction $\omega \in \mathbb{S}^{d-1}$ between stochastic reorientation events, where a new direction is sampled from the von Mises distribution $M_J(\omega) = e^{J\cdot\omega}/Z(J)$, where $Z(J)$ is a normalization factor. The position density ρ_F and the flux J_F are the zeroth and, respectively, first order moments of F with respect to ω. As a position domain the flat torus is chosen: $x \in \mathbb{T}^d$.

The spatially homogeneous version of the model shows the bifurcation behaviour described above with the scaled total mass (or, equivalently, the scaled temperature in the von Mises distribution) as bifurcation parameter. The main goal of this work is the extension of the bifurcation result to the model with position dependence. The basic step is a stability analysis of the linearization at the spatially homogeneous steady states. Due to the lack of an entropy this is not feasible by standard hypocoercivity approaches. A new spectral approach based on the Fourier-Laplace transformed model will be presented. It turns out that the results for the spatially homogenous model only carry over for sufficiently strong transport effects, i.e. for sufficiently large Knudsen number γ. For smaller Knudsen numbers the spatially homogeneous steady states can be unstable under spatially inhomogeneous perturbations. This possibly explains traveling bands observed in simulations with a related model.

The stability result holds in Sobolev spaces with large enough differentiability order such that local stability for the nonlinear problem can be shown. A difficulty is caused by the fact that conservation of mass is the only conservation law, whence the dependence of the equilibrium state on the initial data is not explicit. For large data existence of smooth solutions is only guaranteed locally in time. A global-in-time existence result of weak solutions is shown similarly to the gas dynamics BGK model, based on an estimate for the logarithmic entropy, which grows at most exponentially with time.

[MS15] On a structure-preserving numerical method for fractional Fokker-Planck equations

Nathalie Ayi

Sorbonne Université

Wednesday, 15:15, 1B306

This is a joint work with Maxime Herda (INRIA, Lille, France), Hélène Hivert (INRIA, Rennes, France) and Isabelle Tristani (CNRS, Nice, France).

In this talk, we introduce and discuss a numerical scheme for the Lévy-Fokker-Planck equation. After dealing with the continuous case, I will present the discretizations adopted, designed to preserve the main features of the continuous model such as conservation of mass, heavy-tailed equilibrium and hypocoercivity properties. New tools of discrete functional analysis will be then necessary in order to perform a thorough analysis of the numerical scheme and show exponential stability. We will illustrate our theoretical findings with numerical simulations.

[MS15] Quasistationary distributions for the Langevin process

Julien Reygner

École des Ponts ParisTech

Wednesday, 15:45, 1B306

This is a joint work with Tony Lelièvre (École des Ponts ParisTech) and Mouad Ramil (Seoul National University).

The quasistationary distribution (QSD) of a Markov process $(X_t)_{t\geq 0}$ in a subset D of its state space describes the long time limit of the conditional distribution of trajectories which do not exit D. In PDE terms, it is the left eigenvector associated with the Dirichlet eigenvalue of the infinitesimal generator of $(X_t)_{t\geq 0}$ in D. QSDs play a central role in the theoretical understanding and numerical simulation of systems exhibiting metastability, which are ubiquitous in statistical physics, chemistry and biology.

For uniformly elliptic diffusion processes in bounded domains, the existence and uniqueness of a QSD is well understood. The goal of the talk will be to present recent works extending these results to a class of degenerate diffusion processes, called the Langevin dynamics in molecular dynamics, whose infinitesimal generator is the kinetic Fokker–Planck operator.

[1] T. Lelièvre, M. Ramil, J. Reygner, *A probabilistic study of the kinetic Fokker–Planck equation in cylindrical domains.* J. Evol. Equ. 22, 38 (2022).

[2] T. Lelièvre, M. Ramil, J. Reygner, *Quasi-stationary distribution for the Langevin process in cylindrical domains, Part I: Existence, uniqueness and long-time convergence.* Stochastic Process. Appl. 144, 173–201 (2022).

[MS15] Contractivity for kinetic Fokker-Planck equations with distribution-depedent forces

Katharina Schuh

TU Wien

Thursday, 9:15, 1D236

We study the long-time behaviour of the kinetic Fokker-Planck equation by analysing the probabilistic counterpart, the second-order Langevin dynamics. We consider both the classical Langevin dynamics and the nonlinear Langevin dynamics of McKean-Vlasov type containing distribution-dependent forces.

Instead of the classical hypocoercivity methods we use a coupling approach to establish global contraction in an L^1 Wasserstein distance with an explicit dimension-free rate for pairwise weak interactions. For external forces corresponding to a κ-strongly convex potential, a contraction rate of order $\mathcal{O}(\sqrt{\kappa})$ is obtained in certain cases. But the contraction result is not restricted to these forces. It rather includes multi-well potentials and non-gradient-type external forces as well as non-gradient-type repulsive and attractive interaction forces. The proof is based on a novel distance function which combines two contraction results for large and small distances and uses a coupling approach adjusted to the distance. By applying a componentwise adaptation of the coupling we show uniform in time propagation of chaos bounds for the corresponding mean-field particle system.

[1] K. Schuh, Global contractivity for Langevin dynamics with distribution-dependent forces and uniform in time propagation of chaos. arXiv:2206.03082, 2022, to appear in Ann. Inst. Henri Poincaré Probab. Stat.

[MS15] Hypocoercivity for SPDEs

Martin Grothaus

RPTU in Kaiserslautern

Thursday, 9:45, 1D236

Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and were applied to equations in linear finite dimensional spaces. Meanwhile we can treat equations in manifolds or even infinite dimensional spaces. The condition giving micro- and macroscopic coercivity we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

[MS15] Partially dissipative hyperbolic systems: hypocoercivity and hyperbolic approximations

Timothée Crin-Barat

Friedrich-Alexander Universitätg

Thursday, 10:15, 1D236

In this talk, we review recent results on so-called *partially dissipative* hyperbolic systems. Such systems model physical phenomena with degenerate dissipative terms and appear in many applications. For example, in gas dynamics where the mass is conserved during the evolution, but the momentum balance includes a diffusion (viscosity) or a damping (relaxation) term.

First, using tools from the hypocoercivity theory and precise frequency decompositions, we derive sharp stability estimates for linear systems satisfying the Kalman rank condition. This linear analysis allows us to establish new global-in-time existence and large-time behaviour results in a critical regularity framework for nonlinear systems.

Then, we interpret partially dissipative systems as hyperbolic approximations of parabolic systems, in the context of the paradox of infinite speed of propagation. In particular, we focus on a hyperbolic approximation of the multi-dimensional compressible Navier-Stokes-Fourier system and establish its hyperbolic-parabolic strong relaxation limit.

[MS16] Mathematical Modeling of Traffic Flow — Discrete versus Continuous

Helge Holden

Norwegian University of Science and Technology

Monday, 15:15, 1D222

This is a joint work with N.H. Risebro (University of Oslo, Norway).

Vehicular traffic is one of most serious problems facing modern urban life. We will describe some classical mathematical models for traffic flow. There are two rather distinct ways to model traffic. On the one hand one can track individual vehicles, often called Follow-the-Leader models (FtL). This leads to systems of ordinary differential equations. However, if traffic is dense, a classical model is the so-called Lighthill–Whitham–Richards model (LWR), which is a nonlinear partial differential equation, more specifically, a hyperbolic conservation law. We study these models, and, in particular, the connection between the discrete (FtL) and the continuous (LWR) when traffic becomes dense. We will also briefly discuss traffic on a network or roads, and traffic on multilane roads.

The talk is based in part on the papers [1–4]. Further references can be found in these papers.

[1] H. Holden, N.H. Risebro, *Follow-the-Leader models can be viewed as a numerical approximation to the Lighthill–Whitham–Richards model for traffic flow*. Networks & Heterogeneous Media **13**(3) (2018) 409–421.

[2] H. Holden, N.H. Risebro, *Continuum limit of Follow-the-Leader models — a short proof*. Discrete and Continuous Dynamical Systems **38**(2) (2018) 715–722

[3] H. Holden, N.H. Risebro, *Models for dense multilane vehicular traffic*. SIAM J. Math. Anal. **51** (5)

(2019) 3694–3713.

[4] H. Holden, N.H. Risebro, *The continuum limit of higher-order Follow-the-Leader models*. arXiv:2312.00606, 2023.

[MS16] Evolution of first variations for scalar conservation laws

Elio Marconi

Università di Padova

Monday, 15:45, 1D222

This is a joint work with Prof Ancona and Luca Talamini (University of Padova, Italy).

We consider scalar conservation laws in one space dimension

$$\partial_t u + \partial_x f(u) = 0 \tag{18}$$

and we discuss the evolution of small perturbations of a given solution u of (18). We adopt a Lagrangian/kinetic point of view, which is suitable to address this problem without assuming the structure of the variations a priori. Qualitative properties of perturbations at positive times are obtained: in specific situations they imply or disprove the emergence of the particular class of variations named shifts.

The motivation of this analysis is to provide necessary conditions for the optimality of u for a class of cost functionals.

[MS16] Unconditional flocking for weak solutions to self-organized systems of Euler-type

Debora Amadori

University of L'Aquila

Monday, 16:15, 1D222

This is a joint work with Prof. Cleopatra Christoforou (University of Cyprus).

In this talk, we present some results obtained in [1, 2] on the existence and time-asymptotic flocking of weak solutions to a hydrodynamic model of flocking-type with all-to-all interaction kernel, in one-space dimension:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0, \\ \partial_t(\rho v) + \partial_x \left(\rho v^2 + \alpha^2 \rho\right) = K \int_{\mathbb{R}} \rho(x,t)\rho(x',t) \left(v(x',t) - v(x,t)\right) \, dx' \end{cases}$$

for positive constants α, K.

An appropriate notion of entropy weak solutions with bounded support is given to capture the behavior of solutions to the Cauchy problem with any BV initial data that has finite total mass confined in a bounded interval and initial density uniformly positive therein. The existence of entropy weak solutions with bounded support for a class of initial data having finite, but possibly large, total variation, has been proved in [1].

We will discuss the long time behavior of these solutions, which are shown to experience "flocking" for large time: their support is uniformly bounded in time, and the velocity converges to the mean value. The rate of convergence is exponential. The proof is based on the decay of positive waves and on cancellation properties between positive and negative waves.

[1] D. Amadori and C. Christoforou, *BV solutions for a hydrodynamic model of flocking-type with all-to-all interaction kernel*, Math. Models Methods Appl. Sci. **32**, (2022), 2295–2357.

[2] D. Amadori and C. Christoforou, *Unconditional flocking for weak solutions to self-organized systems of Euler-type with all-to-all interaction kernel*, preprint 2023, https://arxiv.org/abs/2309.01939

[MS16] Inverse Design and Hyperbolic Conservation Laws

Vincent Perrollaz

Université de Tours

Monday, 16:45, 1D222

It is now well known that hyperbolic conservation laws admit generic singularities through shock wave generation. To get a continuous semigroup — acting on $L^\infty(\mathbb{R})$ — out of solutions $t \mapsto S_t u_0$ for the equation

$$\begin{cases} \partial_t u + \partial_x f(x,u) = 0, \\ u(0) = u_0, \end{cases}$$

one has to work within the framework of weak entropy solutions.

In this context however, the time evolution becomes irreversible. The inverse design problem thus asks for the characterization of the set

$$I_T(w) := \{u_0 \in L^\infty(\mathbb{R}) \,:\, S_T u_0 = w\},$$

given a positive time T and a target profile w in $L^\infty(\mathbb{R})$. In the first part of the talk, we will describe the rather surprising properties of this set when the flux f is convex in the variable u.

This problem happens to be also motivated by investigations in data assimilation, for instance in the context of traffic flow. We thus need to understand, on one hand how partial information on w impacts our knowledge of $I_T(w)$ and on the other hand how one can make use of additional information on the model. For instance when dealing with traffic flow, we know a priori that the density of traffic takes value in a given interval. But on the other hand, we might have access to only localized — in space — measurement of the profile w. In the second part of the talk we will provide some first results in this direction.

This talk is based on joint works with Rinaldo Colombo and Abraham Sylla.

[MS16] Hyperbolic Techniques and Biological Models

Francesca Marcellini

University of Brescia

Tuesday, 9:15, 1A305

This is a joint work with Rinaldo M. COLOMBO (University of Brescia), Mauro GARAVELLO (University of Milano–Bicocca) and Elena ROSSI (University of Modena–Reggio Emilia).

The recent pandemic lead to the development of a variety of new epidemiological models. The standard starting point in this area is the so called SIR model, that dates back to the classical work [6]:

$$\begin{array}{c} \boxed{S}\to\boxed{I}\to\boxed{R} \\ \downarrow \end{array} \qquad \left\{\begin{array}{rcl} \dot S &=& -\rho\, I\, S \\ \dot I &=& \rho\, I\, S - (\vartheta+\mu) I \\ \dot R &=& \vartheta\, I \end{array}\right. \tag{19}$$

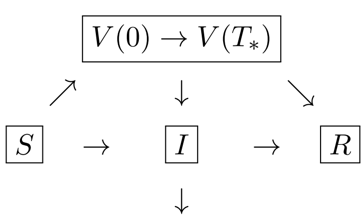
where, as usual, the independent variable is time, S, I and R are the Susceptible, Infected and

Recovered individuals; ρ describes the spreading of the disease, ϑ the recovery rate and μ the mortality rate of infected individuals. First, we discuss the extension of (19) presented in [4],

$$\begin{cases} \partial_t S + \partial_a S + \mathrm{div}_x(v_S\, S) + \mu_S\, S = -\int_{\mathbf{R}_+}\int_{\mathcal{X}} \rho(t,a,\alpha,x,\xi)\, I(t,\alpha,\xi)\, d\xi\, d\alpha\; S(t,a,x) \\ \partial_t I + \partial_a I + \mathrm{div}_x(v_I\, I) + \mu_I\, I = \int_{\mathbf{R}_+}\int_{\mathcal{X}} \rho(t,a,\alpha,x,\xi)\, I(t,\alpha,\xi)\, d\xi\, d\alpha\; S(t,a,x) - \kappa\, I - \vartheta\, I \\ \partial_t H + \partial_a H + \mu_H\, H = +\kappa\, I - \eta\, H \\ \partial_t R + \partial_a R + \mathrm{div}_x(v_R\, R) + \mu_R\, R = +\vartheta\, I + \eta\, H\,. \end{cases} \tag{20}$$

Here H stands for those infected individuals that are isolated, possibly Hospitalized, and do not propagate the infection. The independent variables are, besides time, x for the spatial position and a for age. The other parameters and functions above are typically age, time and space dependent and describe, as usual, mortality rates $(\mu_s, \mu_I, \mu_H, \mu_R)$, movements (v_S, v_I, v_R), infection propagation among different age classes (ρ), lockdown rate (κ), recovery rates (ϑ, η). Note that in the equation for H, clearly, no spatial movement is allowed. Lockdown policies can be simulated and compared through numerical integrations that will be discussed during the talk.

Then, on the basis of [3], we tackle the description of vaccination strategies and of their effects within the more general framework of compartmental models with intra–compartmental dynamics:



$$\begin{cases} \dot S = -\rho_S\, I\, S - p(t,S) \\ \partial_t V + \partial_\tau V = -\rho_V\, I\, V \\ \dot I = (\rho_S\, S + \int_0^{T_*} \rho_V\, V) I - \vartheta\, I - \mu\, I \\ \dot R = \vartheta\, I + V(t, T_*) \\ V(t,0) = p\,(t, S(t))\ . \end{cases} \tag{21}$$

Above, we use essentially the same notation as in (32). Note the introduction of the variable τ ranging in $[0, T_*]$ and describing the time since vaccination, with T_* breng the time necessary for the vaccine to provide full immunization. The compartmental scheme on the left above shows that the evolution of the immunization effect is an example of a dynamics within the compartment of the V variable. The extension of (21) to the case of a vaccination campaign consisting of different concurrent vaccines allows to compare different vaccination strategies.

Finally, we deal with the wellposedness and stability estimates for a wide class of equations of interest in epidemiology, such as

$$\begin{cases} \partial_t u^h + \operatorname{div}_x \left(v^h(t,x)\, u^h \right) = g^h\left(t, x, u(t,x), u(t)\right) & (t,x) \in \mathbf{R}_+ \times \mathcal{X} \\ u^h(t,\xi) = u_b^h\left(t, \xi, u(t)\right) & (t,\xi) \in \mathbf{R}_+ \times \partial\mathcal{X} \\ u^h(0,x) = u_o^h(x) & x \in \mathcal{X}\,, \end{cases} \tag{22}$$

where $h = 1, \ldots, k$, see [2]. The above initial boundary value problem contains nonlocal terms (above, the dependence on $u(t)$ is assumed to of a nonlocal, i.e. functional, nature) in the right hand side as well as in the boundary conditions. We provide detailed wellposedness and stability estimates that apply to various instances of (22) considered in the literature, in the framework of epidemiological models such as (19), as well as in cell division models [1, 9] or age and phenotypically structured population models [7, 8].

[1] G. I. Bell and E. C. Anderson. Cell growth and division: I. A mathematical model with applications to cell volume distributions in mammalian suspension cultures. *Biophysical Journal*, 7(4):329 – 351, 1967.

[2] R. M. Colombo, M. Garavello, F. Marcellini, and E. Rossi. General Renewal Equations Motivated by Biology and Epidemiology. *Journal of Differential Equations*, 354: 133–169, 2023.

[3] R. M. Colombo, F. Marcellini, and E. Rossi. Vaccination strategies through intra-compartmental dynamics. *Networks and Heterogeneous Media*, 17(3): 385–400, 2022.

[4] R. M. Colombo, M. Garavello, F. Marcellini, and E. Rossi. An age and space structured SIR model describing the COVID-19 pandemic. *Journal of Mathematics in Industry*, 10(1), 2020.

[5] G. Giordano, F. Blanchini, R. Bruno, P. Colaneri, A. Di Filippo, A. Di Matteo, and M. Colaneri. Modelling the COVID-19 epidemic and implementation of population-wide interventions in italy. *Nature Medicine*, 26(6):855–860, 2020.

[6] W. O. Kermack and A. G. McKendrick. Contributions to the mathematical theory of epidemics. II. *Proc. R. Soc. Lond. A*, 138:55–83, 1932.

[7] S. Mischler, B. Perthame, and L. Ryzhik. Stability in a nonlinear population maturation model. *Math. Models Methods Appl. Sci.*, 12(12):1751–1772, 2002.

[8] S. Nordmann, B. Perthame, and C. Taing. Dynamics of concentration in a population model structured by age and a phenotypical trait. *Acta Applicandae Mathematicae*, 155(1):197–225, Dec 2017.

[9] S. L. Tucker and S. O. Zimmerman. A nonlinear model of population dynamics containing an arbitrary number of continuous structure variables. *SIAM J. Appl. Math.*, 48(3):549–591, 1988.

[MS16] A multi-scale multi-lane model for traffic regulation via autonomous vehicles

Paola Goatin

Inria

Tuesday, 9:45, 1A305

This is a joint work with Prof. Benedetto Piccoli (University of Rutgers, Camden, New Jersey, USA).

We extend the multi-scale model of traffic flow with moving bottlenecks proposed in [1] to include multi-lane dynamics. The model is intended to capture the macroscopic dynamics of multi-lane traffic, where the presence of slow-moving vehicles on some lanes hinders the flow by reducing locally the road capacity. It consists of a system of hyperbolic balance equations describing the spatio-temporal evolution of the bulk traffic density, coupled with the ordinary differential equations of the bottlenecks' trajectories. The coupling is realized though the bottlenecks' velocities and local flux constraints at the slow vehicles' positions. Lane exchanges are modeled by a general relaxation source term, as commonly proposed in the literature.

We prove existence of solutions by providing suitable compactness estimates on wave front tracking approximations. Next, we investigate the limit as the lane exchange relaxation parameter tends to zero [2].

[1] M. L. Delle Monache and P. Goatin. *Scalar conservation laws with moving constraints arising in traffic flow modeling: an existence result.* J. Differential Equations, 257(11):4015–4029, 2014..

[2] P. Goatin and B. Piccoli. *A multi-scale multi-lane model for traffic regulation via autonomous vehicles.* In preparation.

[MS16] A convergent numerical method for the Hunter–Saxton equation

Thomas Christiansen

Norwegian University of Science and Technology

Tuesday, 10:15, 1A305

This is based on joint work with Katrin Grunert (NTNU, Trondheim, Norway), Anders S. Nordli (UiT, Tromsø, Norway), and Susanne Solem (NMBU, Ås, Norway).

Solutions to the Hunter–Saxton (HS) equation

$$u_t(t,x) + uu_x(t,x) = \frac{1}{4}\int_{-\infty}^{x} u_x^2(t,z)dz - \int_{x}^{\infty} u_x^2(t,z)dz, \tag{23}$$

might experience pointwise blow up of their spatial derivative in finite time, while the solution itself remains bounded. This phenomenon is known as *wave breaking*, see [1]. A consequence is that even classical solutions to (23) may fail to exist globally in time. Furthermore, energy concentrates on sets of zero measure when wave breaking takes place, and the continuation of solutions beyond wave breaking is therefore nonunique and depends on how this concentrated energy is manipulated.

The focus of this talk will be on the family of α-dissipative solutions, which allows us to uniformly treat solutions with nonincreasing energy, by varying the parameter $\alpha \in [0,1]$. As the name suggests, such solutions are obtained by removing an α-fraction of the concentrated energy at each wave breaking occurrence.

After reviewing some properties enjoyed by α-dissipative solutions, we propose a new numerical method. This method is based on applying a novel projection operator to the initial data and thereafter solving exactly with a generalized method of characteristics, see [2]. This projection operator is tailor-made to the HS equation, taking advantage of the fact that piecewise linear initial data is naturally preserved by (23), as well as preserving some of its essential underlying structure.

We prove that initial convergence of the projected data ensures convergence towards α-dissipative solutions at all later times.

We end the talk by examining two explicit examples, which provide two distinct numerical challenges, and for which a convergence rate is acquired.

[1] John K. Hunter and Ralph Saxton, *Dynamics of director fields.* SIAM J. Appl. Math. **51(6)** (1991), 1498–1521.

[2] Thomas Christiansen, Katrin Grunert, Anders Nordli and Susanne Solem, *A convergent numerical algorithm for α-dissipative solutions of the Hunter–Saxton equation.* (In press), J. Sci. Comput.

[MS16] Boundary conditions for first-order hyperbolic relaxation systems

Yizhou Zhou

RWTH Aachen University

Friday, 8:45, 1D226

The first-order hyperbolic relaxation system is a class of time-dependent partial differential equations which model various non-equilibrium phenomena. For such systems, the main interest is to understand the zero relaxation limit. The initial-value problem for the relaxation system has been well-developed and a systematical framework has been built. However, the initial-boundary value problem of the relaxation system is still in the developing stage. In this talk, I will introduce the theory of boundary conditions for general relaxation systems. Under a so-called generalized Kreiss condition, well-posed boundary conditions for the corresponding equilibrium system can be derived by resorting to the asymptotic analysis with boundary-layers. Particularly, the results can be extended to the case with characteristic boundaries. Moreover, applications of this theory in different problems will be presented.

[MS16] Nonlocal Multi-D Systems of Hyperbolic Equations

Mauro Garavello

University of Milano Bicocca

Friday, 9:15, 1D226

This is a joint work with Rinaldo M. Colombo (University of Brescia, Brescia, Italy).

We consider the following multi-D non linear system of hyperbolic equations

$$\begin{cases} \partial_t \rho_i + \nabla \cdot (\rho_i \, V_i(\nabla \rho * \eta)) = 0 & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n \\ \rho_i(0,x) = \rho_{o,i}(x) & x \in \mathbb{R}^n \end{cases} \qquad i \in \{1,\ldots,m\} \tag{24}$$

with non-local terms in the flux functions. Here $t \in \mathbb{R}_+$ is the time, $x \in \mathbb{R}^n$ is the space variable, $\rho = (\rho_1, \ldots, \rho_m)$ denotes the vector of time dependent densities of m populations defined on the whole space $\mathbb{R}^n$, $(\rho_{o,1}, \ldots, \rho_{o,m})$ are the initial conditions, and V_i, for $i \in \{1,\ldots,m\}$, represents the velocity function for the i-th population. We assume that each V_i depends on the $n \times m$ matrix $(\nabla \rho * \eta)$, where the ji entry is

$$(\nabla \rho(t) * \eta(x))_{ji} = \partial_{x_j} (\rho_i(t) * \eta)(x) = \partial_{x_j} \int_{\mathbb{R}^n} \rho_i(t, x-y) \eta(y) dy,$$

for $j \in \{1,\ldots,n\}$ and $i \in \{1,\ldots,m\}$ and $\eta : \mathbb{R}^n \to \mathbb{R}$ is a smooth kernel.

The model (24) is of macroscopic type and is able to describe different behaviors typically emerging in population dynamics. Indeed different shapes of the kernel function η and of the velocity vectors V_i may result for example in an aggregation phenomenon, with the possible formation of clusters (or opinions), or in a segregation of the various populations. An important role is played by the support of the kernel function, which correspond to the visual range of the individuals. In the talk we present several numerical integrations showing various behaviors of (24), together with its well posedness and some analytic qualitative properties.

[MS16] Mixed hyperbolic–parabolic systems: traffic-generated pollution and predator-prey-dynamics

Elena Rossi

University of Modena and Reggio Emilia

Friday, 9:45, 1D226

This is a joint work with Rinaldo M. Colombo (University of Brescia, Italy) and Paola Goatin (Inria, Université Côte d'Azur, France).

Mixed systems consisting of Partial Differential Equations (PDEs) of different types, also together with Ordinary Differential Equations (ODEs), are nowadays widely employed to model various

phenomena, such as in traffic flows, porous media, biology or epidemiology. In this talk, we focus on the coupling of a hyperbolic and a parabolic equation in two different applications.
The first is concerned with traffic flows and the related air pollution. In particular, we consider a merge regulated by traffic lights, the hyperbolic part of the problem, and the evolution of the traffic-generated pollution, governed by a parabolic equation. The outcome is a weakly coupled system, where the roads in the traffic model are allowed to be also 2–dimensional. Once the well posedness is proved, the goal is to manage the traffic lights, as well as inflows and outflows, to minimize the presence of pollutant in given regions, also far away from the merge. We consider a conservative form for the parabolic equation governing the propagation of the pollutant and develop an $\mathbf{L}^1$–well posedness theory for its weak solutions. Moreover, despite the use of a first order model for traffic, the classical LWR, we comprise the effects of *stop & go waves* in the production of pollutant, by means of measure source term in the parabolic equation.
In the second application we couple a non linear and non local hyperbolic equation to a parabolic one to describe two competing populations. In particular, the movement of the first population is directed towards the regions where the concentration of the other one is greater, while this second population is assumed to diffuse. The source terms of the two PDEs generalize Lotka–Volterra equations, and also include control functions. This mixed system provides therefore a usable structure for the search of an optimal control strategy in biological pest control problems.

[1] R.M. Colombo, P. Goatin, E. Rossi, *A hyperbolic–parabolic framework to manage traffic-generated pollution*, submitted, 2024.

[2] R.M. Colombo, E. Rossi, *Hyperbolic predators vs. parabolic prey*, Commun. Math. Sci., **13** (2): 369-400, 2015.

[3] R.M. Colombo, E. Rossi, *Non linear hyperbolic-parabolic systems with Dirichlet Boundary conditions*, Differ. Integral Equ., **37** (7/8), 443 - 478, 2024.

[4] R.M. Colombo, E. Rossi, *Well posedness and control in a hyperbolic-parabolic parasitoid-parasite system*, Studies in Applied Mathematics, **147**(3): 839-871, 2021.

[MS16] Uniqueness for the Camassa–Holm equation

Katrin Grunert

NTNU Norwegian University of Science and Technology

Friday, 10:15, 1D226

Solutions of the Camassa–Holm (CH) equation might enjoy wave breaking in finite time. This means that even classical solutions, in general, do not exist globally, but only locally in time, since their spatial derivative might become unbounded from below pointwise in finite time, while the solution itself remains continuous and bounded. Furthermore, energy concentrates on sets of measure zero when wave breaking occurs. Thus the prolongation of solutions beyond wave breaking is non-unique and depends heavily on how the concentrated energy is manipulated. The two most prominent ones are called dissipative, i.e., the concentrated energy is taken out, and conservative, i.e., the energy is given back into the system. The existence of both types of solutions has been established with the help of a generalized method of characteristics, which allows to rewrite the Camassa–Holm equation as a system of ODEs in Lagrangian coordinates, see e.g. [3].

Here we will focus on the uniqueness of dissipative solutions [2], for which all of the concentrated energy is removed upon wave breaking. The overall idea is to establish a bijection between the properties satisfied by each dissipative solution and the solutions operator defined via a generalized method of characteristics. The main ingredients are measure transport equations, ideas from sub- and supersolutions as well as a good understanding of the characterisation of equivalence classes in Lagrangian coordinates. Furthermore, we will compare the necessary properties guaranteeing uniqueness for dissipative solutions to those for the conservative solutions from [1].

[1] A. Bressan, G. Chen, and Q. Zhang, *Uniqueness of conservative solutions to the Camassa–Holm equation*, Discrete Contin. Dyn. Syst. **35** (2015), 25–42.

[2] K. Grunert, *Uniqueness of dissipative solutions for the Camassa–Holm equation*, arXiv:2311.15344.

[3] K. Grunert, H. Holden, and X. Raynaud, *A continuous interpolation between conservative and dissipative solution for the two-component Camassa–Holm system*, Forum Math. Sigma **3** (2015), Paper

No. e1, 73pp.

[MS17] Weak solvability and convergence of numerical solutions for moisture transport model with non-monotone boundary conditions

Akiko Morimura

Japan Women's University

Monday, 15:15, 1D226

We consider the initial boundary value problem for nonlinear parabolic equations with non-monotone boundary conditions. The problem was already investigated as a mathematical model for moisture transport phenomena in porous materials. It is difficult to show the existence of a strong solution to this problem since the boundary condition is a non-monotone. On this problem we showed that there exists a unique strong solution in [1], when the boundary conditions is simplified. Also, in our recent work, we have established existence and uniqueness of weak solutions by the Schauder fixed point theorem and the dual equation method, respectively. Since the proof for the existence is not constructive, it is important to show convergence of numerical solutions. In the present result, by applying the finite volume method, we approximate the partial differential equations and prove that the sequence of approximation solutions converges to the weak solution. The key to the proof is a compact theorem for discretized functions. This research was partially supported by EBARA Corporation, Japan.

[1] A. Morimura, T. Aiki, Solvability of the moisture transport model for porous materials (submitted).

[MS17] Macrosegregation in a solidifying binary alloy

Michael Vynnycky

University of Limerick

Monday, 15:45, 1D226

The one-dimensional transient solidification of a binary alloy undergoing shrinkage is well-known as an invaluable benchmark for the testing of numerical codes that model macrosegregation, i.e.

variations in composition that occur in alloy castings or ingots as they solidify and can range in scale from several millimetres to centimetres or even metres [1, 2]. This talk gives an overview of our recent work on this topic [1, 2]. Mathematically, we arrive at a degenerate triple-moving-boundary problem, whose behaviour depends on the degree of solidification shrinkage and on the solute transport on the microscale. In addition to small-time analysis to determine the initial conditions, numerical computations are required to determine the solute profile across the entire solidified domain; these are carried out for the often-cited Al-Cu system.

Acknowledgments: The first author acknowledges the financial support of Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for the researcher grant (grant number 2016/12678-0). The second author acknowledges the financial support of FAPESP for the award of a visiting researcher grant (grant number 2018/07643-8).

[1] M. C. Flemings, G. N. Nereo, Macrosegregation. I. Trans. TMS-AIME 239, 1449–1461 (1967).

[2] C. R. Swaminathan, V. R. Voller, Towards a general numerical scheme for solidification systems. Int. J. Heat Mass Transf. 40, 2859–2868 (1997).

[3] M. Assunção, M. Vynnycky, S. L. Mitchell, On small-time similarity-solution behaviour in the solidification shrinkage of binary alloys, Eur. J. Appl. Maths 32, 199-225 (2021).

[4] M. Assunção, M. Vynnycky, On macrosegregation in a binary alloy undergoing solidification shrinkage, Eur. J. Appl. Maths 35, 40-61 (2024).

[MS17] Two-phase micropolar fluids: Phase field models and their analysis

Kei Fong Lam

Hong Kong Baptist University

Monday, 16:15, 1D226

Micropolar fluids are among the simplest cases of fluids with microstructures, where each fluid particle has its own internal rotations. Examples include ferrofluids, blood flows, bubbly liquids and liquid crystals, all of which play significant and important roles in various industries and also in the human body. Combining the seminal work of A. Cemal Eringen and coworkers, with the diffuse interface approach for multiphase fluid flow, we present some new diffuse interface models for binary mixtures of micropolar fluids that seem to be better amenable to further analysis. Using recent advances in the mathematical analysis of such types of models, we present some novel analytical results on existence of weak solutions for a particular diffuse interface model and, if time permits, we will discuss some asymptotic limits. This is a joint work with Baoli Hao, Bjorn Stinner and Kin Shing Chan.

Acknowledgments: Research Grants Council of the Hong Kong Special Administrative Region, China [Project No.: HKBU 14303420].

[MS17] Modeling morphology formation in organic solar cells via a nonlocal Cahn–Hilliard-type system

Rainey Lyons

Karlstad University

Monday, 16:45, 1D226

We utilize a nonlocal Cahn–Hilliard-type system to model morphology formation in a ternary mixture of interacting particles. The model studied was rigorously derived as the hydrodynamic limit of the Kawasaki dynamics for the three-state Blume–Capel model with long-range Kac interactions. In this talk, we discuss the existence, uniqueness, and regularity of solutions to the model. We will then discuss how one can adjust the equations to account for the evaporation of one of the components. Additionally, we develop a bound-preserving finite volume scheme for a simplified version of the model. We prove that the scheme not only preserves the analytical bounds on the solution but is also energy-stable.

Acknowledgments: I acknowledge the financial support of Carl Tryggers Stiftelse via the grant CTS 21:1656.

[MS17] Solvability of free boundary problems representing a bread baking process

Hana Kakiuchi

Japan Women's University

Friday, 8:45, 1D236

We consider a free boundary problem for a system of two parabolic equations on the one-dimensional space interval. The problem has been proposed as a mathematical model for a baking bread process in a hot oven (see [1, 2]). In their modeling they assume that the region consists of crumb, crust, and the evaporation front. For their problem, we add a free boundary condition obtained from the energy conservation law. The unknown functions of the problem are the position of the evaporation front which is regarded as the free boundary, the temperature field, and the mass distribution. Also, the temperature field and the mass distribution satisfy the standard parabolic equations in the crumb and crust regions, which are separated by the free boundary. Here, we note that the mass distribution appears in our free boundary condition and the mass may vanish. Moreover, the boundary condition for the mass contains the temperature field. By these difficulties, we discuss about solvability of the approximation problem in the present talk.

[1] E. Purlis and V. O. Salvadori, Bread baking as a moving boundary problem. Part1: Mathematical Modeling, Journal of Food Engineering, 91(2009), 434-442.

[2] B. Zanoni, S. Pierucci, C. Peri, Study of the bread baking process-II. Mathematical modeling, Journal of Food Engineering, 23(1994), 321-329.

[MS17] Solvability of a model for elastic curves in the energy conservation system

Chiharu Kosugi

Yamaguchi University

Friday, 9:15, 1D236

We consider the initial and boundary value problem for the beam equation. The problem was already proposed as a mathematical model for dynamics of elastic curves on the plane in [1]. The feature of our model is that the strain is non-linear and non-smooth with respect to the derivative of the unknown function. In order to deal with this strain, we assume that the stress function has a singular point and can obtain the lower boundedness of the strain. We have already proved the existence and uniqueness of a weak and strong solution to the problem with the viscosity term in [2]. For weak solutions, we have also proved weak solutions of the problem with the viscosity term converge to a weak solution of the problem without viscosity term in [3]. In this talk, we consider a convergence of strong solutions to the problem with viscosity term when the viscosity coefficient tends to zero. The key to this proof, is a priori estimate for solutions independent of the viscosity coefficient.

[1] C. Kosugi, T. Aiki, M. Anthonissen and M. Okumura, *Numerical Results for ordinary and partial differential equations describing motions of elastic materials,* Advances in Mathematical Sciences and Applications, **30**, 387-414(2021).

[2] C. Kosugi, *Solvability of a PDE model with nonlinear stress function having singularity for compressible elastic curve,* Advances in Mathematical Sciences and Applications, **32**, 155-177 (2023).

[3] C. Kosugi, *Existence and uniqueness of weak solutions for compressible elastic curves in the energy conservation system,* Advances in Mathematical Sciences and Applications, **32**, 447-454 (2023).

[MS17] Torsion-free frames and pressure-driven flow in thin curved tubes

John Fabricius

Luleå University of Technology

Friday, 9:45, 1D236

We analyze the asymptotic behavior of solutions to the Stokes system posed in a thin curved tube as the parameter ε, defined as the ratio between the radius of the cross section and the total length of the tube, tends to zero. An important aspect of the problem concerns the choice of "natural coordinates" that can be associated to a given simple smooth curve in space. To this end we develop a new concept called *torsion-free frame* which defines a coordinate system along the central line of the tube that considerably simplifies the analysis of the equations. The flow is assumed to be driven by an external pressure gradient which is modeled as a normal stress condition on the tube's ends. On the lateral boundary a no-slip condition is imposed for the velocity. We assume asymptotic expansions for the velocity and pressure fields, whose terms can be identified through local problems posed in the uniform cross section of the tube and geometrical invariants of the central line. From the leading terms of these expansions we construct an approximate solution to the problem which is verified by error estimates in H^1- and L^2-norms. The *frame invariance* of the obtained approximation is also discussed. The present study complements and expands on previous work [1–3].

Acknowledgments:

[1] E. Marušić-Paloka. *The effects of flexion and torsion on a fluid flow through a curved pipe,* Appl. Math. Optm. 44(3):245–272 (2001).

[2] E. Maršić-Paloka, I. Pažanin. *Fluid flow through a helical pipe.* Z. Angew. Math. Phys. 58(1):81–99 (2007).

[3] J. Fabricius, E. Miroshnikova, A. F. Tsandzana, P. Wall. Pressure-driven flow in thin domains. Asymptot. Anal. 116(1):1–26, (2020).

[MS17] Random walk approximation for a moving-boundary problem describing the penetration of a diffusant concentration into rubber

Surendra Nepal

Karlstad University

Friday, 10:15, 1D236

We propose a moving-boundary scenario in a one-dimensional setting to model the penetration of diffusants into rubbers. Driven by our interest in estimating how a finite number of diffusant molecules penetrate through a dense rubber, we propose a random walk algorithm to approximate numerically both the concentration profile and the location of the sharp penetration front. Our approach involves decoupling the evolution system in two steps: first, solving the ordinary differential equation corresponding to the evaluation of the speed of the moving-boundary using an explicit Euler method, and second, employing a random walk method to solve the diffusion problem. To verify the correctness of our random walk algorithm, we compare the resulting random walk approximations to computational results obtained via a finite element approach with a controlled convergence rate. Our numerical results recover well penetration depth measurements of a controlled experiment designed specifically for this setting.

Acknowledgments: This research was funded by the Swedish Research Council (VR – project nr. 2018-03648) and the Knowledge Foundation (KK – project nr. 2020-0152).

[MS18] A new hysteretic 1D-2D Richards model for fluid displacement in reservoirs

Catherine Choquet

La Rochelle Université

Tuesday, 8:45, 1B306

A classic model of fluid displacement in porous media is the Richards equation. While various approaches, especially numerical ones, exist to manage the inherent complexities of the equation (particularly its non-linearities and potential degeneracy as a function of saturation), it remains a challenging model for large-scale applications, especially those that involve the interaction of the Richards equation with fast-changing processes (*e.g.*, aquifer studies and management, underground hydrogen storage).

This talk presents an alternative to the three-dimensional Richards model. Although developed from an asymptotic analysis, this model is valid at any time scale in shallow aquifers. The novel approach couples a vertical 1D equation governing fast dynamics with a horizontal 2D equation for slow dynamics, effectively replacing the computationally expensive 3D model.

In asymptotic analysis, neglecting higher-order terms does lead to information loss. This is essential for simplification and tractability. But mathematically, this loss can manifest as the need to close the system with additional assumptions or when the resulting limit problem is not well-posed. In our case, both of these situations occur. In this talk, we'll see that by making sense of the model's weak solution we can effectively close the problem. Our mathematical analysis suggests incorporating a hysteresis phenomenon into the model closure. While this might be surprising from a purely mathematical standpoint, it aligns well with the established framework for multiphase models in porous media, leading to a coherent and physically consistent structure. We will showcase simulation examples demonstrating the performance of this theory. A remarkably simple code achieves results with excellent agreement with the Richards model, while significantly reducing computation time.

[MS18] Traveling waves for the porous medium equation in the incompressible limit

Charlotte Perrin

CNRS, I2M

Tuesday, 9:15, 1B306

In this talk, I will introduce a one-dimensional porous medium equation modeling the growth of living tissues. I will analyze the behavior and the stability of traveling waves solutions to this model in the so-called "incompressible limit". In this asymptotics, the pressure, which governs the diffusion process and limits the creation of cells in the tissue, becomes very stiff and the original PDE degenerates towards a free boundary problem of Hele-Shaw type. This is a joint work with Anne-Laure Dalibard and Gabriela Lopez-Ruiz.

[1] Dalibard, A.-L., Lopez-Ruiz, G., and Perrin, C. "Traveling waves for the porous medium equation in the incompressible limit: asymptotic behavior and nonlinear stability." Indiana University Mathematics Journal (2023).

[MS18] Richards flow in porous media with cross-diffusion

Pina Milisic

University of Zagreb

Tuesday, 9:45, 1B306

This is a joint work with Esther Daus (D-fine, Vienna, Austria) and Nicola Zamponi (University of Ulm, Germany).

We present a model describing the infiltration of contaminants into the upper layers of an unsaturated soil, where the water phase consists of a mixture of N chemical components subject to cross-diffusion effects. We show the existence of variational entropy solutions for a non-isothermal, immiscible, compressible Richards model with dynamic capillary pressure in porous media with large data. The primary variables are the mass concentrations of the components, the temperature and the water saturation of the porous medium. To overcome the lack of parabolicity in the equations due to cross-diffusion, we begin by proving the existence of solutions for an approximate system that is uniformly elliptic with respect to the entropy variables, i.e., the chemical potentials and the logarithm of temperature. A priori estimates are derived from the entropy balance and the total energy balance equation, while compactness is achieved using the div-curl lemma. We point out that the dynamic capillary pressure allows us to obtain the bound for the time derivative of saturation, which is crucial for the compactness argument.

[MS18] On the linear growth of the mixing zone in a semi-discrete model of Incompressible Porous Medium (IPM) equation

Iuliia Petrova

PUC-Rio

Tuesday, 10:15, 1B306

This is a joint work [1] with Prof. Sergey Tikhomirov (PUC-Rio, Rio de Janeiro, Brazil) and Prof. Yalchin Efendiev (Texas A&M University, College Station, USA).

In the talk we will discuss viscous / gravitational fingering phenomenon [4] — the unstable displacement of miscible liquids in porous media with the speed determined by Darcy's law. Laboratory and numerical experiments show the linear growth of the mixing zone [3,5]. Motivated by applications in enhanced oil recovery [2], we are interested in determining the exact speed of propagation of fingers. We consider two models: incompressible porous medium equation (IPM) and Transverse Flow Equilibrium (TFE). The existing theoretical upper bounds for the growth rate of the mixing zone (e.g. [6]) are higher than the observed speed from the numerical simulations. We believe that one of the possible mechanisms of slowing down the fingers' growth is due to convection in the transversal direction, and explain it by introducing a semi-discrete model of IPM and TFE. The model consists of a system of advection-reaction-diffusion equations on concentration, velocity and pressure in several vertical tubes (real lines) and interflow between them. In the simplest setting of two tubes our analysis reveals the structure of gravitational fingers — the mixing zone consists of space-time regions of constant intermediate concentration and the profile of propagation is characterized by two consecutive travelling waves which we call a terrace. We prove the existence of such a propagating terrace for the parameters corresponding to small distances between the tubes. The main tool in the proof is the reduction of IPM travelling wave dynamical system to TFE using geometric singular perturbation theory.

[1] Y. Petrova, S. Tikhomirov, Ya. Efendiev, *Propagating terrace in a two-tubes model of gravitational fingering* (2024), arXiv:2401.05981.

[2] F. Bakharev, A. Enin, K. Kalinin, Y. Petrova, N. Rastegaev and S. Tikhomirov, *Optimal polymer slugs injection profiles,* J. Comput. Appl. Math., **425** (2023), p.115042.

[3] F. Bakharev, A. Enin, A. Groman, A. Kalyuzhnyuk, S. Matveenko, Y. Petrova, I. Starkov and S. Tikhomirov, *Velocity of viscous fingers in miscible displacement: Comparison with analytical models,* J. Comput. Appl. Math., **402** (2022), p.113808.

[4] G.M. Homsy, *Viscous fingering in porous media,* Annu. Rev. Fluid Mech., **19(1)** (1987), pp.271-311.

[5] J.S. Nijjer, D.R. Hewitt, and J.A. Neufeld, *The dynamics of miscible viscous fingering from onset to shutdown,* J. Fluid Mech., **837** (2018), pp.520-545.

[6] G. Menon and F. Otto, *Dynamic scaling in miscible viscous fingering,* Comm. Math. Phys., **257** (2005), pp.303-317.

[MS18] Multiscale and numerical analysis of models for water flow through vegetated soil

Mariya Ptashnyk

Heriot-Watt University

Thursday, 15:15, 1D237

This is a joint work with A. Mair and L.X. Dupuy (NEIKER, Bilbao, Spain)

For modelling the water flow through vegetated soil we consider two approaches: the microscopic description of the water flow and update by plant roots on the scale of a single root branch and the macroscopic description by the generalised Richards equation. For a simple root system architecture we derive the macroscopic model from the microscopic description of the flow using the homogenization techniques. For a general plant root system we formulate the macroscopic preferential flow model that incorporates the impact of plant root architecture and the orientation of root branches on the flow and distribution of water in soil. Along with the multiscale analysis, the numerical analysis and simulation results for the preferential flow model (the generalised Richards equation) will be presented and different application scenarios for the new model will be discussed.

[1] A. Mair, L.X. Dupuy, M. Ptashnyk, *Model for water infiltration in vegetated soil with preferential flow oriented by plant roots.* Plant Soil, **478** (2022), 709–729.

[2] A. Mair, L.X. Dupuy, M. Ptashnyk, *Can root systems redistribute soil water to mitigate the effects of drought?* Field Crops Research **300** (2023), 109006.

[MS18] Asymptotic analysis of parabolic convection-dominated transport problems in thin graph-like networks

Taras Mel'nyk

University of Stuttgart / Institute of Applied Analysis and Numerical Simulation

Thursday, 15:45, 1D237

This is a joint work with Prof. C. Rohde (Institute of Applied Analysis and Numerical Simulation, University of Stuttgart, Stuttgart, Germany).

The presentation will be based on our new results and those currently published in [1–3].

The main object is time-dependent convection-diffusion problems with a high Péclet number in thin three-dimensional graph-like networks consisting of cylinders that are interconnected by small domains (nodes) with diameters of order $\mathcal{O}(\varepsilon)$. Inhomogeneous Robin boundary conditions with different intensity factors are considered on the lateral surfaces of the thin cylinders and the boundaries of the nodes. The main goal is to study the asymptotic behavior of solutions of such problems as ε tends to zero, i.e. when a thin network shrinks to a graph and the diffusion operator disappears in the limit.

A new asymptotic approach, which can take into account various factors, e.g., variable thickness of thin cylinders, inhomogeneous boundary conditions and different geometric characteristics of networks, is proposed.

The asymptotic expansions are constructed for solutions of linear problems and the energetic and uniform pointwise estimates are proved in [1, 2]. For solutions of semilinear [3] and quasilinear problems, asymptotic approximations are constructed and justified.

[1] T. Mel'nyk, C. Rohde, *Asymptotic expansion for convection-dominated transport in a thin graph-like junction.* Anal. Appl. (2024) https://doi.org/10.1142/S0219530524500040

[2] T. Mel'nyk, C. Rohde, *Puiseux asymptotic expansions for convection-dominated transport problems*

in thin graph-like networks: Strong boundary interactions. Asymptot. Anal. **137** (2024), 1–26. DOI: 10.3233/ASY-231876

[3] T. Mel'nyk, C. Rohde, *Asymptotic approximations for semilinear parabolic convection-dominated transport problems in thin graph-like networks.* J. Math. Anal. Appl. **529** (2024), 127587 https://doi.org/10.1016/j.jmaa.2023.127587

[MS18] Quantitative derivation of the porous medium equation from the compressible Navier-Stokes equations

Richard Höfer

Regensburg University

Thursday, 16:15, 1D237

This is a joint work with Šárka Nečasová and Florian Oschmann (Czech Academy of Sciences, Czech Republic).

We consider the solutions $\rho_\varepsilon, u_\varepsilon$ to the compressible Navier-Stokes equations (NSE) in a domain periodically perforated by holes of diameter $\varepsilon > 0$. We focus on the case where the diameter of the holes is of the same order as the distance between neighboring holes. This is the same setting investigated in the paper by Masmoudi [*ESAIM: COCV, 8, 885-906 (2002)*], where convergence $\rho_\varepsilon, u_\varepsilon$ of the system to the porous medium equation has been shown. We prove a quantitative version of this convergence result provided that the solution of the limiting system is sufficiently regular. The proof builds on the relative energy inequality satisfied by the compressible NSE.

[MS18] Solvability and numerical study of two-scale system with nonlinear dispersion

Vishnu Raveendran

Karlstad University

Thursday, 16:45, 1D237

This is a joint work with S. Nepal (Karlstad University, Sweden), R. Lyons (Karlstad University, Sweden), M. Eden (Karlstad University, Sweden), and A. Muntean (Karlstad University, Sweden).

We discuss the solvability and numerical simulations of a strongly coupled two-scale system with nonlinear dispersion. The two-scale system consists of a nonlinear reaction-dispersion problem coupled through a dispersion tensor to a system of elliptic cell problems. The problem is motivated by a rigorously derived effective model of a reaction-diffusion equation with large nonlinear drift, originating from an interacting particle system crossing a regular porous media.

[MS18] **Existence of Weak Solutions to a Cahn–Hilliard–Biot System**

Jonas Haselböck

Universität Regensburg

Friday, 8:45, 1D237

This is a joint work with Helmut Abels and Harald Garcke (Universität Regensburg, Regensburg, Germany).

We prove existence of weak solutions to a diffuse interface model describing the flow of a fluid through a deformable porous medium consisting of two phases. The system non-linearly couples Biot's equations for poroelasticity, including phase-field dependent material properties, with the Cahn–Hilliard equation to model the evolution of the solid, and is further augmented by a visco-elastic regularization consistent with secondary consolidation. The strategy of the proof is to approximate the problem in two steps, where first a semi-Galerkin ansatz is employed to show existence of weak solutions to regularized systems, for which later on compactness arguments allow limit passage. Notably, we also establish a maximal regularity theory for linear visco-elastic problems.

[MS18] **Interaction Between a Fluid and a Multilayered Poroelastic Structure with Membrane**

Andrew Scharf

University of California, Berkeley

Friday, 9:15, 1D237

This is a joint work with Sunčica Čanić (University of California, Berkeley, USA) and Martina Bukač (University of Notre Dame, Notre Dame, USA).

Multilayered poroelastic structures are found in many biological tissues, such as cartilage and the cornea, and find use in the design of bioartificial organs and other bioengineering applications. Motivated by these applications, we analyze the interaction of a free fluid flow modeled by the time-dependent Stokes equation and a multilayered poroelastic structure consisting of a thick Biot layer and a thin, linear, poroelastic membrane separating it from the Stokes flow. The resulting equations are linearly coupled across the thin structure domain through physical coupling conditions such as the Beavers-Joseph-Saffman condition. Previous work has been done in which weak solutions were shown to exist by constructing approximate solutions using Rothe's method. While a number of partitioned numerical schemes have been developed for the interaction of Stokes flow with a thick Biot structure, the existence of an additional thin poroelastic plate in the model presents new challenges related to finite element analysis on multiscale domains. We develop a novel, fully discrete partitioned method for the multilayered poroelastic fluid-structure problem based on the fixed strain Biot splitting method and show stability of the splitting scheme under a time-step condition. We also present numerical results showing convergence plots for manufactured solutions and validate the solver with a comparison to the Stokes-Biot model with no thin interface.

[MS18] An adaptive solution strategy for Richards' equation

Jakob Seierstad Stokke

University of Bergen

Friday, 9:45, 1D237

This is a joint work with Koondanibha Mitra (Eindhoven University of Technology, Eindhoven, Netherlands), Erlend Storvik (Western Norway University of Applied Sciences, Førde, Norway), Jakub W. Both (University of Bergen, Bergen, Norway) and Florin A. Radu (University of Bergen, Bergen, Norway).

Flow in variably saturated porous media is typically modeled by Richards' equation, which is a nonlinear elliptic-parabolic equation. Despite its many important hydrogeological applications, this equation has a reputation for being difficult to solve numerically. Indeed common linearization techniques like Newton's or Picard's methods may not converge, while more robust methods like the L-scheme may converge unacceptably slow.

This work proposes a robust and efficient switching algorithm for Richards' equation. The algorithm is based upon the L-scheme and Newton's method, thus utilizing the robustness of the L- scheme and quadratic convergence of Newton's method. We derive reliable a posteriori indicators for the algorithm's switching criteria. The algorithm's performance is illustrated through realistic numerical examples and compared to the L-scheme and Newton's method in terms of the number of iterations and computational time.

[1] Stokke, J.S., Mitra, K., Storvik, E., Both, J.W., Radu, F.A.: An adaptive solution strategy for Richards' equation. Comput. Math. Appl. **152**, 155–167 (2023)

[2] List, F., Radu, F.A.: A study on iterative methods for solving Richards' equation. Comput. Geosci. **20**(2) 341–353 (2016)

[MS18] A morpho-poroelastic model applied to soft tissues

Fred Vermolen

University of Hasselt

Friday, 10:15, 1D237

This is a joint work with Ginger Egberts (UHasselt & Dutch Burns Foundation) and Sabia Asghar (UHasselt).

Biological tissues are often subject to regeneration or deterioration. For this reason, we consider an extension of a poro-elastic model (Biot) to the growth and shrinkage of tissues. The model equations are given by

$$\begin{cases} \dfrac{D}{Dt}(\rho\mathbf{v}) + \rho(\nabla\cdot\mathbf{v})\mathbf{v} - \nabla\cdot\sigma = \mathbf{f}, \\ \sigma = 2\mu\varepsilon + \lambda\ \mathrm{tr}(\varepsilon)I + \mu_1\ \mathrm{sym}(\nabla\mathbf{v}) + \mu_2\ \mathrm{tr}(\mathrm{sym}(\nabla\mathbf{v}))I - pI, \\ \dfrac{D\varepsilon}{Dt} + \varepsilon\ \mathrm{skw}(\nabla\mathbf{v}) - \mathrm{skw}(\nabla\mathbf{v})\varepsilon + (\mathrm{tr}(\varepsilon)-1)\mathrm{sym}(\nabla\mathbf{v}) = -G, \qquad \text{in } \Omega,\ t>0. \\ \dfrac{D\mathbf{u}}{Dt} = \mathbf{v}, \\ \nabla\cdot\mathbf{v} = \nabla\cdot(\kappa\nabla p). \end{cases} \tag{25}$$

This set of equations is completed with initial and boundary conditions and the problem is solved for the displacement velocity $\mathbf{v}$ and strain tensor ε. The (third) equation for the strain is characteristic for morpho-elasticity. The G–tensor models growth or shrinkage of tissue as a result of microstructural changes. The rest of the equations correspond to Biot's poro-elasticity model with inertia. This model extension has important applications in tumor growth and in the contraction of skin after deep tissue injury.

The talk will present the model equations, which as far as we know, provides an original problem setting. Furthermore, the talk will address the symmetry and its stability of the strain tensor ε. Stability with respect to perturbations around the steady–state solutions of the solutions to both the fully continuous and semi-discrete problems will be presented. In a more elaborate model where the biochemical processes are modelled in injured skin, this find its relevance in determining the stability of the undamaged versus the damaged states. The linear stability assessment is based on Fourier analysis. It is further known that numerical approximations based on finite element methods applied to Biot's model are sensitive to loss of monotonicity. This loss of monotonicity results due to the fact that the linear system does not always provide an M–matrix. Application of conforming finite element strategies based on Taylor–Hood, Crouzeix–Raviart or the mini–element mitigate the condition for monotonicity only to a very limited, not even significant extent. Though loss of monotonicity does not occur in the purely morphoelastic system, loss of monotonicity of the numerical solution can also occur in the current morpho-poroelastic problem. The talk will address conditions for monotonicity of the numerical solution, as well as conditions for stabilization of the finite element solution that lead to a guarantee of the monotonicity of the numerical solution.

[MS19] Measure differential equations with applications to model-based analysis of single cell data of adult neurogenesis

Anna Marciniak-Czochra

Heidelberg University

Tuesday, 15:15, 21A342

High-dimensional data arise in the description of the dynamics and control of heterogeneous systems. Models that describe the evolution of heterogeneous systems, particularly those based on local and non-local growth and transport processes, are used in various applications, such as crowd dynamics, tissue regeneration, tumour growth, and coagulation-fragmentation processes. The processes can be described using the language of measure evolution on a multidimensional state space, which presents mathematical challenges related to model formulation, asymptotic analysis, and numerical simulation and parameter estimation. This talk is devoted to formulation and numerical approximation of measure structured population models on Polish metric spaces. Understanding the structure of the state space is currently one of the major challenges in the analysis of single cell data, which is becoming one of the main tools in the analysis of cell-based processes. Model-based approaches can be a useful tool to complement the topology-based, geometric or statistical approaches currently used for data analysis. We present first transcriptome-structured population model and discuss its applicability in context of single cell data of adult neurogenesis.

[MS19] Monotone vector fields in the Wasserstein space

Nickolay Pogodaev

Università degli Studi di Padova

Tuesday, 15:45, 21A342

The concept of a maximal monotone operator is well-established in the Hilbert setting, where such operators are closely related to partial differential equations generating nonexpansive semigroups [1]. In many cases, maximal monotone operators have a gradient structure, meaning that they can be represented as subdifferentials of proper convex lower semicontinuous functions. The corresponding partial differential equations are usually referred to as gradient flows. The theory of gradient flows was extended from the Hilbert to the Wasserstein setting in the early 2000s. Initiated in the papers [2,3], it took its classical form in the monograph [4]. In the present talk, we explore the prospect of defining a monotone vector field in the Wasserstein space without necessarily adhering to a gradient form. We discuss the corresponding variant of the JKO scheme, provide examples of such operators, and explore potential applications.

[1] H. Brézis, Opérateurs maximaux monotones et semi-groupes de contractions dans les espaces de Hilbert. North-Holland Pub. Co, Amsterdam, 1973.

[2] R. Jordan, D. Kinderleherer, F. Otto, *The Variational Formulation of the Fokker–Planck Equation.* SIAM J. Math. Anal. **29** (1998), 1–17.

[3] F. Otto, *The geometry of dissipative evolution equations: The porous medium equation.* Comm. in Partial Differential Equations, **26** (2001), 101–174.

[4] L. Ambrosio, N. Gigli, G. Savarè, Gradient flows: in metric spaces and in the space of probability measures. Birkhäuser, Boston, 2005.

[MS19] Dissipative measure-valued solutions to the Euler-Poisson equation

Tomasz Debiec

University of Warsaw

Tuesday, 16:15, 21A342

We consider several pressureless variants of the compressible Euler equation driven by nonlocal repulsion-attraction and alignment forces with Poisson interaction. Under an energy admissibility criterion, we prove existence of global measure-valued solutions, i.e., very weak solutions described by a classical Young measure together with appropriate concentration defects. We then investigate the evolution of a relative energy functional to compare a measure-valued solution to a regular solution emanating from the same initial datum. This leads to a weak-strong uniqueness type of result.

Based on joint work with J. A. Carrillo, P. Gwiazda, and A. Świerczewska-Gwiazda [1].

[1] J. A. Carrillo, T. Debiec, P. Gwiazda, A. Świerczewska-Gwiazda *Dissipative measure-valued solutions to the Euler-Poisson equation.* SIAM J. Math. Anal. **56**(1) (2024), 304–335.

[MS19] Computationally efficient simulation of cells that release diffusing compounds in their environment

Qiyao Peng

Leiden University

Tuesday, 16:45, 21A342

This is a joint work with Dr. Sander C. Hille (Leiden University, Leiden, The Netherlands).

In biomedical applications, there are many interactions between single objects and the direct environment, and one of the most important interactions is conducted by the diffusion of the compounds, such as signalling molecules. For example, in wound healing, immune cells release cytokines to attract skin cells to migrate towards the wound region; in drug delivery, microbubbles release the drug molecules to the targeted region.

For the sake of computational efficiency, in mathematical modelling and for theoretical purposes, the Dirac Delta distributions are often utilized as a replacement for cells or vesicles, since the size of cells or vesicles is much smaller than the size of the surrounding tissues. Here, we consider the scenario that the cell or the vesicle releases the diffusive compounds to the immediate environment, which is modelled by the diffusion equation. Typically, one separates the intracellular and extracellular environment and uses homogeneous Neumann boundary condition for the cell boundary (so-called spatial exclusion approach), while the point source approach neglects the intracellular environment. We show that extra conditions are needed such that the solutions to the two approaches are consistent. We prove a necessary and sufficient condition for the consistency. Suggested by the numerical results, we conclude that an initial condition in the form of a Gaussian kernel in the point source approach compensates for a time-delay discrepancy between the solutions to the two approaches in the numerical solutions. Various approaches determining optimal intensity and variance of the Gaussian kernel have been discussed; see [1] for more details.

For the homogeneous flux density from the cell membrane, one can easily replace each cell with one Dirac point at the cell centre. However, for inhomogeneous flux density, for instance, the sinusoidal function, multiple Dirac points are needed besides the cell centre to represent the cell [2], which forms a dipole or tripole system – similar to the cases in electromagnetism. In this part, the challenge lies on determining the intensities of the Dirac delta points to minimize the discrepancy between the solutions to the two models, i.e. to ensure the consistency between the flux density over the cell boundary in the two models.

[1] Q. Peng and S.C. Hille. Comput. Math. Appl. **151** (2023) 491–507. Ordinary differential equations. John Wiley, New York, 1964.

[2] Q. Peng and S.C. Hille. ArXiv 2401.16261 (2024).

[MS19] Discretization of the granular medium equation via gradient flows

Błażej Miasojedow

University of Warsaw

Friday, 8:45, 21A342

In this study, we examine the discretization of the non-linear Fokker-Planck equation. By analyzing the second Wasserstein distance, we establish uniform in time error bounds for particle approximation. We explore the variational formulation of the non-linear Fokker-Planck equation and demonstrate that particle approximation can be viewed as an inexact gradient flow of the free energy functional. To address non-local interaction kernels, we draw upon insights from the propagation of chaos theory. This presentation is based on joint research with Matej Benko, Iwona Chlebicka, and Jorgen Endal.

[MS19] Computation of distances between measures in Fortet-Mourier metric

Sander Hille

Leiden University

Friday, 9:15, 21A342

This is joint work with former master student at Leiden University Esmée Theewis
(now: Delft University of Technology, Delt, The Netherlands).

The Fortet-Mourier metric is a distance on the convex cone of all finite positive Borel measures on a Polish space that metrizes the weak topology defined by pairing with the continuous bounded functions that is commonly used in probability theory and statistics. Although it has been around since the 1953 paper by Fortet and Mourier [1], there has been no general algorithms to compute its value for specific classes of measures, like positively weighted sums of Dirac measures (so-called '*molecular measures*' in the terminology of Jan Pachl). On computing distances among molecular measures on the real line or an interval thereof only a preprint appeared [4].

The ability to compute such distances becomes relevant when one starts to compare Interacting Particle Systems (IPSs) with continuum models in a measure-framework, or if one wants to compare different IPSs within such a framework. We shall briefly introduce these settings and then disucuss recent progress that was achieved in [2]. It provided some novel exact formulas for the Fortet-Mourier distance and allowed to compute effectively the distance between molecular measures on any Polish space. Moreover, it provided methods to compute the distance between a molecular measure and one and a non-molecular measure, e.g. one given by a density relative to a reference measure. Also, some novel explicit exact formulas for Fortet-Mourier distances were obtained.

If time permits it will be sketched how these computational questions relate to the question of characterising a suitably dense set of extreme point of the unit ball of Banach spaces of Lipschitz functions [3].

[1] R. Fortet, E. Mourier, *Convergence de la répartition empirique vers la répartition théorique*, Ann. Sci. Éc. Norm. Supér. (3) **70**(3) (1953), 267–285.

[2] S.C. Hille, E.S. Theewis, *Explicit expressions and computational methods for the Fortet-Mourier distance of positive measures to finite weighted sums of Dirac measures*, J. Approx. Theory **294** (2023), 105947.

[3] S.C. Hille, E.S. Theewis, *Norming and dense sets of extreme points of the unit ball in spaces of bounded Lipschitz functions*, J. Math. Anal. Appl. **536** (2024), 128200.

[4] J. Jabłoński, A. Marciniak-Czochra, *Efficient algorithms computing distances between Radon measures on* $\mathbb{R}$. arXiv preprint.

[MS19] Strong convergence of the vorticity. From Navier-Stokes to Euler in a bounded domain

Jakub Woźnicki

University of Warsaw

Friday, 9:45, 21A342

This is a joint work with Christian Seis (University of Munster, Germany), Emil Wiedemann (University of Erlangen-Nuremberg, Germany).

In the vanishing viscosity limit from the Navier-Stokes to Euler equations on domains with boundaries, a main difficulty comes from the mismatch of boundary conditions and, consequently, the possible formation of a boundary layer. Within a purely interior framework, Constantin and Vicol showed that the two-dimensional viscosity limit is justified for any arbitrary but finite time under the assumption that on each compactly contained subset of the domain, the enstrophies are bounded uniformly along the viscosity sequence. Within this framework, we upgrade to local strong convergence of the vorticities under a similar assumption on the p-enstrophies, $p > 2$. The key novel idea is the analysis of the evolution of the weak convergence defect.

[MS20] A global bifurcation diagram for a one-parameter family of nonautonomous scalar ODE's driven by a minimal flow

Roberta Fabbri

Università di Firenze

Tuesday, 15:15, 1D226

This is a joint work with Carmen Núñez (Universidad de Valladolid, Spain).

The description of nonautonomous bifurcation patterns is of growing interest in the scientific community, both for its theoretical interest and for its possible applications to the analysis of mathematical models. In the talk, in the wake of [1] and using results and methods of [2], conditions on the coefficients of the one-parameter family

$$x'(t) = \varepsilon\,(a(t) + b(t)\,x(t)) + c(t)\,x^2(t) - x^3(t) \tag{26}$$

as ε varies are established to describe the global diagram of the motion. Such conditions include the recurrency of the coefficients and the analysis is based on the study of the number and the structure of the minimal invariant subsets of the extended state space for the corresponding skew-product flow induced by the solutions of equations (26).

[1] R. Fabbri, R. Johnson, F. Mantellini, *A nonautonomous saddle-node bifurcation pattern*, Stoch. Dyn. **4** (2004), no 3, 335–350.

[2] J. Dueñas, C. Núñez., R. Obaya, *Bifurcation theory of attractors and minimal sets in d-concave nonautonomous scalar ordinary differential equations*, J. Differential Equations, **361** (2023) 138–182.

[MS20] Critical transitions for asymptotically concave ordinary differential equations

Carmen Núñez

Universidad de Valladolid

Tuesday, 15:45, 1D226

This is a joint work with Jesús Dueñas and Rafael Obaya (Universidad de Valladolid, Spain).

The occurrence of tracking or tipping situations for a transition equation $x' = f(t, x, \Gamma(t, x))$ is analyzed under the assumption on concavity in x of the maps giving rise to the asymptotic equations $x' = f(t, x, \Gamma_{\pm}(t, x))$, but without assuming this condition on the transition equation itself. The approaching condition is just $\lim_{t\to\pm\infty}(\Gamma(t, x) - \Gamma_{\pm}(t, x)) = 0$ uniformly on compact real sets, and so there is no restriction to the dependence on time of the limit equations. The analysis provides a powerful tool to analyze the occurrence of critical transitions for one-parametric families $x' = f(t, x, \Gamma^{c}_{\pm}(t, x))$. The new approach significatively widens the field of application of the results, since the evolution law of the transition equation can be essentially different from those of the limit equations. As an application, a scalar population dynamics model subject to non trivial predation and migration is analyzed.

[MS20] Evans function, parity and nonautonomous bifurcation

Christian Poetzsche

University of Klagenfurt

Tuesday, 16:15, 1D226

This is a joint work with Iacopo Longo (Imperial College, London, UK) and Robert Skiba (Nicolaus Copernicus University, Toruń, Poland).

We provide an approachable and yet flexible sufficient condition for the bifurcation of bounded entire solutions to nonautonomous ordinary differential equations. This requires to relate the parity, which is a crucial tool in the abstract bifurcation theory of nonlinear Fredholm operators to the Evans function, an established concept for the stability analysis of traveling waves to evolutionary differential equations.

Our approach covers both single and multiparameter problems. Based on topological methods, in the latter case we additionally obtain information concerning the Lebesgue covering dimension of the parameter set yielding bifurcations.

[1] J. Pejsachowicz, P. Rabier. *Degree theory for C^1 Fredholm mappings of index* 0, Journal d'Analyse Mathématique 76(1), 289–319, 1998.

[2] C. Pötzsche. *Nonautonomous bifurcation of bounded solutions: Crossing curve situations.* Stoch. Dyn. 12(2), 1150017, 2012.

[3] B. Sandstede. *Stability of travelling waves*, in Handbook of Dynamical Systems 2 (ed. B. Fiedler), Elsevier Science, Amsterdam, 983–1055, 2002.

[MS20] Multiplicity results for a class of asymptotically linear systems of second-order ordinary differential equations

Francesca Dalbono

University of Palermo

Tuesday, 16:45, 1D226

We study multiplicity of solutions to a Dirichlet problem associated with a planar system of the form

$$\begin{cases} u''(t) + A(t, u(t))u(t) = 0, & t \in [0, \pi], \\ u(0) = u(\pi) = 0, \end{cases} \tag{27}$$

where $A : [0, \pi] \times \mathbb{R}^2 \to GL_s(\mathbb{R}^2)$ is a continuous function satisfying asymptotically linear conditions.

The existence of two sign-preserving component-wise solutions is guaranteed when the Morse indexes of the linearizations at zero and at infinity do not coincide, and one of the asymptotic problems has zero-index, cf. [1].

We highlight that, under suitable extra assumptions involving the sign of the entries of the nonlinearity $A(t, x)$, the gap between the Morse indexes of the linearizations at zero and infinity provides a lower estimate on the number of solutions of problem (27), cf. [2].

Our result represents a first step in the direction of studying multiplicity of solutions to asymptotically linear Dirichlet systems in absence of a Hamiltonian structure and in absence of symmetric assumptions on the space variable. The proof is developed in the framework of topological and shooting methods. Our approach is based on conjugate points theory and on the concept of phase angles: we develop a constructive argument based on some elementary tools of phase plane analysis to determine an explicit expression of the two phase angles which allows us to visualize their reciprocal motions in the phase plane.

Acknowledgements: This work was partially supported by the PRIN project 2017JPCAPN "*Qualitative and quantitative aspects of nonlinear PDEs*", and by GNAMPA-INdAM.

[1] F. Dalbono, *Sign-preserving solutions for a class of asymptotically linear systems of second-order ordinary differential equations.* Topol. Methods Nonlinear Anal. **59** (2022), 163–191.

[2] F. Dalbono, C. Rebelo, *Multiplicity of solutions of Dirichlet problems associated with second order equations in* $\mathbb{R}^2$. Proc. Edinb. Math. Soc. (2) **52** (2009), 569–581.

[MS20] Oscillation theory and instability of nonlinear waves

Peter Howard

Texas A& M University

Thursday, 8:45, 1D341

In recent work, Baird et al. have introduced a generalized Maslov index which allows oscillation techniques that have previously been restricted to eigenvalue problems with underlying Hamiltonian structure to be extended to the non-Hamiltonian setting [1]. We show that this approach can be implemented in the analysis of spectral instability for nonlinear waves, taking as our setting a class of equations previously investigated by Pego and Weinstein via the Evans function [2].

[1] T. J. Baird, P. Cornwell, G. Cox, C. Jones, and R. Marangell, *Generalized Maslov indices for non-Hamiltonian systems*, SIAM J. Math. Anal. **54** (2022) 1623-1668.

[2] R. L. Pego and M. I. Weinstein, *Eigenvalues, and instabilities of solitary waves*, Phil. Trans. R. Soc.

Lond. A **340** (1992) 47-94.

[MS20] Renormalized oscillation theory for Hamiltonian pencils

Alim Sukhtayev

Miami University

Thursday, 9:15, 1D341

This is a joint work with Peter Howard (Texas A&M University, College Station, USA).
Working with a general class of linear Hamiltonian systems with nonlinear dependence on the spectral parameter, we show that renormalized oscillation results can be obtained in a natural way through consideration of the Maslov index associated with appropriately chosen paths of Lagrangian subspaces. By reduction to a generalized nonlinear eigenvalue problem, we apply our results to a class of models such as magneto-hydrodynamics systems and the Saint-Venant equations.

[MS20] New directions in oscillation theory of linear Hamiltonian systems

Peter Šepitka

Masaryk University

Thursday, 9:45, 1D341

This is a joint work with Roman Šimon Hilscher (Masaryk University, Brno, Czech Republic).
We present a new approach for the study of the oscillation properties of linear differential equations, in particular of linear Hamiltonian systems. We introduce a new notion of a generalized focal point as well as its multiplicity, which do not depend on the validity of the traditionally assumed Legendre condition. We provide a local version of the Sturmian separation theorem, which involves a lower bound and an upper bound for the multiplicity of a generalized focal point for any conjoined basis of the system. As a main tool we use the comparative index, which was originally applied in the discrete oscillation theory.

[1] O. Došlý, J. V. Elyseeva, R. Šimon Hilscher, Symplectic Difference Systems: Oscillation and Spectral Theory. Pathways in Mathematics, Birkhäuser/Springer, Cham, 2019.

[2] J. V. Elyseeva, *Comparative index for solutions of symplectic difference systems*, Differential Equations **45** (2009), no. 3, 445–459.

[3] P. Šepitka, R. Šimon Hilscher, *Comparative index and Sturmian theory for linear Hamiltonian systems*, J. Differential Equations **262** (2017), no. 2, 914–944.

[4] P. Šepitka, R. Šimon Hilscher, *Generalized focal points and local Sturmian theory for linear Hamiltonian systems*, Discrete Contin. Dyn. Syst. **43** (2023), no. 12, 4139–4173.

[MS20] Generalized oscillations for linear Hamiltonian systems via Maslov index theory

Roman Simon Hilscher

Masaryk University

Thursday, 10:15, 1D341

This is a joint work with Peter Šepitka (Masaryk University, Brno, Czech Republic).
We present a generalized oscillation theory for linear Hamiltonian systems, also called canonical systems of differential equations, without assuming the Legendre condition. We show that this theory can be effectively developed via the comparative index of two Lagrangian planes. We present a connection of the comparative index with the Maslov index or the Hörmander index or the triple index, for which we use the Lidskii angles of a symplectic fundamental matrix of the system. We analyze and explain the exact role of the Legendre condition and the role of the minimal multiplicities of focal points in the (generalized) oscillation theory.

[1] O. Došlý, J. V. Elyseeva, R. Šimon Hilscher, Symplectic Difference Systems: Oscillation and Spectral Theory. Pathways in Mathematics, Birkhäuser/Springer, Cham, 2019.

[2] J. Elyseeva, P. Šepitka, R. Šimon Hilscher, *Oscillation numbers for continuous Lagrangian paths and Maslov index*, J. Dynam. Differential Equations **35** (2023), no. 3, 2589–2620.

[3] J. Elyseeva, P. Šepitka, R. Šimon Hilscher, *Comparative index and Hörmander index in finite dimension and their connections*, Filomat **37** (2023), no. 16, 5243–5257.

[4] P. Šepitka, R. Šimon Hilscher, *Comparative index and Sturmian theory for linear Hamiltonian systems*, J. Differential Equations **262** (2017), no. 2, 914–944.

[5] P. Šepitka, R. Šimon Hilscher, *Comparative index and Lidskii angles for symplectic matrices*, Linear

Algebra Appl. **624** (2021), 174–197.

[6] P. Šepitka, R. Šimon Hilscher, *Generalized focal points and local Sturmian theory for linear Hamiltonian systems*, Discrete Contin. Dyn. Syst. **43** (2023), no. 12, 4139–4173.

[MS21] Numerical approximations to nonlinear dispersive equations in non-smooth regimes

Yvonne Alama Bronsard

Sorbonne Université, LJLL

Tuesday, 15:15, 1D236

This talk deals with the numerical approximation to nonlinear dispersive equations, such as the prototypical nonlinear Schrödinger equation. We introduce novel integration techniques allowing for the construction of schemes which perform well both in smooth and non-smooth settings. We obtain symmetric low-regularity schemes with very good structure preserving properties over long times, see [1] and [2].

Higher order extensions will be presented, following new techniques based on decorated trees series inspired by singular stochastic PDEs via the theory of regularity structures. We will consider both the case of deterministic and randomized initial data.

These new schemes, together with their optimal local error, allow for convergence under lower regularity assumptions than required by classical methods, such as exponential integrator or splitting methods.

This includes joint work with Yvain Bruned (Université de Lorraine, France), Georg Maierhofer (University of Cambridge, UK), and Katharina Schratz (Sorbonne Université, France).

[1] Y. Alama Bronsard, *A symmetric low-regularity integrator for the nonlinear Schrödinger equation.* IMA J. Numer. Anal. (2023) drad093.

[2] Y. Alama Bronsard, Y. Bruned, G. Maierhofer, and K. Schratz, *Symmetric resonance based integrators and forest formulae.* arXiv preprint arXiv:2305.16737, 2023.

[MS21] Spectral approaches to solutions that are non-trivial at infinity

Nikola Stoilov

Institut de Mathematiques de Bourgogne

Tuesday, 15:45, 1D236

This is a joint work with Christian Klein (Université de Bourgogne).
We present several methods for numerically finding solutions to partial differential equations as well as fractional differential equations, that have non-trivial behaviour at infinity. Generally, the compactified real line is divided into a number of intervals, thus amounting to a multidomain approach; after transformations ensuring analyticity of the respective integrands, the integrals over the different domains are computed with a Clenshaw–Curtis algorithm. As examples, we consider solutions the NLS and KdV as well as solitary waves for fractional Korteweg-de Vries equations.

[1] C. Klein, J. Prada-Malagon, N. Stoilov, *On numerical approaches to nonlinear Schrödinger and Korteweg–de Vries equations for piecewise smooth and slowly decaying initial data*, Phys. D **454**, (2023), 133885

[2] C. Klein, N. Stoilov, *Multidomain spectral approach to rational-order fractional derivatives* Stud. Appl. Math. (2024), 1–23.

[MS21] Collision of two solitary waves for the Zakharov-Kuznetsov equation

Frédéric Valet

CY Cergy Paris Université

Tuesday, 16:15, 1D236

This is a joint work with Prof Didier Pilod (University of Bergen, Norway).

The Zakharov-Kuznetsov (ZK) equation in dimension 2 is a generalization in plasma physics of the one-dimensional Korteweg-de Vries equation. Both equations admit solitary waves, that are solutions moving in one direction at a constant velocity, vanishing at infinity in space. Consider a solution behaving at $-\infty$ as a sum of two solitary waves for ZK, with nearly equal size and almost same abscissa. As time goes by, the two solitary waves strongly interact and collide. In particular, the non-linear effects rule the behaviour of the solitary waves. In this talk based on [1], I will give a precise approximation of the structure of the solution on the whole time interval and then answer the question of stability of this phenomenon.

[1] D. Pilod, F. Valet, *Dynamics of the collision of two nearly equal solitary waves for the Zakharov-Kuznetsov equation.* Submitted, arXiv:2403.02262, (2024).

[MS21] Soliton Gas for Toda Lattice, rigorous long-time asymptotics

Bingying Lu

SISSA

Tuesday, 16:45, 1D236

We analytically study the long time asymptotics of a class of solution that describes then N-soliton gas, where $N \to \infty$. We formulate this problem by a Riemann-Hilbert problem that we can rigorously analyze. Using the soliton gas formulation, we study the dynamics of the solution with step-like initial data. Dispersive shock or rarefraction region ensues, depending on the initial step, and in long time the solution decomposes to three different regions, where their asymptotics are described with the Riemann-Hilbert problem. This work is joint with Tamara Grava.

[MS21] Spectral estimates for Schrödinger operators with Neumann boundary conditions on Hölder domains

Charlotte Isabel Dietze

LMU Munich

Wednesday, 8:45, 1D327

We prove a universal bound for the number of negative eigenvalues of Schrödinger operators with Neumann boundary conditions on bounded Hölder domains, under suitable assumptions on the Hölder exponent and the external potential. Our bound yields same semiclassical behaviour as the Weyl asymptotics for smooth domains. We also discuss different cases where Weyl's law holds and fails.

[MS21] Quantitative controllability and stability for KdV equations

Jingrui Niu

Sorbonne Université

Wednesday, 9:15, 1D327

This is a joint work with Ludovick Gagnon (Université de Lorraine, Nancy, France) and Shengquan Xiang (Peking University, Beijing, China).

We study the following KdV system

$$\begin{cases} \partial_t y + \partial_x^3 y + \partial_x y = 0 & \text{in } (0,T)\times(0,L), \\ y(t,0) = y(t,L) = 0 & \text{in } (0,T), \\ \partial_x y(t,L) = u(t) & \text{in } (0,T), \\ y(0,x) = y^0(x) & \text{in } (0,L). \end{cases} \tag{28}$$

First, we establish a quantitative null controllability result with the control cost

$$\|u\|_{L^2(0,T)} \leq Ce^{\frac{C}{T^{1/2}}} \|y^0\|_{L^2(0,L)}. \tag{29}$$

On the other hand, we also establish the following quantitative exponential energy decay for (28) with $u = 0$:

$$\|y(t)\|_{L^2(0,L)} \leq e^{-C(L)t} \|y^0\|_{L^2(0,L)}, \tag{30}$$

where $C(L)$ is an effectively computable constant and $C(L) \sim dist(L, \mathcal{N})$ as L approaches the critical length set $\mathcal{N}$.

[MS21] A variational approach to a fuzzy Boltzmann equation

Zihui He

Bielefeld University

Wednesday, 9:45, 1D327

We study a fuzzy Boltzmann equation where the particles interact via delocalized collisions.

We discuss the existence and uniqueness of solutions and their convergence to the solutions of classical Boltzmann equations. We also provide a variational characterization casting the fuzzy Boltzmann equation into the framework of GENERIC (General Equations for Non-Equilibrium Reversible-Irreversible Coupling) systems.

[MS21] Nonlinear Schrödinger equations on compact surfaces

Nicolas Camps

Nantes Université

Wednesday, 10:15, 1D327

This talk is devoted to the general study of the long-time dynamics of solutions to nonlinear Schrödinger equations (NLS) on compact surfaces. In this context, weak dispersion and nonlinear resonances can cause energy cascades from low to high frequency scales of oscillations. Meanwhile, one can use the Galerkin approximation and extend methods from the study of finite-dimensional Hamiltonian systems to show stability in certain regimes.

We present a dynamical approach based on Birkhoff normal forms to prove long-time stability on Diophantine tori, as well as a statistical approach in which we prove the invariance of the Gibbs measure for the cubic NLS on the sphere.

The results are based on a joint work with Joackim Bernier, and ongoing joint works with Gigliola Staffilani, and Nicolas Burq, Chenmin Sun, Nikolay Tzvetkov.

[MS22] The effect of protest control on traveling waves of protesting activity

Nancy Rodriguez

University of Colorado Boulder

Tuesday, 15:15, 1D237

Protesting is a powerful form of collective behavior that has been used throughout time and has led to many political changes. Real-life protesting data shows traveling wave-like solutions and previous mathematical models have developed to study this phenomenon. In this talk, we introduce the effect of different management strategies to the model, with a particular emphasis on two prevalent approaches in the United States. We discuss some quantitative and qualitative changes to the traveling wave solutions to this system.

[MS22] The effect of auto-toxicity in plant-growth dynamics: a cross-diffusion model

Cinzia Soresina

University of Trento

Tuesday, 15:45, 1D237

This is joint work with Francesco Giannino (University of Naples Federico II, Naples, Italy) and Annalisa Iuorio (Parthenope University of Naples, Naples, Italy).

In population dynamics, cross-diffusion describes the influence of one species on the diffusion of another and, surprisingly, even though the reaction part does not present the activator-inhibitor structure, cross-diffusion terms are often the key ingredient for the appearance of spatial patterns [1]. Furthermore, from the modelling perspective, cross-diffusion terms naturally appear in the fast-reaction limit of a "microscopic" model (in terms of time scales) presenting only standard diffusion and fast-reaction terms, thus incorporating processes occurring on different time scales [4].

We exploit this technique to model the auto-toxicity effect in plant growth dynamics [2], i.e. negative plant-soil feedback due to the decomposed biomass of the plant on its growth. The "macroscopic" model presents a cross-diffusion term that allows the formation of spatial patterns without introducing water as a variable [3]. A deeper understanding of the conditions required for non-homogeneous steady states to exist is provided by combining a detailed linear analysis with advanced numerical bifurcation methods via the continuation software pde2path and numerical simulations.

[1] Breden, M., Kuehn, C., Soresina, C. *On the influence of cross-diffusion in pattern formation.* J. Comput. Dyn. **8(2)** (2021), 213–240.

[2] Cartení, F., Marasco, A., Bonanomi, G., Mazzoleni, S., Rietkerk, M., & Giannino, F. *Negative plant soil feedback explaining ring formation in clonal plants.* J. Theoret. Biol. **313** (2012), 153–161.

[3] Giannino, F., Iuorio, A., Soresina, C. (in preparation). *The effect of auto-toxicity in plant-growth dynamics: a cross-diffusion model.*

[4] Kuehn, C., Soresina, C. *Numerical continuation for a fast-reaction system and its cross-diffusion limit.* Partial Differ. Equ. Appl. **1(2)** (2020), 7.

[MS22] A rigorous derivation of the asymptotic wavenumber in spiral wave solutions of the complex Ginzburg-Landau equation

Inmaculada Baldomá Barraca
Universitat Politècnica de Catalunya
Tuesday, 16:15, 1D237

This is joint work with Prof. Maria Aguareles (Universitat de Girona, Girona, Spain) and Prof Teresa Martínez-Seara (Universitat Politècnica de Catalunya, Barcelona, Spain).

Spiral patterns are commonly observed in certain chemical, biological and physical systems as, for instance, the Belousov-Zhabotinskii reaction, in social amoebas Dictyosteliu and in the cardiac muscle tissue.

These systems are governed by a chemical or biological reaction and a spatial diffusion, and they are called reaction-diffusion systems:

$$\partial_\tau U = D\Delta U + F(U, a).$$

Here D is the diffusion matrix, F is the reaction nonlinearity, a is a parameter (for instance some catalyst concentration) and the solution $U = U(\tau, x, y) \in \mathbb{R}^2$.

Our work focuses on the existence of rotating archimedean spirals for the Ginzburg-Landau systems which corresponds to the first order of a reaction-diffusion equation near a Hopf bifurcation:

$$\begin{aligned} u_t &= \Delta u + \lambda(\sqrt{u^2+v^2})u - \omega(\sqrt{u^2+v^2})w, \\ w_t &= \Delta w + \omega(\sqrt{u^2+v^2})u + \lambda(\sqrt{u^2+v^2})w, \end{aligned}$$

where the functions λ, ω are $\lambda(z) = 1 - z^2$, $\omega(z) = \omega_0 + qz^2$ and q the small twist parameter. Another relevant parameter is the asymptotic wavenumber k defined by $q(1-k^2) = \Omega - \omega_0$. It can be seen that the rotating archimedean spirals are solutions of the form $U(t, r, \theta) = f(r)e^{i\ [\Omega t + n\theta - \chi(r)]}$ where $f, f', v = \chi'$ are solutions of a first order system of ordinary differential equations depending on q and k, satisfying $f(r) > 0$, $v(r) < 0$ and the boundary conditions:

$$f(0) = v(0) = 0, \qquad \lim_{r\to\infty} f(r) = \sqrt{1-k^2}, \qquad \lim_{r\to\infty} v(r) = -k.$$

That is, four boundary conditions in a three dimensional system of ordinary differential equations have to be imposed and this suggest a selection mechanism for k with respect to q. Indeed we have proven that, in order to exist rotating archimedean spirals for the Ginzburg-Landau systems, the asymptotic wavenumber has to be

$$k = k(q) = \frac{2}{q} A_n e^{-\frac{\pi}{2nq}} (1 + \mathcal{O}(q))$$

with A_n a constant that only depends on the solutions for $q = 0$.
These systems have been previously studied by many authors, Koppel, Hagan, Greenberg, Chapman, etc. using different techniques as Fenichel's theory, asymptotic methods, numerical methods, shooting methods. Our study is based on functional and complex analysis techniques.

[MS22] Chaotic fronts in a generalised Allen-Cahn-type model through a Shil'nikov bifurcation

Martina Chirilus-Bruckner

Leiden University

Tuesday, 16:45, 1D237

This is a joint work with Prof. Jens Rademacher (University of Hamburg, Hamburg, Germany) and Peter van Heijster (Wageningen University, Wageningen, the Netherlands).

We give an overview of the complicated dynamics exhibited by front solutions in reaction-diffusion systems. Along the example of an Allen-Cahn-type multi-component system, we show how the dynamics of the speed of a front can be described by a set of ODEs, which - under certain conditions - can even support chaotic motion emerging from nil-potent singularities. Furthermore, we demonstrate how one can embed arbitrary singularities into this reduced ODE by an appropriate choice of system parameters and coupling function.

[MS22] Jumping oscillons and other exotic oscillatory patterns in an excitable reaction-diffusion system

Arik Yochelis

Ben-Gurion University

Thursday, 15:15, 1D328

Oscillons, i.e., immobile spatially localized but temporally oscillating structures, are the subject of intense study since their discovery in Faraday wave experiments. However, oscillons can also disappear and reappear at a shifted spatial location, becoming jumping oscillons (JOs). After about two decades since they were first observed in simulations, the origin of this behavior is explained by employing a three-variable reaction-diffusion system, numerical continuation, and bifurcation theory: JOs are created via a modulational instability of excitable traveling pulses (TPs). We also reveal the presence of bound states of JOs and TPs and patches of such states (including jumping periodic patterns) and determine their stability. Potential applications are also briefly addressed.

[MS22] Pattern selection via marginal stability of pushed fronts in the FitzHugh-Nagumo system

Montie Avery

Boston University

Thursday, 15:45, 1D328

This is joint work with Paul Carter (UC Irvine, Irvine, CA, US) and Björn de Rijk (KIT, Karlsruher, Germany).

Complex coherent structures in physical systems often form after a homogeneous background state becomes unstable. When the transition out of the unstable state is seeded by a spatially localized perturbation, this perturbation grows and forms an invasion front, which propagates into the unstable state and selects a new state in its wake. The marginal stability conjecture asserts that the propagation speed is the unique speed for which the associated invasion front solution is marginally spectrally stable. In many cases, propagation at a fixed speed combines with oscillatory dynamics in either the leading edge or the wake of the invasion process to generate a spatially periodic pattern. Universal wavenumber selection laws predict the wavelength of this pattern through an appropriate combination of the selected speed and the frequency of the temporal oscillations. We explore this phenomenon in the FitzHugh-Nagumo system, a prototypical model for large amplitude pattern formation. In this setting, we give the first rigorous proof of the marginal stability conjecture and associated wavenumber selection laws for any pattern-forming invasion process. Our proof relies on a nonlinear stability analysis of a pushed front, which is in many ways similar to a generic source defect.

[MS22] Different concepts of stability in a stochastic Klausmeier model with a Busse balloon

Christian Hamster

Universiteit van Amsterdam

Thursday, 16:15, 1D328

This is joined work with Peter van Heijster (Wageningen University, Wageningen, the Netherlands) and Eric Siero (Wageningen University, Wageningen, the Netherlands)

We study a classic one-dimensional Klausmeier model for dryland vegetation patterning with stochastic forcing. For the deterministic equation, it is well known that there is a Busse balloon, i.e. for a fixed set of parameters there is a band of stable periodic patterns alongside a range of unstable periodic patterns. Stability here is defined in the classic $t \to \infty$ sense. When we add stochastic forcing, this notion becomes meaningless as we are forced to look at much shorter time scales because stable patterns can destabilize due to the noise. We will discuss several notions of (in)stability in the stochastic model and discuss the implications this can have on the observability of patterns in real-world ecosystems.

[MS22] Far-from-threshold dynamics in sloped semi-arid environments driven by autotoxicity effects

Annalisa Iuorio

Parthenope University of Naples

Thursday, 16:45, 1D328

Joint work with Gabriele Grifò (University of Messina, Messina, Italy) and Frits Veerman (Leiden University, Leiden, The Netherlands)

In this talk, we focus on the investigation of far-from-threshold vegetated dynamics in sloped semi-arid environments in the presence of autotoxicity. To this aim, an extension of the 1D Klausmeier model that accounts for the toxicity compounds is considered and the occurrence of travelling stripes is analysed. Numerical simulations are first carried out to capture the qualitative behaviour of the pulse-type solutions and, then, geometric singular perturbation theory is used to prove the existence of such travelling pulses by constructing the corresponding homoclinic orbits in the associated 4-dimensional system. A scaling analysis on the investigated model is performed to identify the asymptotic scaling regime in which travelling pulses can be constructed. Some biological observations are extracted from the analytical results, in particular enlightening the role played by autotoxicity in the structure of the travelling patterns. Finally, the analytically constructed solutions are compared with the numerical ones, leading to a good agreement that confirms the validity of the conducted analysis. Numerical continuation with respect to the main system parameters linked to autotoxicity are also performed using the software AUTO in order to gain additional information on the emerging vegetation dynamics.

[MS24] Convective Turing bifurcation with conservation laws, and applications to modern biomorphology

Kevin Zumbrun

Indiana University Bloomington

Tuesday, 8:45, 1D237

Modern biomorphology models such as Murray-Oster and Scianna-Bell-Preziosi involve pattern formation in systems with mechanical/hydrodynamical effects taking the form of convection-reaction-diffusion models with conservation laws. Here, extending previous work of Matthews-Cox and Häcker-Schneider-Zimmerman in pattern formation with conservation laws, and of Eckhaus, Mielke, and Schneider on stability of Turing patterns in reaction diffusion models, we investigate diffusive stability of Turing patterns for convection-reaction-diffusion models with conservation laws. Formal multiscale expansion yields a singular system of amplitude equations coupling Complex Ginzburg Landau with a singular convection-diffusion system, similar to partially coupled systems found by Häcker-Schneider-Zimmerman in the context of thin film flow, but with the singular convection part now fully engaged in long term stability and behavior rather than transient as in the (triangular) partially coupled case.

The resulting complicated two-parameter matrix perturbation problem governing spectral stability can nonetheless be solved, yielding (m+1) simple stability criteria analogous to the Eckhaus and Benjamin-Feier-Newell criteria of the classical (no conservation law) case, where m is the number of conservation laws. It is to be hoped that these can play the same important role in the study of biopattern formation as the classical ones in myriad other applications.

[MS24] Stability of Pulled Pattern-Forming Fronts

Björn de Rijk

Karlsruhe Institute of Technology

Tuesday, 9:15, 1D237

This is joint work with Montie Avery (Boston University, Boston MA, USA), Paul Carter (University of California, Irvine CA, USA) and Arnd Scheel (University of Minnesota, Minneapolis MN, USA).

Pattern formation is often nucleated when a localized disturbance grows and spreads, generating a pattern in the wake of this spreading. The marginal stability conjecture postulates that only those invasion speeds and patterns are selected for which the corresponding invasion front, connecting the ground state to this pattern, is marginally spectrally stable. Recently, the marginal stability conjecture was proven for the selection of spatially constant patterns by adapting sharp local stability results of the associated invasion front. In anticipation of extending the marginal stability conjecture to the selection of spatially inhomogeneous patterns, we established a sharp stability theory for pattern-forming invasion fronts. In this talk I focus on the case of pulled pattern-forming fronts, where marginal spectral stability manifests itself by two curves of essential spectrum touching the origin: one originating from the invaded ground state and the other from the periodic pattern in the wake. Such pulled pattern-forming fronts arise in the FitzHugh-Nagumo system, which was originally proposed as a simplification of the Hodgkin-Huxley model for nerve propagation and has since then attracted much interest as a phenomenological model for pattern formation. I will present a sharp nonlinear stability result for these fronts against suitably localized perturbations.

[MS24] Localized patterns in inhomogeneous reaction-diffusion systems: front dynamics in a spatially driven Allen-Cahn equation

Arjen Doelman

Leiden University

Tuesday, 9:45, 1D237

This is joint work with Robbin Bastiaansen (Utrecht University, the Netherlands) and Tasso Kaper (Boston University, USA).

Ecosystem models typically are of reaction-diffusion type, with components representing (various kinds of) vegetation, soil and overland water, etc.. Naturally, the local topography of the terrain is an important factor and it appears as a heterogeneous spatially varying term in such models. To build a conceptual understanding of the impact of such a heterogeneity on the patterns exhibited by a reaction-diffusion model, we consider the most simple setting of an explicit scalar bi-stable Allen-Cahn model driven by a small spatially heterogeneous effect, $U_t = U_{xx} + U - U^3 + \varepsilon F(U, U_x, x)$ for $x \in \mathbb{R}$ (and with $0 < \varepsilon \ll 1$), where we will focus on the case in which the spatial inhomogeneity $F(U, U_x, x)$ is localized in x. Unlike earlier studies of more realistic models, we do not focus on the persistence under such a perturbation of localized structures that already exist (and are stable) in the homogeneous $\varepsilon \to 0$-limit. Instead we consider the question: What kinds of novel multiple-front dynamics may be triggered by the (small) heterogeneous effects? More specifically, in the homogeneous case it is well-known that multiple-front patterns evolve (for $t \to \infty$) either into one of the trivial background states $\bar{U}_{\pm} = \pm 1$ or into a solitary standing one-front (of tanh-type). We will show that a small (localized) spatial inhomogeneity $F(U, U_x, x)$ has a strong impact on this, it may for instance generate slowly diverging 'trains' of N-fronts that collectively travel to + or $-\infty$ (with slowly decreasing speeds). It is shown that the decay rate of $F(U, U_x, x)$ (as function of x) – and for instance whether $F(U, U_x, x)$ decays exponentially or algebraically – is a crucial factor determining the character of the long time behavior of multi-front patterns.

[MS24] Spots the difference: Approximating fully localised planar patterns

Dan Hill

Saarland University

Tuesday, 10:15, 1D237

This is joint work with Prof. David J. B. Lloyd (University of Surrey, Guildford, UK) and Dr Jason J. Bramburger (Concordia University, Montréal, Canada).

Spatially localised patterns are known to emerge in a variety of physical settings, ranging from dryland vegetation to vibrating fluids to the buckling of cylinders. While there are a number of mathematical tools for studying localised patterns in one spatial dimension, developing equivalent approaches in higher spatial dimensions remains a major challenge in the area of pattern formation.

In this talk, we will focus on 2D patterns that are localised in the radial direction and explore the theory of radial spatial dynamics, introduced by Scheel in 2003. We present recent results for approximating fully localised patterns with dihedral symmetries, highlighting connections to the 1D and axisymmetric cases. Finally, we present a novel formal approach to derive radial amplitude equations, which provides new insight into the emergence of fully localised planar patterns. As an example, we derive radial amplitude equations for fully localised hexagons and quasipatterns in the Swift–Hohenberg equation, for which we can obtain explicit localised solutions.

[MS24] Dynamics of solutions to a rimming-flow equation

Christina Lienstromberg

University of Stuttgart

Thursday, 8:45, 1D237

This is joint work with Juri Joussen (University of Stuttgart, Germany) and Juan J.L. Velázquez (University of Bonn, Germany).

We study the dynamic behaviour of a thin viscous fluid film coating the inner wall of a rotating cylinder – a so-called *rimming flow*. The resulting PDE

$$h_t + \big(h + \gamma h^3(h_\theta + h_{\theta\theta\theta}) - \delta h^3 \cos(\theta)\big)_\theta = 0, \quad t > 0,\ \theta \in S^1, \tag{31}$$

for the height $h > 0$ of the fluid film is quasilinear, degenerate parabolic and of fourth order. Three effects drive the dynamics of the interface – viscosity, surface tension, and gravity: If the surface-tension parameter $\gamma > 0$ is of order one and gravitational effects are neglected ($\delta = 0$), then positive steady states of (31) are characterised by positive constants, describing circles centered at the origin. These constant steady states are orbitally stable. The existence of positive steady states can further be guaranteed for small positive gravitational influence ($\delta > 0$ small enough). Finally, going for small $\delta > 0$ to a rotating coordinate frame $\xi = \theta - t$, we find that circles not centered at the origin are further steady states. General solutions converge exponentially fast to a neighborhood of the manifold of steady states as long as they stay bounded away from zero. On this manifold the dynamics takes place on a rather slow time scale $\tau = \delta^2 t$, compared to off the manifold.

[MS24] Stability of Periodic Waves for the Defocusing Fractional Cubic Nonlinear Schrödinger Equation

Handan Borluk

Ozyegin University

Thursday, 9:15, 1D237

This is a joint work with Prof. Gulcin M. Muslu (Istanbul Technical University, Türkiye) and Prof. Fabio Natali (State University of Maringa, Brazil).

In this study, the spectral instability of periodic odd waves for the defocusing fractional cubic nonlinear Schrödinger equation is studied. Real periodic waves are constructed by minimizing a suitable constrained problem. Our approach for the stability problem is based on periodic perturbations that have the same period as the standing wave solution. The spectral information for the solutions is obtained by using tools related to the oscillation theorem for fractional Hill operators. Then, Newton's iteration method is presented to generate the odd periodic standing wave solutions and numerical results have been used to apply the spectral stability theory.

[MS24] Stability of shock waves of the 1D isentropic compressible Navier Stokes equations

Benjamin Melinand

Univ. Paris Dauphine

Thursday, 9:45, 1D237

This is a joint work with Miguel Rodrigues (Univ Rennes, France).

This talk concerns the nonlinear stability of viscous shock waves of the one dimensional Navier-Stokes equations in Eulerian coordinates. Since the works of Mascia and Zumbrun in the early 2000s it is known that a shock wave is nonlinear stable if the linearized operator about the shock wave does not have any nonzero eigenvalue with nonnegative real part. We show that the previous spectral condition is satisfied for any shock wave thanks to the use of an adhoc effective velocity. We then explain how one can use Mascia and Zumbrun strategy to obtain the nonlinear stability with a parabolic-type decay rate. For that we consider localized initial data and we emphasize that no zero mass condition is assumed. We also study the asymptotic behavior close to the shock wave.

[MS24] Stability of a nulli-stable front in presence of pattern formation.

Louis Garénaux

Karlsruhe Institute of Technology

Thursday, 10:15, 1D237

In this talk, I will present a stability result for a front interface connecting two unstable states. We will discuss the possible interactions of such structure with another slow pattern-forming interface. The precise system under study is a coupling between the Swift-Hohenberg and the KPP equation. We will see how weighted spaces are necessary to handle the different instabilities. Such structure composed of successive propagating fronts is often referred to as terraces. In this setting, the interface dynamics is up to date not properly described, see for example [1].

[1] L. Girardin, K.-Y. Lam, *Invasion of open space by two competitors: spreading properties of monostable two-species competition-diffusion systems.* Proc. Lond. Math. Soc. (3) **119** (2019), 1279–1335.

[MS24] Some Approximation Results for Quasilinear Systems and Systems with Marginally Stable Long Modes

Dominik Zimmermann

University of Stuttgart

Friday, 8:45, 1D324

Surface tension driven convection, also called Bénard-Marangoni convection, is a well-known pattern forming system. In order to describe its dynamics at the onset of pattern formation, a Ginzburg-Landau-like system of amplitude equations can be derived via multiple scaling analysis. When trying to establish and approximation theorem for this amplitude system, we encounter two problems, which are both connected to the free surface. First, the equations become quasilinear, which prevents the direct application of the existing semilinear techniques. Second, conservation of mass leads to marginally stable long modes, such that the interaction of critical modes cannot be controlled by exponential damping. We show how to overcome these difficulties with some rather general techniques that can also be applied independently to other problems.

[MS24] Global existence for long wave Hopf unstable spatially extended systems with a conservation law

Anna Logioti

University of Stuttgart

Friday, 9:15, 1D324

This is a joint work with N. Gauss, G. Schneider and D. Zimmermann (Universität Stuttgart , Stuttgart, Germany).

We are interested in reaction-diffusion systems, with a conservation law, exhibiting a Hopf bifurcation at the spatial wave number $k = 0$. With the help of a multiple scaling perturbation ansatz a Ginzburg-Landau equation coupled to a scalar conservation law can be derived as an amplitude system for the approximate description of the dynamics of the original reaction-diffusion system near the first instability. We use the amplitude system to show the global existence of all solutions starting in a small neighborhood of the weakly unstable ground state for original systems posed on a large spatial interval with periodic boundary conditions.

[MS24] Thin fluid films heated from below

Gabriele Brüll

Lund University

Friday, 9:45, 1D324

This is a joint work with B. Hilder (TU Munich, Germany) and J. Jansen (Lund University, Sweden).

Thin liquid films find various applications in industrial processes. These films, ranging from nanometers to micrometers in thickness, are often exposed to temperature gradients, which compete with stabilizing surface tension effects. A mathematical model describing the evolution of thin fluid film's height resting on a horizontal, heated plate can be derived via lubrication approximation, constituting a quasilinear, degenerate, fourth-order partial differential equations. When a critical temperature disparity between the heated plate and the surrounding environment is reached, the flat steady state becomes spectrally unstable and periodic stationary solutions bifurcate. Using a Hamiltonian formulation of the stationary problem and analytic bifurcation theory, we prove the existence of a global bifurcation curve converging to a weak stationary periodic solution exhibiting film rupture.

[MS24] Nonlocal equations describing cold plasmas

Rafael Granero Belinchon

Universidad de Cantabria

Friday, 10:15, 1D324

This is a joint work with Prof. Diego Alonso-Orán (Universidad de La Laguna, La Laguna, Spain) and Prof. Ángel Durán (Universidad de Valladolid, Valladolid, Spain).

The motion of a cold plasma in a magnetic field consisting of singly-charged particles can be described by the following system of PDEs

$$n_t + (un)_x = 0, \tag{32a}$$

$$u_t + uu_x + \frac{bb_x}{n} = 0, \tag{32b}$$

$$b - n - \left(\frac{b_x}{n}\right)_x = 0 \tag{32c}$$

In this talk we will present new asymptotic models to approximate the dynamics of the solutions of (32). The PDEs that we will introduce are nonlinear and nonlocal dispersive PDEs having a Hamiltonian structure.

Furthermore, we will also establish a number of rigorous well-posedness and finite time singularity results.

These result have appeared in [1].

[1] Diego Alonso-Orán, Angel Durán, Rafael Granero-Belinchón, *Derivation and well-posedness for asymptotic models of cold plasmas* Nonlinear Analysis, **244** (2024), 113539.

[MS25] Ergodicity and random dynamical systems for conservative SPDEs

Benjamin Gess

Bielefeld University and MPI MiS Leipzig

Monday, 15:15, 1D236

The dynamics of the solutions to a class of conservative SPDEs will be discussed from two perspectives: Firstly, a purely probabilistic construction of a corresponding random dynamical system is presented for the first time. Secondly, the existence and uniqueness of invariant measures, as well as mixing for the associated Markov process is shown.

This is based on joint work with Benjamin Fehrman, and Rishabh S. Gvalani.

[MS25] Quantitative instability estimates for order-preserving reaction-diffusion equations

Alexandra Blessing
University of Konstanz
Monday, 15:45, 1D236

This is a joint work with Tommaso Rosati (University of Warwick, UK).

We study long-time properties of nonlinear stochastic partial differential equations (SPDEs) driven by a noise which is white in time and correlated in space. Provided that the SPDE admits a positive Lyapunov exponent we obtain uniform bounds on negative moments of the solution. To this aim we first derive for the linearized equation a Lyapunov function which contains a corrector that is constructed implicitly through the solution of a Poisson problem involving the generator of the projective dynamics. In order to extend such bounds to the nonlinear case we introduce a cut-off process which is comparable to the linearization and bounds the solution from below.

[MS25] Early-Warning Signs for SPDEs and Applications on a Climate Mode

Paolo Bernuzzi
Technische Universität München (TUM)
Monday, 16:15, 1D236

This is a joint work with Prof Christian Kuehn (Technische Universität München, München, Germany) and Prof Henk Dijkstra (Universiteit Utrecht, Utrecht, Netherlands).

Early-warning signs able to predict the approach of a parameter to a deterministic bifurcation threshold on a stochastic partial differential equation (SPDE) present useful tools in a wide variety of applications, as they can anticipate major changes in its solution. In our study we assume the noise as Gaussian perturbation on the domain or on its boundary, in accordance to its role in the original model. We specifically focus on the qualitative behaviour of the time-asymptotic autocovariance and autocorrelation, along general choices of functions, of the solutions of the linearized system. The properties of such signs are highly dependent on the shape of the spectrum of the linear operator present in the drift component. For this reason, various cases are considered and compared. We also discuss the reliability of the tools from an analytic perspective and through different examples. Among those, the application of the early-warning signs on a climate science Boussinesq model is deeply discussed through the use of numerical simulations of its solution. Such two-dimensional SPDE system has been developed in order to study the behaviour and collapse of the Atlantic Meridional Overturning Circulation (AMOC). The analytic results presented in the discussed work expand on previous work by the authors [1, 2].

[1] Bernuzzi, P. and Kuehn, C., 2023. Bifurcations and Early-Warning Signs for SPDEs with Spatial Heterogeneity. J. Dynam. Differential Equations, pp.1-45.

[2] Bernuzzi, P., Düx, A. and Kühn, C., 2023. Early-Warning Signs for SPDEs with Continuous Spectrum. arXiv preprint arXiv:2307.14080.

[MS25] Diffusion and mixing in random flows

Michele Coti Zelati

Imperial College London

Monday, 16:45, 1D236

We prove that certain randomization of classical flows in $2d$ and $3d$ lead to exponential mixing and optimal enhanced dissipation results. This settles the question of the existence of smooth, exponential mixing fields in the two and three dimensional periodic box. One of the consequences of our work is that a suitable modification of the ABC flow provides an example of a dynamo in inviscid passive vectors, modeling the evolution of magnetic fields.

[MS25] Transition to positive finite-time Lyapunov exponents

Alex Blumenthal

Georgia Institute of Technology

Tuesday, 15:15, 1D328

This is a joint work with Professors Alexandra Blessing (Universität Konstanz, Germany), Maxime Breden (École Polytechnique, France) and Maximilian Engel (University of Amsterdam, Netherlands).

I will discuss some recent results and an outlook on a scenario in stochastic dynamical systems characterized by a change from negative to positive finite-time Lyapunov exponents. This scenario, a kind of stochastic bifurcation, is expected in a variety of circumstances, notably including: stochastic pitchfork bifurcations, and as a "warning sign" on the route to chaos. I will describe recent work establishing this transition for the Chaffee-Infante equation, as well as recent theoretical work and computer-assisted work characterizing this transition in terms of large deviations principles.

[MS25] A study in randomly perturbed Hamiltonian systems

Anna Maria Cherubini

University of Salento

Tuesday, 15:45, 1D328

This is a joint work with Prof Marian Gidea (Yeshiva University, New York City, USA).
Integrable Hamiltonian systems subjected to small, time-dependent perturbations, generate, in general, some orbits that experience significant energy growth. In this work we explore the effect of random time-dependent perturbations on integrable Hamiltonian systems, in order to extend results on the *Arnold diffusion problem* to the context of random dynamical systems.

[MS25] Markov RDS in discrete space and discrete time

Robin Chemnitz

Freie Universität Berln

Tuesday, 16:15, 1D328

This is a joint work with M. Engel (University of Amsterdam) and G. Olicón-Méndez (Universität Potsdam).

In this talk, we consider random dynamical system (RDS) representations of positive recurrent Markov chain on a countable state space in discrete time, and study their random attractors. A key feature of discrete space RDS is that once the trajectories of two initial conditions meet, they stay together for all future times, i.e. they synchronize. While Markov RDS on general state spaces may exhibit a wide range of attraction phenomena, like attractor-repeller pairs, forward attractors, pullback attractors, the discrete space setting is less intricate. Our main result is that for discrete Markov RDS there exists a unique random attractor that is both a forward and pullback attractor. In particular, this shows that in our setting forward and pullback attraction (for compact deterministic sets) are equivalent notions of attraction. Additionally, we study the expected time it takes for an initial state to enter the random attractor, in particular under which conditions the expected time is finite.

[MS25] Iterated function systems of affine contracting and expanding maps

Ale Jan Homburg

University of Amsterdam

Tuesday, 16:45, 1D328

I'll discuss a class of iterated function systems generated by piecewise affine contracting and expanding maps on the unit interval. The research is connected to heterochaos baker maps. My focus will be on the transition of the Lyapunov exponent crossing zero when varying probabilities of taking the maps.

This is a joint work with Charlene Kalle (Leiden University).

[MS25] Capacity estimates for Fokker-Planck equations

Benny Avelin

Uppsala University

Thursday, 8:45, 1D327

In this talk I will present an extension to our previous result regarding the geometric characterization of capacity for the Fokker-Planck equation related to overdamped Langevin dynamics. The extension is that we are able to treat general configuration of saddle points by using certain localization methods and relating the problem to a discrete problem on a graph.

[MS25] Asymptotically unbiased approximation of the QSD of diffusion processes with a decreasing time step Euler scheme

Julien Reygner

École des Ponts ParisTech

Thursday, 9:15, 1D327

This is a joint work with Fabien Panloup (Université d'Angers).

The quasistationary distribution (QSD) of a Markov process $(X_t)_{t\geq 0}$ in a subset D of its state space describes the long time limit of the conditional distribution of trajectories which do not exit D. In PDE terms, it is the left eigenvector associated with the Dirichlet eigenvalue of the infinitesimal generator of $(X_t)_{t\geq 0}$ in D. QSDs play a central role in the theoretical understanding and numerical simulation of systems exhibiting metastability, which are ubiquitous in statistical physics, chemistry and biology.

The practical approximation of the QSD of a process is a nontrivial question. The talk will be dedicated to the presentation of a stochastic algorithm which achieves this task. The idea of the algorithm dates back to [1] for Markov chains, and was recently adapted to the setting of uniformly elliptic diffusion processes in [2]. Working with such processes requires an additional time discretisation, which was addressed in [3].

[1] D. Aldous, B. Flannery, J.-L. Palacios, *Two applications of urn processes: the fringe analysis of search trees and the simulation of quasi-stationary distributions of Markov chains.* Probab. Eng. Inf. Sci., 2(3):293–307, 1988.

[2] M. Benaïm, N. Champagnat, D. Villemonais, *Stochastic approximation of quasi-stationary distributions for diffusion processes in a bounded domain.* Ann. Inst. Henri Poincaré Probab. Stat., 57(2):726–739, 2021.

[3] F. Panloup, J. Reygner, *Asymptotically unbiased approximation of the QSD of diffusion processes with a decreasing time step Euler scheme.* Preprint arXiv:2308.16196.

[MS25] Metastability in the joint limit of small noise and large volume

Giacomo Di Gesù

Sapienza University

Thursday, 9:45, 1D327

We consider the Allen-Cahn equation on a finite interval perturbed by space-time white noise. Keeping the size of the spatial domain fixed, the dynamics becomes metastable in the limit of vanishing noise. I will review some sharp metastability estimates in this regime (joint work with M. Brooks) and discuss how the invariant measure and long-time behavior is affected if one allows the system size to grow while the noise vanishes (joint work with L. Bertini and P. Buttà).

[MS25] Interacting Dynamical Systems on Random Graphs: the Law of Large Numbers and the Large Deviation Principle

Georgi Medvedev

Drexel University

Thursday, 10:15, 1D327

The problem of the macroscopic description of motion of interacting particles has a long history [2]. When the number of particles is large, the analysis of individual trajectories becomes intractable and one is led to study statistical distribution of particles in the phase space. This is done using the Vlasov equation or other kinetic equations describing the state of the system in the continuum limit as the size of the system goes to infinity [1, 3, 6].

Modern applications ranging from neuronal networks to power grids feature models with spatially structured interactions. The derivation of the continuum limit for such models has to deal with the fact that in contrast to the classical setting used in [1, 3, 6], the particles are no longer identical, and it has also to take into account the limiting connectivity of the network assigned by the underlying graph sequences. This problem was addressed in [7, 8], where the ideas from the theory of graph limits [5] were used to formulate and to justify the continuum limit for interacting dynamical systems on certain convergent graph sequences. In particular, in [7] and in the followup paper [9], solutions of coupled dynamical systems on a sequence of W-random graphs were approximated by those of a deterministic nonlocal diffusion equation on a unit interval, representing a continuum of nodes in the spirit of the theory of graph limits. This result can be interpreted as a Law of Large Numbers for the solutions of the initial value problems of interacting dynamical systems on W-random graphs. Futhermore, the large deviation principle for dynamical systems on W-random graphs, established in [4], implies that the probability of $O(1)$ deviations of the solutions of the discrete system on random graph from their typical behavior is exponentially small.

[1] W. Braun and K. Hepp, *The Vlasov dynamics and its fluctuations in the $1/N$ limit of interacting classical particles*, Comm. Math. Phys. **56** (1977), no. 2, 101–113.

[2] F. Golse, *On the dynamics of large particle systems in the mean field limit*, Macroscopic and large scale phenomena: coarse graining, mean field limits and ergodicity, Lect. Notes Appl. Math. Mech., vol. 3, Springer, [Cham], 2016, pp. 1–144.

[3] R. L. Dobrušin, *Vlasov equations*, Funktsional. Anal. i Prilozhen. **13** (1979), no. 2, 48–58, 96.

[4] P. Dupuis and G. S. Medvedev, *The large deviation principle for interacting dynamical systems on random graphs*, Comm. Math. Phys. **390** (2022), no. 2, 545–575.

[5] L. Lovász, *Large networks and graph limits*, AMS, Providence, RI, 2012.

[6] H. Neunzert, *Mathematical investigations on particle - in - cell methods*, vol. 9, 1978, pp. 229–254.

[7] G. S. Medvedev, *The nonlinear heat equation on dense graphs and graph limits*, SIAM J. Math. Anal. **46** (2014), no. 4, 2743–2766.

[8] G. S. Medvedev, *The nonlinear heat equation on W-random graphs*, Arch. Ration. Mech. Anal. **212** (2014), no. 3, 781–803.

[9] G. S. Medvedev, *The continuum limit of the Kuramoto model on sparse random graphs*, Communications in Mathematical Sciences **17** (2019), no. 4, 883–898.

[MS26] The thin-film equation with thermal noise

Rishabh Gvalani

ETH Zürich

Tuesday, 8:45, 1D227

This is based on joint works with Benjamin Gess (MPI-MiS, Leipzig, Germany), Florian Kunick (MPI-MiS, Leipzig, Germany), and Felix Otto (MPI-MiS, Leipzig, Germany) and with Markus Tempelmayr (University of Münster, Münster, Germany).

In this talk, we will discuss the lubrication approximation of the well-known fluctuating hydrodynamics model introduced by Landau and Lifschitz. The corresponding system is a fourth-order, degenerate, quasilinear singular PDE commonly referred to as the thin-film equation with thermal noise. We start by presenting an alternative derivation of the equation from thermodynamic principles using as inputs the correct invariant measure for the dynamics (the 1D Gaussian free field restricted to positive functions) and the correct dissipation mechanism (a weighted version of the H^{-1} inner product). Next, we propose a natural structure-preserving discretisation which we will show preserves the strict positivity of the film height for large enough mobility exponents.

Finally, we study the equation in the framework of the theory of regularity structures: we estimate (uniformly in the regularisation parameter) the appropriately renormalised centered model associated to the equation completing the first step in obtaining a solution theory for the equation. The counterterm is closely related to a correction term which shows up naturally in our discretisation.

[MS26] Sensitive dependence on initial data as measured by different norms

Alex Blumenthal

Georgia Institute of Technology

Tuesday, 9:15, 1D227

This is a joint work with Prof. Sam Punshon-Smith (Tulane University, USA).

Lyapunov exponents are a general tool for the description of the asymptotic behavior of linear partial differential equations. I will discuss some general results for linear PDE of dissipative parabolic type, in particular implying that the Lyapunov exponent is equivalent across a variety of H^s norms. I will then discuss particular implications for two natural classes of linear PDE arising in fluid mechanics: the passive scalar advection equation, and the first variation along a trajectory of the Navier-Stokes equations. These results are drawn from the paper [1].

[1] A. Blumenthal and S. Punshon-Smith, *On the norm equivalence of Lyapunov exponents for regularizing*

linear evolution equations. ARMA **247.5** (2023), 97.

[MS26] Large time behaviour of the 2D thermally non-diffusive Boussinesq equation

Camilla Nobili

University of Surrey

Tuesday, 9:45, 1D227

This is a joint work with Fabian Bleitner (University of Hamburg) and Elizabeth Carlson (CalTech).

In this talk we consider the two-dimensional Boussinesq equations without thermal diffusion on a bounded domain with Navier-slip boundary conditions and study large-time asymptotics. We prove that, in suitable norms, the solution converges to the hydrostatic equilibrium and show linear stability for the hydrostatic equilibrium when the temperature is an increasing affine function of the height, i.e. the temperature is vertically stably stratified.

[MS26] Annealed mixing for advection by stochastic velocity fields

Sam Punshon-Smith

Tulane University

Tuesday, 10:15, 1D227

We consider the long-time behavior of a passive scalar advected by an incompressible velocity field. If the velocity field is autonomous or time periodic, long-time behavior follows by studying the spectral properties of the transfer operator associated with the finite time flow map. When the flow is uniformly hyperbolic, it is well known that it is possible to construct anisotropic Sobolev spaces where the transfer operator becomes quasi-compact with a spectral gap, yielding exponential decay in these spaces. In the non-autonomous and non-uniformly hyperbolic case this approach breaks down. In this talk, I will discuss how in the stochastic ergodic velocity setting one can recover analogous results under the expectation using pseudo differential operators to obtain exponential decay of solutions to the transport equation from $H^{-\delta}$ to $H^{-\delta}$ – a property we call annealed mixing. As a result, we show that the Markov process obtained by considering advection with a source term has a unique stationary measure. This is a joint work with Jacob

Bedrossian and Patrick Flynn.

[MS26] Spontaneous stochasticity for 2D autonomous flows

Massimo Sorella

EPFL

Thursday, 15:15, 1D340

This is a joint work with Carl Johansson (EPFL, Lausanne, Switzerland).
In this talk, we present a new result of a 2d autonomous divergence free velocity field in C^α (for $\alpha < 1$ arbitrary but fixed) for which solutions of the advection diffusion equation exhibit anomalous dissipation for some initial data. The proof relies on proving spontaneous stochasticity using a stochastic Lagrangian approach.

[MS26] Hydrodynamic limit of incompressible Euler equation

Joonhyun La

KIAS

Thursday, 15:45, 1D340

This is a joint work with Professor Chanwoo Kim (University of Wisconsin-Madison).
In this talk, we discuss recent progress in the hydrodynamic limit of incompressible Euler equation from the Boltzmann equation.

[MS26] A glimpse of ideal turbulence

Hyunju Kwon

ETH Zurich

Thursday, 16:15, 1D340

Based on recent joint works with Matthew Novack (Purdue University, West Lafayette, IN, USA) and Vikram Giri (ETH Zürich, Zürich, Switzerland), I will discuss solutions to the 3D incompressible Euler equations that reflect phenomena in ideal turbulence.

[MS26] First integrals of the 3D Euler equations and non-mixing

Francisco Torres de Lizaur

University of Seville

Thursday, 16:45, 1D340

This is a joint work with Robert Cardona (University of Barcelona).

Abstract: The following is, roughly speaking, the non-mixing problem for the time-dependent 3D Euler equations of hydrodynamics: show that there are pairs of smooth velocity fields, with the same energy and helicity, such that no initial velocity field close to one (in some appropriate distance) can ever evolve into a field close to the other.

Khesin, Kuksin and Peralta-Salas proved this in 2014 for the C^k distance, with $k \geq 4$, using KAM theorem. Proving the same result for smaller values of k remained an open problem. I will present joint work with Robert Cardona where we prove non-mixing for all $k \geq 1$. To do this, we construct new integrals of motion of the Euler equation on any 3-manifold with any fixed metric, using ideas from contact and symplectic topology.

[MS27] Analytic weak-stable manifolds in unfoldings of saddle-nodes

Kristian Uldall Kristiansen

Technical University of Denmark

Monday, 15:15, 1D237

This is joint work with Peter Szmolyan (Technical University of Vienna, Austria).

Any attracting, hyperbolic and proper node of a two-dimensional analytic vector-field has a unique strong-stable manifold. This manifold is analytic. The corresponding weak-stable manifolds are, on the other hand, not unique, but in the nonresonant case there is a unique weak-stable manifold that is analytic. As the system approaches a saddle-node (under parameter variation), a sequence of resonances (of increasing order) occur.

In this talk, we will (based upon [1]) provide a detailed description of the analytic weak-stable manifolds during this process. In particular, we will relate a “flapping-mechanism”, corresponding to a dramatic change of the position of the analytic weak-stable manifold as the parameter passes through the infinitely many resonances, to the lack of analyticity of the center manifold at the saddle-node.

Our work is motivated and inspired by the work of Merle, Raphaël, Rodnianski, and Szeftel [2], where this flapping mechanism is the crucial ingredient in the construction of C^∞-smooth self-similar solutions of the compressible Euler equations.

[1] Kristiansen, K. U. and Szmolyan, P. Analytic weak-stable manifolds in unfoldings of saddle-nodes, arXiv pre-print: 2403.01488 (2024).

[2] Merle, F. and Raphaël, P. and Rodnianski, I. and Szeftel, J, On the implosion of a compressible fluid I: Smooth self-similar inviscid profiles, Ann. of Math. **192** (2022), 567–778.

[MS27] Geometric Blow-up for Dynamic Bifurcations in Reaction-Diffusion Equations

Samuel Jelbart

Vienna University of Technology

Monday, 15:45, 1D237

This is a joint work with Alejandro Martinez Sanchez (Technical University of Munich, Garching, Germany), Prof. Christian Kuehn (Technical University of Munich, Garching, Germany) and Prof. Peter Szmolyan (Vienna University of Technology, Vienna, Austria).

Many authors have shown that the geometric blow-up method can provide a rigorous and geometrically informative description of so-called *dynamic bifurcations* in systems of ODEs. By comparison, there have only been a handful of attempts to generalise the method for applications to PDEs. This talk will focus on the development and application of the method to describe a known exchange of stability property in a forced Allen-Cahn equation with slow parameter drift, namely

$$u_t = u_{xx} + \mu(t)u - u^3 + \varepsilon\lambda, \qquad \mu(t) = \mu(0) + \varepsilon t, \tag{33}$$

where $t > 0$, $x \in \mathbb{R}$, $u = u(x,t) \in \mathbb{R}$, $\mu(0) < 0$, $0 < \varepsilon \ll 1$ and $\lambda \in \mathbb{R}$ is a parameter. We show that the singularity which occurs at $u = \mu = 0$ in the frozen system with $\varepsilon = 0$ due to the presence of continuous spectrum along the negative real axis up to and including zero, can be 'resolved' using a variant of the geometric blow-up method which involves a time-dependent stretching of space. The equations in the blown-up space have a spectral gap property, which allows us to show that nearby solutions are exponentially attracted to an invariant manifold of spatially independent solutions, which we then extend through a neighbourhood of the (dynamic) bifurcation point using center manifold theory and direct estimates. Our findings allow for a rigorous and geometrically informative proof that the exchange of stability occurs as λ changes sign.

Many of the obstacles to the use of the geometric blow-up method in the study of dynamic bifurcations in PDEs in general are already present in this simple example, and we are hopeful the techniques used to overcome them here will also be useful in more complicated problems.

[MS27] A multi-parameter singular perturbation analysis of the Robertson Model

Lukas Baumgartner

TU Wien

Monday, 16:15, 1D237

This is based on a joint work [1] with Peter Szmolyan (TU Wien, Austria).

The Robertson Model

$$\begin{aligned} \dot{x} &= -k_1 x + k_3 yz \\ \dot{y} &= k_1 x - k_2 y^2 - k_3 yz \\ \dot{z} &= k_2 y^2, \end{aligned} \tag{34}$$

with $k_1 = 4 \cdot 10^{-2}$, $k_2 = 3 \cdot 10^7$, $k_3 = 10^4$, see [2], is one of the classical examples for stiffness in ODEs. The model has been widely used as a numerical test problem. Surprisingly, no analytical results seem to exist.

In this talk we will provide a full asymptotic analysis of (34) under the assumption $k_1, k_3 \ll k_2$, which covers the classical choice above. We rewrite the equations as a two-parameter singular perturbation problem in the rescaled parameters $(k_1/k_2,\ k_3/k_2) \approx (0, 0)$, which we then analyze using Geometric Singular Perturbation Theory (GSPT).

To deal with the multi-parameter singular structure, we perform blow-ups in parameter- and variable space. This reveals three distinct regimes in the singular limit $(k_1/k_2,\ k_3/k_2) \to (0,0)$. Within these three regimes we use GSPT and additional blow-ups to analyze the dynamics and the structure of solutions. Our asymptotic results are in excellent qualitative and quantitative agreement with the numerics.

[1] L. Baumgartner, P. Szmolyan, *A multi-parameter singular perturbation analysis of the Robertson model.* In preparation.

[2] H.H. Robertson, *The solution of a set of reaction rate equations.* In: *Numerical Analysis: An Introduction.* Academic Press, 1966, pp. 178-182.

[MS27] Travelling fronts between spatially heterogeneous background states: a new approach to a fully spatiotemporal problem

Frits Veerman

University of Leiden

Tuesday, 15:15, 1D340

We study travelling fronts in a two-component singularly perturbed reaction-diffusion system of FitzHugh-Nagumo type with spatially heterogeneous coefficients. The spatial heterogeneity induces non-constant background states, which renders a comoving frame practically useless. The travelling wave problem therefore becomes fully spatiotemporal. Moreover, the singular structure cannot be exploited in a straightforward manner by applying geometric singular perturbation theory, as the spatial heterogeneity causes the critical manifolds to be non-compact.

We show that the existence of a travelling front is encoded in the solution to a delay differential equation for the time-dependent wave speed. We discuss the extension and application of Fenichel's results to the non-compact setting of our travelling wave problem, that are necessary to prove the existence of the travelling front. We compare the results of our analysis with numerical simulations, and show that the speed of the front is determined by the shape of the background states.

[MS27] A geometric singular perturbation analysis of generalised shock selection rules in reaction-nonlinear diffusion models

Robert Marangell

University of Sydney

Tuesday, 15:45, 1D340

This is a joint work with B.H. Bradshaw-Hajek, I. Lizarraga, and M. Wechselberger

Reaction-nonlinear diffusion (RND) partial differential equations (PDEs) are a fruitful playground to model the formation of sharp travelling fronts. In this talk, I will demonstrate the utility and scope of regularisation as a technique to investigate shock fronted solutions of RND PDEs, using geometric singular perturbation theory (GSPT) as the mathematical framework. In particular, I will show how composite regularisations can be used to construct families of monotone shock-fronted travelling waves sweeping out distinct generalised area rules, which interpolate between the equal area and extremal area (i.e. algebraic decay) rules that are well-known in the shockwave literature.

If time, I will consider the spectral stability of these new interpolated shockwaves. Using techniques from geometric spectral stability theory, one can determine whether or not our RND PDE admits spectrally stable shock-fronted travelling waves. The multiple-scale nature of the regularised RNDPDE continues to play an important role in the analysis of the spatial eigenvalue problem.

[MS27] Geometric analysis of Travelling Waves in the Zeldovich-Frank-Kamenetskii Equation

Peter Szmolyan

TU Wien

Tuesday, 16:15, 1D340

This is a joint work with S. Jelbart (TU Wien), K. Kristiansen (DTU).

We prove the existence of a family of travelling wave solutions in a variant of the Zeldovich-Frank-Kamenetskii equation, a reaction-diffusion equation which models premixed flame propagation [1]. Our results are valid in an asymptotic regime which corresponds to a reaction with high activation energy, and provide a rigorous counterpart to formal asymptotic results that have been obtained for similar problems using high activation energy asymptotics.

In the high activation energy setting the Arrhenius type reaction rate can be written as the exponential nonlinearity $\exp\left(\frac{\Theta-1}{\varepsilon}\right)$ which depends singularly on the small parameter $0 < \varepsilon \ll 1$. Here $0 < \Theta \leq 1$ is the scaled temperature. This leads to two different scaling regimes $1-\Theta = \mathcal{O}(1)$ and $1 - \Theta = \mathcal{O}(\varepsilon)$ as $\varepsilon \to 0$, which are known as the convective-diffusive and diffusive-reactive zones. We identify and analyse a (c, ε)-family of heteroclinic orbits which traverse both of these regimes, and correspond to travelling waves with speed c in the original ZFK equation. We also go beyond the existing results by proving smoothness of the minimum wave speed function $c_{\min}(\varepsilon)$. The main idea of the proof is to use geometric singular perturbation theory and the blow-up method to rigorously control and match the two regimes.

[1] S. Jelbart, K. Kristiansen, P. Szmolyan, *Travelling Waves and Exponential Nonlinearities in the Zeldovich-Frank-Kamenetskii Equation,* arXiv:2405.10076, (2024).

[MS27] Conformally symplectic perturbations of Hamiltonian systems

Marian Gidea

Yeshiva University

Tuesday, 16:45, 1D340

This is a joint work with Samuel Akingbade (Yeshiva University, New York, USA) and Tere M-Seara (Polytechnical University of Catalonia, Barcelona, Spain).

We study the effect of dissipative, conformally symplectic perturbations on Hamiltonian systems. We consider a model of a pendulum-rotator system subject to a small, time-periodic Hamiltonian perturbation of a generic type. The system possesses a normally hyperbolic invariant manifold (NHIM), which contains a Cantor family of KAM tori. There also exist transverse homoclinic orbits, whose effect on the dynamics can be described via the scattering map, which relates the past asymptotic of any homoclinic orbit to the future asymptotic. Previous works show that the scattering map is symplectic.

To this system we add a small, dissipative, conformally symplectic perturbation. This is a singular perturbation, as it leads to attractors inside the NHIM, which can contain at most one invariant KAM torus. Transverse homoclinic orbit persist, giving rise to a perturbed scattering map. It turns out that, under some conditions on the conformal factor and the hyperbolic rates, the perturbed scattering remains symplectic [2].

The main result is that we can use the two dynamics – the dynamics restricted to the NHIM and the dynamics along homoclinic orbits – to obtain orbits along which the energy of the system grows by an amount independent of the small parameter [1]. This amounts to proving Arnold diffusion in a Hamiltonian system with small dissipation, thus answering a question by Chirikov.

[1] Samuel W Akingbade, Marian Gidea, and Tere M-Seara. Arnold diffusion in a model of dissipative system. *SIAM Journal on Applied Dynamical Systems*, 22(3):1983–2023, 2023.

[2] M. Gidea, R. de la Llave, and T. M-Seara. Conformally symplectic perturbations of Hamiltonian system. *Preprint*, 2024.

[MS27] Front propagation in two-component reaction-diffusion systems with a cut-off

Nikola Popovic

University of Edinburgh

Wednesday, 8:45, 1D328

This work is joint with Zhouqian Miao (China Jiliang University, Hangzhou, China), Mariya Ptashnyk (Heriot-Watt University, Edinburgh, United Kingdom), and Zak Sattar (University of Edinburgh, Edinburgh, United Kingdom).

The Fisher-Kolmogorov-Petrowskii-Piscounov (FKPP) equation with a cut-off was popularised by Brunet and Derrida in the 1990s as a model for many-particle systems in which concentrations below a given threshold are not attainable. While travelling wave solutions in cut-off scalar reaction-diffusion equations have since been studied extensively, the impacts of a cut-off on systems of such equations are less well understood. As a first step towards a broader understanding, we consider various coupled two-component reaction-diffusion equations with a cut-off in the reaction kinetics, such as an FKPP-type population model of invasion with dispersive variability due to Cook [1], a FitzHugh-Nagumo-style model with piecewise linear Tonnelier-Gerstner kinetics [2] and, finally, a more general predator-prey model with a cut-off in both components that is motivated by standard Lotka-Volterra-type dynamics. Throughout, our focus is on the existence, structure, and stability of travelling fronts, as well as on their dependence on model parameters; in particular, we determine the correction to the front propagation speed that is due to the cut-off. Our analysis is for the most part based on a combination of geometric singular perturbation theory [3] and the desingularisation technique known as "blow-up" [4].

[1] J. D. Murray, Mathematical Biology I. An Introduction. Springer-Verlag, Berlin Heidelberg, 2002.

[2] E. P. Zemskov and V. Méndez, *Propagation of fronts in activator-inhibitor systems with a cutoff*. Eur. Phys. J. B **48**(1) (2005), 81–86.

[3] N. Fenichel, *Geometric singular perturbation theory for ordinary differential equations*. J. Differential Equations **31**(1) (1979), 53–98.

[4] M. Krupa and P. Szmolyan, *Extending geometric singular perturbation theory to nonhyperbolic points—fold and canard points in two dimensions*. SIAM J. Math. Anal. **33**(2) (2001), 286–314.

[MS27] Geometric analysis for Galerkin's approximations of fast-slow systems of partial differential equations

Mariya Ptashnyk

Heriot-Watt University

Wednesday, 9:15, 1D328

This is a joint work with M. Engel (FU Berlin, Germany), F. Hummel, C. Kühn (TU Munich, Germany), N. Popović, T. Zacharis (University of Edinburgh, Scotland, UK)

To analyse the dynamics around a singularity of solutions to a system of fast-slow partial differential equations (PDEs) we consider the corresponding Galerkin approximation [1]. The finite-dimensional Galerkin approximation reduces the system of PDEs to a large system of ODEs, with the specific structure resulting from the projection of PDEs on the H^1-basis given by the eigenfunctions of the Laplace operator with the zero Neumann boundary conditions.

Using the geometric singular perturbation techniques the finite-dimensional Galerkin approximation of the slow manifold, which exists in the phase space of the system of PDEs away from the singularity, is extended around the fold singularity. Under appropriate assumptions on initial conditions, corresponding to solutions of PDEs close to spatially homogeneous ones, we show that the dynamics of the finite-dimensional Galerkin approximation in a neighbourhood of the fold singularity at the origin can be reduced to the dynamics of the corresponding ODEs for the singularly perturbed planar fold.

The challenges and novelty of our analysis are related to the spatial dependence of solutions to PDEs, which incurs in the corresponding Galerkin approximation through the domain length, eigenvalues, and complex nonlinear coupling between equations.

[1] M. Engel, F. Hummel, C. Kühn, N. Popović, M. Ptashnyk, T. Zacharis, *Geometric analysis of fast-slow PDEs with fold singularities.* arXiv 2022.

[MS27] Fast-slow systems and slow manifolds: From ODE to PDE

Jan-Eric Sulzbach

Technical University of Munich

Wednesday, 9:45, 1D328

This is a joint work with Christian Kuehn (Technical University of Munich).

Fast-slow dynamical systems as a class of singularly perturbed systems arise in various fields from climate science to biology and chemistry. In finite dimensions these systems have been studied by many authors over the last decades in the framework of the geometric singular perturbation theory. However, it is very challenging to transfer these results to infinite dimensions. In this talk we focus on the following fast-reaction system

$$\begin{aligned} \partial_t u &= Au + \frac{1}{\varepsilon} f(u,v), \\ \partial_t v &= Bv + g(u,v) \end{aligned} \tag{35}$$

in a general Banach space, where A and B are linear unbounded (differential) operators and the parameter $\varepsilon > 0$ denotes the spearation of time-scales. The goal is to obtain a generalization of the Fenichel-Tikhonov theory in this infinite dimensional setting. The key insight is to introduce an additional parameter that stems from a splitting in the slow variable space. Following a Lyapunov-Perron approach we construct the slow manifold and then prove that the slow manifold is close to the critical manifold of the limit problem. Moreover, the semi-flow on the slow manifold converges to the semi-flow on the critical manifold. At the end of this talk we will apply this abstract framework to a predator-prey model and provide an outlook to further generalizations of the theory.

[MS27] Rigorous derivation of Michaelis-Menten kinetics in the presence of diffusion

Bao Ngoc Tran

University of Graz

Wednesday, 10:15, 1D328

This is a joint work with Prof Bao Quoc Tang (University of Graz, Austria).

Reactions with enzymes are critical in biochemistry, where the enzymes act as catalysis in the process. One of the most used mechanisms for modeling enzyme-catalyzed reactions is the Michaelis-Menten (MM) kinetics. In the ODE level, i.e. concentrations are only on time-dependent, this kinetics can be rigorously derived from mass action law using quasi-steady-state approximation. This issue in the PDE setting, for instance when molecular diffusion is taken into account, is considerably more challenging and only formal derivations have been established. We present interesting results in the paper [1], where the derivation is proved rigorously and MM kinetics is obtained in the presence of spatial diffusion. In particular, we show that, in general, the reduced problem is a cross-diffusion-reaction system. Besides, the outlook will be discussed with ralated problems of singular limits and fast-slow systems.

[1] Bao Quoc Tang, Bao-Ngoc Tran, *Rigorous derivation of Michaelis-Menten kinetics in the presence of diffusion* https://arxiv.org/pdf/2303.07913.pdf

[MS28] An application of Talagrand's inequality to the Linear Sigma Model on $\mathbb{T}^2$

Scott Smith

Chinese Academy of Sciences

Monday, 13:45, 1A305

This is a joint work with Matías Delgadino (University of Texas-Austin).

The Linear Sigma Model on $\mathbb{T}^2$ is the N-component and $O(N)$-invariant generalization of the well-known Φ^4_2 model. In the present work, we show that combining Talagrand's inequality with some classical tools from Euclidean field theory leads to a simple proof that the 2-Wasserstein distance between the interacting measure and a massive Gaussian free field obeys CLT scaling in N. In contrast to prior work using stochastic quantization, our proof applies in finite volume without any perturbative assumptions on the mass or the coupling constant.

[MS28] Lower bounds to energy dissipation in passive scalar advection

Tommaso Rosati

University of Warwick

Monday, 14:15, 1A305

We consider a passive scalar advected by a stochastic velocity field. Under some non-degeneracy assumptions on the noise, we prove a lower bound on the energy dissipation that is quantitative in the diffusivity of the scalar. The proof is based on dynamics of energy level sets, a refined short-time and high-frequency expansion and the introduction of suitable concentration norms. Joint work with M. Hairer, S. Punshon-Smith and J. Yi.

[MS28] Weak coupling scaling of critical SPDEs

Giuseppe Cannizzaro

University of Warwick

Monday, 15:15, 1A305

This is a joint work with D. Erhard (Universidad da Bahia, Salvador, Brazil), M. Gubinelli (University of Oxford, Oxford, UK), and F. Toninelli (Technische Universität Wien, Vienna, Austria).

The study of stochastic PDEs has known tremendous advances in recent years and, thanks to Hairer's theory of regularity structures and Gubinelli and Perkowski's paracontrolled approach, (local) existence and uniqueness of solutions of subcritical SPDEs is by now well-understood. The goal of this talk is to move beyond the aforementioned theories and present novel tools to derive the scaling limit (in the so-called weak coupling scaling) for some stationary SPDEs at the critical dimension. Our techniques are inspired by the resolvent method developed by Landim, Olla, Yau, Varadhan, and many others, in the context of particle systems in the supercritical dimension. Time allowing, we will explain how it is possible to use our techniques to study a much wider class of statistical mechanics models at criticality such as (self-)interacting diffusions in random environment.

[MS28] Two-point function of KPZ with Gaussian initial data

Sefika Kuzgun

University of Rochester

Monday, 15:45, 1A305

In this talk, we consider KPZ equation (respectively, KPZ fixed-point) starting from a Gaussian process with stationary increments. Using Malliavin integration by parts, we establish formulas for two-point correlation function of the spatial derivative process in terms of the variance. This talk is based on an ongoing project with Arjun Krishnan.

[MS28] SHE and KPZ with rougher than white noise

Mate Gerenscer

TU Wien

Monday, 16:15, 1A305

We consider the KPZ equation (or after a Cole-Hopf transform, SHE, or nonlinear variants of the latter) in one dimension driven by noise that is rougher than white, e.g. it is derivative of white of order γ. It is known that a change in behavior takes place at $\gamma = 1/4$, as opposed to the criticality threshold $\gamma = 1/2$. We describe some recent contributions in both the $\gamma < 1/4$ and $\gamma > 1/4$ regimes, complementing works of [1,2,3]. Based partly on joint work with Fabio Toninelli.

[1] M. Hairer, *Renormalisation in the presence of variance blowup*, `arXiv:2401.10868` (2024)

[2] M. Hoshino, *KPZ equation with fractional derivatives of white noise*, Stochastics and Partial Differential Equations: Analysis and Computations, **4** (2016), 827–890

[3] Y. Hu, J. Huang, K. Lê, D. Nualart, and S. Tindel, *Stochastic heat equation with rough dependence in space*, Annals of Probability, **45** (2017) 4561-4616

[MS28] On global solutions for fluid dynamics SPDEs

Oana Lang

Imperial College London

Monday, 16:45, 1A305

This is joint work with Prof. Dan Crisan (Imperial College London)

A large class of partial differential equations exists in the literature for which global existence of solutions holds only locally, with several such examples coming from geophysical fluid dynamics. We introduce a stochastic component that ensures that the solutions of the corresponding stochastically perturbed equations are global. The class of partial differential equations amenable for this type of treatment includes the rotating shallow water model (viscous and inviscid), 3D Euler equation (in vorticity form), 2D Burgers' equation and many other fluid dynamics models.

[1] Dan Crisan and Oana Lang, Global Solutions for stochastically controlled fluid dynamics models, arXiv:2403.05923.

[MS28] Analysis of a positivity-preserving splitting scheme for some nonlinear stochastic heat equations

David Cohen

Chalmers University of Technology and University of Gothenburg

Monday, 17:15, 1A305

The talk is based on joint works with Johan Ulander (Chalmers University of Technology and University of Gothenburg, Sweden) and Charles-Edouard Bréhier (Université de Pau et des Pays de l'Adour, France)

We construct and analyze a positivity-preserving Lie–Trotter splitting scheme with finite difference discretization in space for approximating the solutions to a class of nonlinear stochastic heat equations driven by multiplicative noise.

[MS28] The Porous Medium Equation: Rescaled Zero-Range Process, Large Deviations and Gradient Flow

Daniel Heydecker

Max Planck Institute for Mathematics in the Sciences

Monday, 17:45, 1A305

This is joint work with Professor Benjamin Gess (Max-Planck Institute for Mathematics in the Sciences / University of Bielefeld)

We study a rescaling of the zero-range process with homogenous jump rates $g(k) = k^\alpha$ with arbitrary $\alpha \geq 1$. With a simultaneous rescaling of space, time and particle size, we identify the dynamical large deviations from the porous medium equation

$$\partial_t u(t,x) = \frac{1}{2}\Delta u^\alpha(t,x), \qquad x \in \mathbb{T}^d. \tag{36}$$

The method is based on pathwise discretised regularity estimates to prove a version of the superexponential estimate [2] in the spirit of the Aubin-Lions-Simons lemma [3]. Finally, we use the connection of large deviation principles to gradient flow [1] to give an expression of the porous medium equation as the gradient flow of the Boltzmann entropy with respect to a tailor-made Wasserstein-type distance.

[1] S. Adams, N. Dirr, M. Peletier, and J. Zimmer. *Large deviations and gradient flows.* Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, **371** (2013).

[2] C. Kipnis and C. Landim. *Scaling limits of interacting particle systems.* Springer Science & Business Media (2013).

[3] A. Moussa. *Some variants of the classical Aubin-Lions lemma.* Journal of Evolution Equations, 16 (2016), pp.65-93.

[MS28] Regularisation of reaction-diffusion equations by multiplicative noise

Konstantinos Dareiotis

University of Leeds

Tuesday, 8:45, 1D327

This is a joint work with Teodor Holland and Khoa Lê. (University of Leeds, UK).

We consider stochastic reaction-diffusion equations in $1+1$ dimensions driven by multiplicative space-time white noise with distributional drift belonging to the Besov-Hölder space $\mathcal{C}^\alpha$ with $\alpha > -1$. We assume that the diffusion coefficient is sufficiently regular and nondegenerate. By using a combination of stochastic sewing arguments and Malliavin calculus, we show that the equation admits a unique strong solution.

[MS28] Non-equilibrium fluctuations, conservative stochastic PDE, and parabolic-hyperbolic PDE with irregular drift

Ben Fehrman

LSU

Tuesday, 9:15, 1D327

This is a joint work with Prof. Benjamin Gess (Universität Bielefeld / Max Planck Institute for Mathematics in the Sciences, Bielefeld / Leipzig, Germany).

Far-from-equilibrium behavior in physical systems is widespread. A statistical description of these events is provided by macroscopic fluctuation theory, a framework for non-equilibrium statistical mechanics that postulates a formula for the probability of a space-time fluctuation based on the constitutive equations of the system. This formula is formally obtained via a zero noise large deviations principle for the associated fluctuating hydrodynamics, which postulates a conservative, singular stochastic PDE to describe the system out-of-equilibrium. In this talk, we will focus particularly on the fluctuations of certain interacting particle processes about their hydrodynamic limits. We will show how the associated MFT and fluctuating hydrodynamics lead to a class of conservative SPDEs with irregular coefficients, and how the study of large deviations principles for the particles processes and SPDEs leads to the analysis of parabolic-hyperbolic PDEs in energy critical spaces. The analysis makes rigorous the connection between MFT and fluctuating hydrodynamics in this setting, and provides a positive answer to a long-standing open problem for the large deviations of the zero range process.

[MS28] Higher order approximation for nonlinear SPDEs with additive space-time white noise

Ana Djurdjevac

Freie Universität Berlin

Tuesday, 9:45, 1D327

This is a joint work with Máté Gerencsér (TU Wien) and Helena Kremp (TU Wien).

We consider strong approximation of 1 + 1-dimensional stochastic PDEs driven by additive space-time white noise. It has been long proposed [1,2], as well as observed in simulations, that approximation schemes based on samples from the stochastic convolution, rather than from increments of the underlying Wiener processes, should achieve significantly higher convergence rates with respect to the temporal timestep.

The present paper proves this. For a large class of nonlinearities, with possibly superlinear growth, a temporal rate of (almost) 1 is proved, a major improvement on the rate 1/4 that is known to be optimal for schemes based on Wiener increments. The spatial rate remains (almost) 1/2 as it is standard in the literature.

[1] A. M. Davie and J. G. Gaines, *Convergence of numerical schemes for the solution of parabolic stochastic partial differential equations* Mathematics of Computation **233(70)** (2001), 121–135.

[2] A. Jentzen and P. E Kloeden, Overcoming the order barrier in the numerical approximation of stochastic partial differential equations with additive space–time noise,Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, **2102(465)** (2018), 649–667.

[MS28] Stochastic Modified Flows, Mean-Field Limits and Dynamics of Stochastic Gradient Descent

Vitalii Konarovskyi

Hamburg University

Tuesday, 10:15, 1D327

This is a joint work with Benjamin Gess (Bielefeld University, Bielefeld, Germany and Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany) and Sebastian Kassing (Bielefeld University, Bielefeld, Germany).

We will discuss new limiting dynamics for stochastic gradient descent in the small learning rate regime, called stochastic modified flows. These SDEs are driven by a cylindrical Brownian motion and improve the so-called stochastic modified equations by having regular diffusion coefficients and by matching the multipoint statistics. We will also introduce distribution-dependent stochastic modified flows, which we prove to describe the fluctuating limiting dynamics of stochastic gradient descent in the small learning rate – infinite width scaling regime.

[1] Benjamin Gess, Sebastian Kassing, Vitalii Konarovskyi, *Stochastic Modified Flows, Mean-Field Limits and Dynamics of Stochastic Gradient Descent* J. Mach. Learn. Res. **25** (2024) No. 30, 27 pp.

[2] Benjamin Gess, Rishabh S. Gvalani, Vitalii Konarovskyi, *Conservative SPDEs as fluctuating mean field limits of stochastic gradient descent* arXiv:2207.05705. (2022), 65 pp.

[MS30] Controlling Populations of Neural Oscillators

Jeff Moehlis

University of California, Santa Barbara

Monday, 15:15, 1D327

Many challenging problems that consider the analysis and control of neural brain rhythms have been motivated by the advent of deep brain stimulation as a therapeutic treatment for a wide variety of neurological disorders. In a computational setting, neural rhythms are often modeled using large populations of coupled, conductance-based neurons. Control of such models comes with a long list of challenges: the underlying dynamics are nonnegligibly nonlinear, high dimensional, and subject to noise; hardware and biological limitations place restrictive constraints on allowable inputs; direct measurement of system observables is generally limited; and the resulting systems are typically highly underactuated. In this talk, I highlight a collection of recent analysis techniques and control frameworks that have been developed to contend with these difficulties. Particular emphasis is placed on the problem of desynchronization for a population of pathologically synchronized neural oscillators, a problem that is motivated by applications to Parkinson's disease where pathological synchronization is thought to contribute to the associated motor control symptoms. Several different control approaches will be described: maximizing the Lyapunov exponent associated with the neurons' phase dynamics, optimal phase resetting, phase density control, and controlling the population to have clustered dynamics. This talk is based on [1].

[1] D. Wilson and J. Moehlis, *Recent advances in the analysis and control of large populations of neural oscillators* Ann. Rev. Control **54** (2022), 327–351.

[MS30] Phase-amplitude description of oscillatory neural networks via the parameterization method with applications to neural communication

Gemma Huguet

Universitat Politècnica de Catalunya

Monday, 15:45, 1D327

This is a joint work with Prof A. Guillamon (UPC), M. Orieux (UPC), Prof. A. Pérez-Cervera (UPC), D. Reyner-Parra (UPC) and Prof. T. M. Seara (UPC).

Abstract: Oscillations are ubiquitous in the brain, and they have been associated with several cognitive functions, such as perception and attention. The Communication Through Coherence theory [1] proposes that oscillations play a role in neuronal communication, emphasizing the need of synchronization at specific phases for effective communication among oscillating neural groups. In this presentation, we examine oscillations in neural networks using exact mean-field models [2, 3], which provide a precise description of a network's macroscopic activity. Leveraging recent numerical advancements in phase-amplitude descriptions of oscillators [4], we thoroughly analyze oscillatory dynamics and phase-locking patterns between neural populations, beyond the simple phase reduction [5]. We identify conditions for optimal phase-locking and effective communication and we design suitable controls to regulate the frequency of oscillations and set the oscillatory groups in the appropriate phase for communication [6].

Our methodology combines the phase-amplitude description of an oscillator via the parameterization method along with optimal control techniques. The parameterization method allows to design efficient numerical algorithms to obtain a simplified description of the dynamics on the attracting invariant manifold of the limit cycle in terms of the phase-amplitude variables. It also provides straightforwardly the infinitesimal Phase and Amplitude Response Functions, which monitor the phase and amplitude shifts beyond the asymptotic state. We also introduce strategies to reduce the dimensionality of dynamics, including the reduction to the slow stable submanifold. This comprehensive approach advances our understanding of oscillatory dynamics in neural networks, offering insights for targeted control in cognitive processes.

[1] P. Fries. *Rhythms for cognition: Communication through coherence.* Neuron, **88** (2015), 220–232.

[2] E. Montbrió, D. Pazó, and A. Roxin. *Macroscopic description for networks of spiking neurons.* Phys. Rev. X, **5** (2015).

[3] G. Dumont and B. Gutkin. *Macroscopic phase resetting-curves determine os- cillatory coherence and signal transfer in inter-coupled neural circuits.* PLOS Comput. Biol., **15** (2019), 1–34.

[4] A. Pérez-Cervera, T. M.-Seara, and G. Huguet. *Global phase-amplitude description of oscillatory dynamics via the parameterization method.* Chaos, **30** (2020).

[5] D. Reyner-Parra and G. Huguet. *Phase-locking patterns underlying effective communication in exact firing rate models of neural networks.* PLOS Comput. Biol., **18** (2022), 1–41.

[6] M. Orieux, A. Guillamon and G. Huguet. *Optimal control of oscillatory neuronal models with applications to communication through coherence.* Preprint. Arxiv: 2311.08790, 2023.

[MS30] Phase resetting as a boundary-value problem

Peter Langfield

Inria Bordeaux

Monday, 16:15, 1D327

In phase resetting experiments, a biological oscillator is typically subjected to a fixed-amplitude stimulus at a particular moment in its cycle, and the new phase that results is recorded. The graph of the map from application phase to resulting phase is the *phase transition curve* (PTC), which, when modeling the oscillator as an attracting periodic orbit, can be used to predict more complicated behaviours such as synchrony and entrainment. Phase response to perturbations is also studied by considering the *isochrons* of the periodic orbit, where each isochron is the set of points with the same resulting phase, and forms an $(n-1)$-dimensional submanifold [1] [2]. However, the biological models that motivate their study typically have dimension $n \gg 2$, and evolve over vastly different time scales, which imposes numerical challenges when resolving the isochrons [3].

A practical approach to the first problem is to consider that one is typically only interested in the phase response of points perturbed away from the periodic orbit in some fixed direction. In this way, the space of interest is a two-dimensional subspace parametrised by perturbation application phase and amplitude. Moreover, the slices of isochron that exist in this two-dimensional subspace are generally one-dimensional curves.

We first present our method for computing PTCs via the numerical continuation of a two-point boundary-value problem that relates the phase of a point resulting from a perturbation, with the amplitude of the perturbation, and the phase in the cycle at which it is applied [4]. We then show how this method can be adapted to also compute isochron slices parametrised directly in perturbation phase and amplitude. We demonstrate the strength of our method by computing PTCs of an extremely sensitive seven-dimensional sino-atria node cell model. We also compute isochron slices of the 4D Hodgkin–Huxley model, and show crucial detail missed in previously depictions.

[1] A.T. Winfree, *Patterns of phase compromise in biological cycles.* J. Math. Biol., **1** (1974), pp.73-93.

[2] J. Guckenheimer, *Isochrons and phaseless sets.* J. Math. Biol., **1** (1975), 259-273.

[3] P. Langfield, B. Krauskopf, and H. M. Osinga. *Solving Winfree's puzzle: The isochrons in the FitzHugh-Nagumo model.* Chaos **24** (2014).

[4] P. Langfield, B. Krauskopf, and H. M. Osinga, *A continuation approach to computing phase resetting curves,* In Advances in Dynamics, Optimization and Computation, Springer International Publishing, (2020), pp. 3-30.

[MS30] A geometric singular perturbation approach to a coupled oscillator model for wake, REM and non-REM sleep

Anne Skeldon

University of Surrey

Monday, 16:45, 1D327

This is a joint work with Rachel Bernasconi (University of Surrey, Guildford, UK), Panos Kaklamanos (University of Edinburgh, Guildford, UK), Gianne Derks (University of Leiden, Leiden, Netherlands).

Nearly all animals alternate between the two vigilant states of wake and sleep. In many species, sleep is further sub-divided into two distinct categories: rapid eye movement (REM) sleep and non-REM sleep. In healthy adult humans, we have one major sleep episode a day in which we alternative between REM and non-REM sleep in cycles of approximately 90 minutes. REM and non-REM sleep contribute in different ways to human function and one view of the REM–Non-REM cycling is that it is an adaptive mechanism to make sure that even if sleep is curtailed we experience at least some Non-REM and some REM sleep. At the beginning of the night, the majority of the time in the cycle is spent in non-REM sleep, while towards the end of the night the majority of the time in the cycle is spent in REM sleep.

Coupled oscillator models have been developed that include the key neuronal populations that promote wake, non-REM and REM sleep, the oscillations of the biological clock (circadian rhythmicity) and a relaxation oscillator describing homeostatic sleep pressure. Such models are inherently multi-timescale, because we typically remain in a given individual vigilant state for much more time than the time it takes to switch between different states. In this talk, I will discuss the multiple timescale structure of the most widely used model of wake, REM and non-REM sleep [1]. We find that for commonly used parameter values the model has three timescales. Understanding the three timescale structure clarifies the mechanisms underlying the duration, timing and number of REM episodes.

[1] V. Booth and G.G. Diniz Behn, *Physiologically-based modeling of sleep-wake regulatory networks*, Math. Biosc. **250** (2014), 54-68.

[MS30] Integrability of a globally coupled complex Riccati array

Erik Andreas Martens

Lund University

Thursday, 8:45, 1D328

This is joint work with Dr. C. Bick (Vrije Universiteit, Amsterdam, Netherlands) and Dr. R. Cestnik (Lund University, Lund, Sweden).

Biological and neural systems can be seen as networks of interacting periodic processe. Their functionality, i.e., whether these networks can perform their function or not, depends on the emerging collective dynamics of the network. Synchrony of oscillations is one of the most prominent examples of such collective behavior and has been associated with function and dysfunction. Understanding how collective dynamics emerges in oscillatory networks is critical to finding factors that lead to malfunction of the network.

However, many biological systems such as the brain consist of a large number of dynamical units. Hence, their analysis has either relied on simplified heuristic models, or the analysis comes at a huge computational cost. Recently introduced approaches, known as the Ott–Antonsen and Watanabe–Strogatz reductions, valid for sinusoidally coupled phase oscillators, allow to exactly describe the microscopic dynamics in terms of few collective (macroscopic) variables only. I will briefly review these methods [1], and then discuss a generalization of the Watanabe-Strogatz method to globally coupled complex-valued Riccati equations that we discovered recently [2]. This formulation reshapes our understanding of a broad class of coupled system which fall under the category of integrable systems. If time permits, I will also give a short outlook into possibilities to reduce the dynamics of oscillator networks where the coupling strength between oscillators slowly adapts in time [3].

[1] C. Bick, C. Laing, M. Goodfellow and E. A. Martens, *Understanding Synchrony Patterns in Biological and Neural Oscillator Networks through Mean-Field Reductions: A Review*, Journal of Mathematical Neuroscience, **10**(9) (2020); `http://arXiv:1902.05307`

[2] R. Cestnik, E. A. Martens, **I**ntegrability of a globally coupled complex Riccati array: quadratic integrate-and-fire neurons, phase oscillators and all in between, Phys. Rev. Lett. **132** (2024), 057201; `https://arXiv:2305.17683`

[3] E. A. Martens and C. Bick, *Constrained adaptation for oscillator networks* (in preparation).

[MS30] On the stability of coupled cell systems on hypergraphs with polynomial interaction functions

Shaoxuan Cui

University of Groningen

Thursday, 9:45, 1D328

This is a joint work with Prof. Guofeng Zhang (Department of Applied Mathematics, The Hong Kong Polytechnic University, China), Dr. Hildeberto Jardón Kojakhmetov (Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, Netherlands), and Prof. Ming Cao (Engineering and Technology Institute Groningen, University of Groningen, Netherlands).

A coupled cell system is a network of dynamical systems, where the fundamental units, “cells”, are coupled together. Such systems are widely considered in engineering and mathematics. They can be represented schematically by a directed network whose nodes correspond to cells and whose edges represent couplings. Typical examples of coupled cell systems include but are not limited to phase oscillators, diffusion processes, and ecological systems.

Recently, coupled cell systems on hypergraphs have been proposed by [1]. With the introduction of higher-order interaction terms, the dynamics are potentially able to capture the higher-order relationship among cells. However, higher-order terms challenge the mathematical analysis of the system’s behavior.

In this talk, we focus on a coupled cell system on a hypergraph with polynomial-type coupling. Inspired by the analysis of conventional networked systems, where the adjacency matrix plays an important role, we investigate the case when the adjacency tensor of the hypergraph adopts a certain structure. Then, we further use the properties of the tensor to study the stability of equilibria of such a system. Finally, we further illustrate our methodologies with a concrete real-world system, namely an SIS diffusion process on a hypergraph. The talk is based on [2].

[1] C. Bick, E. Gross, H. A. Harrington, and M. T. Schaub, “What are higher-order networks?” SIAM Review, 2023.

[2] S. Cui, G. Zhang, H. Jardón-Kojakhmetov, M. Cao, "On Metzler positive systems on hypergraphs". arXiv preprint arXiv:2401.03652, 2024.

[MS30] Emergent hypernetworks in weakly coupled oscillators

Eddie Nijholt

Imperial College London

Thursday, 10:15, 1D328

Networks of coupled oscillators are important models of complex systems. In particular, model reconstruction from data is an invaluable tool for understanding a wide range of natural phenomena. Nevertheless, in many cases such techniques show the emergence of hyper-connections among oscillators, such as triplet and higher-order interactions, even though the models were originally defined as networks. In other words, only pairwise interactions should appear. We demonstrate that hypernetworks can indeed spontaneously appear in the presence of pairwise, nonlinear coupling under certain frequency resonance conditions. As it turns out, model reconstruction finds the so-called normal form of the system instead, which is indeed often a hypernetwork. This allows us to predict the emergent hypernetwork in terms of appearing and forbidden motifs. Our results are moreover shown in experiments with electrochemical oscillators. This is joint work with Jorge Luis Ocampo-Espindola, Deniz Eroglu, István Z. Kiss and Tiago Pereira.

[MS31] Physics Informed Machine Learning for the approximation of attracting Slow Invariant Manifolds

Dimitrios Patsatzis

Scuola Superiore Meridionale

Monday, 13:45, 1B306

This is a joint work with Gianluca Fabiani (Modelling Engineering Risk & Complexity Dept., Scuola Superiore Meridionale, Naples, Italy), Dr. Lucia Russo (Institute of Science and Technology for Energy and Sustainable Mobility, National Research Center, Naples, Italy) and Prof. Constantinos Siettos (Dept. of Mathematics and Applications "Renato Caccioppoli", University of Naples Federico II, Naples, Italy)

In this work, a Physics Informed Machine Learning (PIML) approach is presented [1], for tracking approximations of Slow Invariant Manifolds (SIMs) [2] that arise from singularly perturbed dynamical systems with explicit timescale splitting. To facilitate the construction and numerical integration of reduced order models (ROMs), the SIM functionals are provided in an explicit form by a ML function. For training the ML function, the loss function minimizes the well known *invariance equation* (IE), within the Geometric Singular Perturbation Theory (GSPT) framework [2]; thus introducing the physics informed character of the approach. Two neural network structures are used as ML functions and symbolic differentiation is employed for the computation of the gradients required for the learning process. The efficiency of the proposed approach is demonstrated in two examples; the target mediated drug disposition reaction mechanism, and the 3D Sel'kov model. The computational costs between symbolic, automatic and numerical approximation of the required derivatives in the learning process are also compared. It is finally shown that the proposed PIML scheme provides approximations, of equivalent or even higher accuracy, than those provided by other traditional GSPT-based methods (details can be found [3]), and more importantly, it is not affected by the magnitude of the perturbation parameter.

[1] D. G. Patsatzis, G. Fabiani, L. Russo, & C. Siettos, *Slow Invariant Manifolds of Singularly Perturbed Systems via Physics-Informed Machine Learning*, SIAM J. Sci. Comput., accepted (2023).

[2] N. Fenichel, *Geometric singular perturbation theory for ordinary differential equations*, J. Differential Equations, **31**(1):53–98, 1979.

[3] C. Kuehn, *Multiple time scale dynamics*, vol 191, 2015, Springer

[MS31] How spatial heterogeneities affect the behaviour of front solutions

Robbin Bastiaansen

Utrecht University

Monday, 14:15, 1B306

This contains joint works with prof. dr. ir. Henk A. Dijkstra (Utrecht University, NL), prof. dr. Arjen Doelman (Leiden University, NL), dr. Anna S. von der Heydt (Utrecht University, NL), and prof. dr. Tasso J. Kaper (Boston University, Boston, USA).

In many natural systems, spatial patterns emerge – such as vegetation patterns in ecosystems or convection patterns in fluids. The behaviour of these patterns is intricate and plays out over multiple time scales, including for example slow pattern adaptation (i.e. movement of localised structures) and fast pattern degradation (i.e. creation or annihilation of localised structures). Moreover, the precise pattern dynamics depends on the details of the domain, such as on the precise topography in case of ecosystems or the precise spatial variation in deep water densities in case of ocean circulation.

In this talk, I will illustrate how spatial heterogeneities (i.e. spatially varying coefficients) can effect the dynamics of (interacting) fronts in reaction-diffusion systems of Allen-Cahn type – and how this differs from the spatial homogeneous case. Specifically, I will highlight how front interactions play out on a small-amplitude spatially periodically forced Allen-Cahn equation, and how spatial heterogeneities alter the bifurcation structure of natural pattern forming systems thus potentially giving rise to a more fragmented tipping response when facing worsening environmental conditions.

[MS31] Cross-diffusion system driven by fast reaction limit: weak solutions and weak-strong stability

Elisabetta Brocchieri

University of Graz

Tuesday, 8:45, 1D328

In this talk, we analyse a class of cross-diffusion systems modeling the evolution of densities of multicomponent populations in interaction. More precisely, we prove the existence of weak solutions for a starvation driven cross-diffusion system, obtained as the limit of a reaction diffusion system with linear diffusion and fast reaction. The main tools used to rigorously pass to the limit consist of a priori estimates, given by the analysis of a family of entropy functionals, and compactness arguments. However, we also investigate the regularity of the obtained solution, by improving the entropy a priori estimates thanks to a bootstrap argument. We conclude the analysis with a weak-strong stability result. This talk is based on [1] and [2, Chapter 2].
This is a joint work with L. Corrias (Université d'Evry Val d'Essonne, Paris, France) and L. Desvillettes (Université Paris Cité, Paris, France).

[1] E. Brocchieri, *Evolutionary dynamics of populations structured by dietary diversity and starvation: cross-diffusion systems.* PhD Thesis, (2023).

[2] E. Brocchieri, L. Corrias, H. Dietert, and Y-J. Kim, *Evolution of dietary diversity and a starvation driven cross-diffusion system as its singular limit.* J. Math. Biol., **83** (2021).

[MS31] Excitability and the role of different noise sources in networks of photonic neurons

Otti D'Huys

Maastricht University

Tuesday, 9:15, 1D328

This is a joint work with S. Barland (Université Côte d'Azur, France) and R. Veltz (Inria Sophia Antipolis, France).

Neuromorphic photonics – the photonic implementation of excitable nodes - can lead to applications in data processing and photonic computing platforms. On the other hand, photonics has long served as a hardware platform for the exploration of complex dynamics. Here, we study the dynamics of a population of several hundred coupled semiconductor lasers, coupled via their mean intensity through a non-linear optoelectronic feedback loop.

The system is well described by a set of stochastic differential equations with three different time scales [1, 2]: the fast (optical) time scales describe the semiconductor laser dynamics and the slow time scale describes the electronic signal in the feedback loop. We consider additive noise terms in the fastest variable - this noise is based on spontaneous emission noise, and is independent for each node. We also include noise in the slowest variable - the electronic noise is common for the whole network.

We study experimentally and numerically the excitable character of the network, by recording the response to an external perturbation. We show that the coupling has a stabilizing effect on the excitability, making the response more consistent, when only considering optical noise in each node. However, such effect is not reproduced in the experiment. We find that the inclusion of electronic noise common for the whole network counteracts this effect, and reproduces the experimental data. In a simplified model, we explain the interplay of the slow-fast network dynamics and the two different noise sources.

[1] K. Al-Naimee K, F. Marino, M. Ciszak, R. Meucci and F.T. Arecchi, *Chaotic spiking and incomplete homoclinic scenarios in semiconductor lasers with optoelectronic feedback*, New J. Phys. **11** 073022 (2009)

[2] A. Dolcemascolo, A. Miazek, R. Veltz, F. Marino F and S. Barland, *Effective low-dimensional dynamics of a mean-field coupled network of slow-fast spiking lasers*, Phys. Rev. E **101**, 052208 (2020)

[MS31] Multiple time-scale dynamics in neural mass models: canard solutions and critical regimes

Elif Köksal-Ersöz

Inserm

Tuesday, 9:45, 1D328

This is a joint work with J. Modolo, F. Wendling (Université de Rennes, INSERM LTSI U1099, Rennes, France) and F. Bartolomei (Aix Marseille University, Inserm, INS, Institut de Neurosciences des Systèmes, Marseille, France; APHM, Timone Hospital, Clinical Neurophysiology, Marseille, France).

Epileptic seizures are very often hallmarked by pre-ictal phases which constitute a possible window of opportunity to trigger electrical stimulation with the objective to prevent seizure initiation. In this study, we consider a typical pre-ictal regime with complex bursting-type oscillations, which can be accurately reproduced by a neural mass model that includes three interacting neuronal subpopulations with different synaptic kinetics. By analyzing the multiple time-scaled structure of the model, we identify the key role of the subpopulations of GABAergic interneurons. We unveil the existence of so-called "canard solutions" that can be encountered in specific non-linear dynamical systems. We detail how they organize the complex oscillations and excitability properties of the model [1, 2]. We show that boundaries between pathological epileptic discharges and physiological background activity are determined by the canard solutions. We report a degenerate folded saddle-node II singularity leading to canard trajectories that appear due to the general structure of neural mass models, which are defined via second-order differential equations.

[1] E. Köksal-Ersöz, J. Modolo, F. Bartolomei and F. Wending PLoS Comput. Biol. **16(11)** (2020),

e1008430.

[2] E. Köksal-Ersöz and F. Wending *Canard solutions in neural mass models: consequences on critical regimes* J. Math. Neurosci. **11** (2021), 11.

[MS31] Effects of multiple timescales in cardiomyocyte models

Jorge Alberto Jover-Galtier

University of Zaragoza

Tuesday, 10:15, 1D328

This is a joint work with Profs. R. Barrio, M.A. Martínez, S. Serrano (University of Zaragoza, Spain) and L. Pérez (University of Oviedo, Spain).

Cardiomyocytes are excitable cells, and they carry out their functions by generating electrical signals that are determined by the change in the potential of the cell membrane. Several mathematical models, including the 27-variable Sato model, have been developed during the last decades to describe these cells with satisfactory results. In particular, the Sato model predicts the appearance, under certain conditions, of early afterdepolarizations: small rises in the potential at abnormal phases of the excitatory cycle that, if large enough and occurring over a substantial tissue area, can lead to arrhythmia and cardiac problems.

In this talk, I will analyze the predictions of the Sato model and compare it with those of the 3-variable reduced Luo-Rudy (LR) model. I will show how the appearance of early afterdepolarizations is connected with the existence of different time scales [1]. Separation of the variables in the LR model according to their respective time scales allow for a mathematical analysis using singular perturbations [2], which I will present.

[1] R. Barrio, M. Á. Martínez, E. Pueyo, and S. Serrano, *Dynamical analysis of early afterdepolarization patterns in a biophysically detailed cardiac model.* Chaos **31** (2021), 1–20.

[2] R. Barrio, J. A. Jover-Galtier, M. Á. Martínez, L. Pérez, and S. Serrano, *Mathematical birth of Early Afterdepolarizations in a cardiomyocyte model.* Math. Biosci. **366** (2023), 109088.

[MS31] Slow passage through the transcritical and Hopf bifurcations by piecewise linear differential systems.

Antonio Teruel

University of Balearic Islands

Wednesday, 15:15, 1B309

This is a joint work with Prof. Alberto Pérez (Polytechnic University of Catalonia, Barcelona, Spain) and Prof. Mathieu Desroches (Inria Center at Université Côte d'Azur, Nice, France)
In this presentation, we examine the phenomenon of slow passage through bifurcations in piecewise linear (PWL) differential systems, where an equilibrium point loses stability. Specifically, we focus on the transcritical bifurcation and the Hopf bifurcation (referred to as Hopf-like within the PWL framework).
Our analysis explores the delay in the loss of stability and places emphasis on describing the maximum delay as a function of the bifurcation parameter and the singular parameter. Our findings encompass all potential maximum delay behaviors across the parameter range, enabling us to identify three scenarios:

i) The trivial scenario, where the maximum delay tends to zero with the singular parameter.

ii) The singular scenario, where the maximum delay is unbounded.

iii) The transitional scenario, where the maximum delay tends to a positive finite value as the singular parameter approaches zero.

Finally, we apply this knowledge to analyze a non-bounded canard explosion, resulting in an enhanced delay phenomenon [1]. Additionally, we apply it to analyze the elliptic bursting behavior in the Doi-Kumagai differential system [2].

[1] A. Vidal,J. P. Françoise, *Canard cycles in global dynamics.* Internat. J. Bifur. Chaos Appl. Sci. Engrg, 22(02), (2012).

[2] S. Doi, S. Kumagai, *Generation of very slow neuronal rhythms and chaos near the Hopf bifurcation in single neuron models.* J. Comput. Neurosci. 19(3), 325–356 (2005).

[MS31] Variational convergence of an approximation scheme for aggregation-diffusion equations

Anastasiia Hraivoronska

Institut Camille Jordan, Universite Claude Bernard Lyon

Wednesday, 15:45, 1B309

This contains joint works with prof. dr. André Schlichting (University of Münster, DE), dr. Oliver Tse (Eindhoven University of Technology, NL).

Aggregation-diffusion equations appear in many mathematical models describing the dynamical behavior of biological systems. Examples of such dynamics are the crowd motion of bacteria driven by chemical potentials and the collective movement of people. In this talk, I will consider a finite-volume discretization of aggregation-diffusion equations based on the Scharfetter-Gummel approach. I will discuss the convergence of the Scharfetter-Gummel semi-discrete scheme using a variational method. Our variational approach allows for studying the discrete-to-continuum and vanishing-diffusion limits simultaneously.

[MS32] Euler-Poisson system and related problems

Bongsuk Kwon

Ulsan National Institute of Science and Technology

Wednesday, 8:45, 1B306

Various plasma phenomena are mathematically studied using a fundamental uid model for plasmas, called the Euler-Poisson system. Among them, plasma solitary waves are of our interest, for which existence, stability, and the time-asymptotic behavior of the solitary wave will be brie y discussed. On the other hand, to study nonlinear stability, a question of existence of smooth global solution naturally arises, which is completely open, to the best of our knowledge. We introduce the finite-time blow-up results for the Euler-Poisson system, and discuss the related open questions. This talk is based on joint work with Junsik Bae and Yunjoo Kim at UNIST.

[MS32] Time-asymptotic stability of generic Riemann solutions to the compressible Navier-Stokes equations

Yi Wang

Institute of Applied Mathematics, AMSS, Chinese Academy of Sciences

Wednesday, 9:15, 1B306

This is based on the joint works with Professor Moon-Jin Kang (KAIST, Korea) and Alexis F. Vasseur (University of Texas at Austin, USA).

The talk is concerned with our recent works on the time-asymptotic stability of basic wave patterns to the compressible Navier-Stokes equations and non-convex scalar conservation laws. First, the time-asymptotic stability of generic Riemann solutions, including different and multiple basic wave patterns, to one-dimensional both isentropic Navier-Stokes equations and full Navier-Stokes-Fourier equations will be shown ([1, 2]). Then the stability of planar viscous shock wave to the multi-dimensional compressible Navier-Stokes equations (joint with Teng Wang) will be presented ([3]). Last but not least, we will talk about our very recent result on the time-asymptotic stability of composite wave of viscous shock wave and rarefaction wave to a non-convex scalar conservation laws.

[1] Moon-Jin Kang, Alexis F. Vasseur, Yi Wang *Time-asymptotic stability of composite waves of viscous shock wave and rarefaction for barotropic Navier-Stokes equations,* Adv. Math. **419** (2023), Paper No. 108963, Pages 66.

[2] Moon-Jin Kang, Alexis F. Vasseur, Yi Wang *Time-asymptotic stability of generic Riemann solutions for compressible Navier-Stokes-Fourier equations,* https://arxiv.org/abs/2306.05604.

[3] Teng Wang, Yi Wang *Nonlinear stability of planar viscous shock wave to three-dimensional compressible Navier-Stokes equations,* to appear in Journal of the European Mathematical Society. https://arxiv.org/abs/2204.09428.

[MS32] Large Data Solutions to 1-D Hyperbolic Systems, Ill-Posedness, and Convex Integration

Sam Krupa

Max Planck Institute – Leipzig

Wednesday, 10:15, 1B306

For hyperbolic systems of conservation laws in one space dimension endowed with a single convex entropy, it is an open question if it is possible to construct solutions via convex integration. Such solutions, if they exist, would be highly non-unique and exhibit little regularity. In particular, they would not have the strong traces necessary for the nonperturbative L^2 stability theory of Vasseur. Whether convex integration is possible is a question about large data, and the global geometric structure of genuine nonlinearity for the underlying PDE. In this talk, I will discuss recent work which shows the impossibility, for a large class of 2×2 systems, of doing convex integration via the use of T_4 configurations. Our work applies to every well-known 2×2 hyperbolic system of conservation laws which verifies what we call the *structural Liu entropy condition.*

This talk is based on joint work with Laslo Szekelihidi (Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany).

[MS32] Uniform Asymptotic Stability for ConvectionReaction-Diffusion Equations in the Inviscid Limit Towards Riemann Shocks

Paul Blochas

University of Texas at Austin

Thursday, 8:45, 1D340

This is a joint work with Professor Miguel Rodrigues (University of Rennes, Rennes, France). In this talk, I will present results obtained in [1,2] on the study of the stability in time of families $(\underline{U}_\epsilon)_{0<\epsilon<\epsilon_0}$ of traveling waves solutions to

$$\partial_t u + \partial_x(f(u)) = g(u) + \epsilon\partial_x^2 u \tag{37}$$

that approximate a given Riemann shock. The aim is to show some uniform asymptotic orbital stability result of these waves under some conditions that guarantee the asympotic orbital stability of the corresponding Riemann shock, as proved in a previous work of V. Duchêne and L. M. Rodrigues, as well as work of L. Garénaux and L. M. Rodrigues. We will present results in weighted and unweighted topologies. Even at the linear level, to ensure uniformity in ϵ, the decomposition of the Green function associated with the (fast-variable) linearization about $\underline{U}_\epsilon$ of the above equation into a decreasing part and a phase modulation is carried out in a highly non-standard way. Furthermore, we introduce a multi-scale norm depending in ϵ that is the usual $W^{1,\infty}$ norm when restricted to functions supported away from the shock location. To avoid the use of arguments based on parabolic regularization that would preclude a result uniform in ϵ, we close nonlinear estimates on this norm through some suitable maximum principle.

[1] P. Blochas and L. M. Rodrigues. *Uniform asymptotic stability for convection-reaction-diffusion equations in the inviscid limit towards Riemann shocks.* Ann. Inst. H. Poincaré C Anal. Non Linéaire, to appear.

[2] P. Blochas. *Uniform convective stability for advection-reaction-diffusion equations in the inviscid limit.* Work in progress.

[MS32] Transverse linear asymptotic stability of one-dimensional solitary gravity water waves

Changzhen Sun

University of Franche-Comte

Thursday, 9:15, 1D340

In this presentation, we are concerned with the transverse linear asymptotic stability of one-dimensional small-amplitude solitary waves of the gravity water-waves system. More precisely, we show that the semigroup of the linearized operator about the solitary wave decays exponentially within a spectral subspace supplementary to the space generated by the spectral projection on continuous resonant modes. The key element of the proof is to establish suitable uniform resolvent estimates. To achieve this, we use different arguments depending on the size of the transverse frequencies. For high transverse frequencies, we use reductions based on pseudodifferential calculus, for intermediate ones, we use an energy-based approach relying on the design of various

appropriate energy functionals for different regimes of longitudinal frequencies and for low frequencies, we use the KP-II approximation. As a corollary of our main result, we also get the spectral stability in the unweighted energy space.
This is a joint work with Frédéric Rousset (University of Paris-Sacaly).

[MS32] Multidimensional stability and transverse bifurcation of hydraulic shocks and roll waves in open channel flow

Zhao Yang

Academy of Mathematics and Systems Science, Chinese Academy of Sciences

Thursday, 9:45, 1D340

This is a joint work with Prof Kevin Zumbrun (Indiana University Bloomington, USA).
We study by a combination of analytical and numerical methods multidimensional stability and transverse bifurcation of planar hydraulic shock and roll wave solutions of the inviscid Saint Venant equations for inclined shallow-water flow, both in the whole space and in a channel of finite width, obtaining complete stability diagrams across the full parameter range of existence. Technical advances include development of efficient multi-d Evans solvers, low- and high-frequency asymptotics, explicit/semi-explicit computation of stability boundaries, and rigorous treatment of channel flow with wall-type physical boundary. Notable behavioral phenomena are a novel essential transverse bifurcation of hydraulic shocks to invading planar periodic roll-wave or doubly-transverse periodic herringbone patterns, with associated metastable behavior driven by mixed roll- and herringbone-type waves initiating from localized perturbation of an unstable constant state; and Floquet-type transverse "flapping" bifurcation of roll wave patterns.

[1] Z. Yang and K.Zumbrun, *Multidimensional stability and transverse bifurcation of hydraulic shocks and roll waves in open channel flow*, arXiv:2309.08870, 2023.

[MS32] Stability of discrete shock profiles for systems of conservation laws

Lucas Coeuret

Institut de Mathématiques de Toulouse

Thursday, 10:15, 1D340

This talk deals with the stability analysis of discrete shock profiles for systems of conservation laws. These profiles correspond to approximations of shocks of systems of conservation laws by conservative finite difference schemes. Such discontinuous solutions appear naturally in the study of systems of conservation laws, which can model many physical situations, such as gas dynamics. Existence and stability of discrete shock profiles for each stable shock of the approximated system of conservation laws is seen as an improved consistency condition and implies that the finite difference scheme should approach discontinuities fairly precisely.

The aim of the talk will be to review some stability results regarding discrete shock profiles and to present a recent effort to extend them. More precisely, most results known up until recently are focused on the stability of discrete shock profiles associated with shocks of small amplitude. The talk will focus on a nonlinear stability result for discrete shock profiles in quite a general setting, where the smallness assumption on the shock's amplitude is replaced by a spectral stability assumption on the linear operator obtained by linearizing the numerical scheme about the discrete shock profile. This nonlinear stability result relies on a precise description of the Green's function of the linearization about discrete profiles presented in [1].

[1] L. Coeuret, *Linear stability of discrete shock profiles for systems of conservation laws*, ArXiV: https://arxiv.org/abs/2311.02507 (2023)

[MS33] Using persistent homology to analyze access to resources with heterogenous quality

Sarah Tymochko

University of California, Los Angeles

Monday, 13:45, 1B309

This is a joint work with Gillian Grindstaff (University of Oxford), Abigail Hickok (Columbia University), Jiajie Luo (University of California, Los Angeles), and Mason Porter (University of California, Los Angeles).

Ideally, all public resources (e.g. parks, grocery stores, hospitals, etc.) should be distributed in a way that is fair and equitable to everyone. However, this is not always the case. Quantifying how much (or little) access individuals have to certain resources is a complex problem. Previous work has shown that tools from topological data analysis (TDA) can be useful in determining "holes" in the locations of resource locations based on geographic locations and travel times [1]. Some resources may necessitate incorporation a notion of quality. As a case study, we look at public parks, which are heterogenous in many ways. Having access to a park that is hundreds of acres with basketball courts, baseball diamonds, and an aquarium is inherently different than having access to a small patch of grass with an overgrown tennis court. Here we present an exploration of the access to public parks in Chicago using persistent homology, a tool from TDA.

[1] A. Hickok, B. Jarman, M. Johnson, J. Luo, and M.A. Porter. Persistent Homology for Resource Coverage: A Case Study of Access to Polling Sites. arXiv:2206.04834, 2022. http://arxiv.org/abs/2206.04834

[MS33] Topology of Spatio-Temporal Trajectories

Heather Harrington

Max Planck Institute of Molecular Cell Biology and Genetics

Monday, 14:15, 1B309

Many processes in the life sciences are inherently multi-scale and dynamic. Spatial structures and patterns vary across levels of organisation, from molecular to multi-cellular to multi-organism. With more sophisticated mechanistic models and data available, quantitative tools are needed to study their evolution in space and time. Topological data analysis (TDA) provides a multi-scale summary of data. We review single and multi-parameter persistent homology. Recent work by Kim and Memoli proposed a filtration for the case of dynamic metric spaces, requiring three parameter persistence. In-progress work by Lesnick, Bender and Gäfvert combines Gröbner bases algorithms to compute minimal presentations of multiparameter persistence. Here we build on this work and present an algorithm, GBlandscapes, which computes 3-parameter persistent homology landscapes. We highlight the utility of two and three parameter persistence landscapes with concrete case studies arising in biological systems.

[MS33] Persistent homology for attractor quantification and description

Eugene Tan

The University of Western Australia

Monday, 15:15, 1B309

This is work done in conjunction with the Complex Systems Group, UWA.

Phase space attractors are mathematical structures that contain useful information regarding the dynamics and invariant properties of dynamical systems. This can be useful in the field of time series analysis for performing a variety of tasks such as model assessment and selecting embedding parameters. The geometric nature of phase space attractors also lend themselves well to the application of topological data analysis (TDA) methods. This has led to an increased interest in the development of TDA-inspired methods for nonlinear time series analysis applications. In this talk, I will briefly cover some ways in which persistent homology can be employed in time series analysis. To supplement this, this talk will also include some recent persistent homology methods that have been developed for studying strange attractors and their applications for nonlinear time series analysis. Specifically, I will discuss a recently proposed statistic, conformance, based on persistent homology ideas that aims to quantify the goodness of fit between the topology of attractors. There will also be discussion on how persistent homology can be used to assess the quality of embeddings and inform the selection of delay embedding parameters when performing nonlinear time series analyses.

[MS33] Integrating topological data analysis with the equation-free method for studying the dynamics of complex random networks

Konstantinos Spiliotis

Democritus University of Thrace

Monday, 15:45, 1B309

The presentation is based on joint work with Nikos Kavallaris and Ole Sönnerborn at the

Department of Mathematics and Computer Science, Karlstad University, and Haralampos Hatzikirou at the Department of Mathematics, Khalifa University.

Topological methods provide new insights into analyzing dynamical systems, including characterizing phase transitions and detecting criticalities [1, 2]. In this direction, we propose a method for analysing dynamics of complex random networks based on a topological data analysis (TDA) approach [3] and equation-free method (EFM) [4]. Precisely, using a TDA filtration connected to the activated node density, we determine the minimum radius required for the first persistent Betti number to emerge. Employing a simulated annealing optimisation, we develop a *lifting* procedure which transforms the minimum radius to a consistent microscopic network realisation. Going beyond the standard analysis, within the EFM framework, we characterize macroscopic network dynamics based on the minimum radius. This also enables us to conduct numerical bifurcation and stability analysis of these dynamics [5].

[1] İ. Güzel, E. Munch, F. A. Khasawneh *Detecting bifurcations in dynamical systems with CROCKER plots.* Chaos **32**(9) (2022) 093111.

[2] C.M Topaz, L. Ziegelmeier, T. Halverson, *Topological Data Analysis of Biological Aggregation Models.* PLOS ONE **10**(5) (2015), e0126383.

[3] G. Carlsson, *Topology and data* Bull. Amer. Math. Soc.**46** (2009), 255-308.

[4] I. Kevrekidis, G. Samaey, *Equation-free multiscale computation: Algorithms and applications* Annual Review of Physical Chemistry **68** (2009), 321-344.

[5] K. Spiliotis, O. Sönnerbornb, N. I. Kavallaris, H. Hatzikirou, *Exploring complex network dynamics through integrating topological data analysis and the equation-free method.* work in progress.

[MS33] Topological techniques for classification of agent-based tumour-immune model

Gillian Grindstaff

University of Oxford

Monday, 16:15, 1B309

We address the problem of classifying time series of synthetic 2-d spatial data from an agent-based model of tumour growth that includes tumour cells, macrophages, and blood vessels. We implement and compare the predictive power of four topological vectorizations specialized to such cell data: persistence images of Vietoris-Rips and radial filtrations at static time points, and persistence images for zigzag filtrations and persistence vineyards varying in time.

[MS33] Why Topological Data Analysis Detects Financial Bubbles

Marian Gidea

Yeshiva University

Monday, 16:45, 1B309

This is a joint work with Samuel W. Akingbade (Yeshiva University), Matteo Manzi (CrunchDAO), and Vahid Nateghi (Politecnico di Milano).

This works is concerned with applications of Topological Data Analysis (TDA) to critical transitions in financial time series, particularly market crashes and financial bubbles. A positive bubble represents a rapid increase of the price of an asset – which does not reflect the asset's intrinsic value – followed by a crash. A negative bubble represents a rapid decay followed by a quick rebound. In this context, the asset price peak in a positive bubble prior to a crash, and the trough in a negative bubble prior to a rebound, represent tipping points of the system. There is a practical interest in the detection of early warning signals of critical transitions, which would allow minimizing the losses from a bubble crash, or maximize the profit from a rebound.

One common method for employing TDA with financial time series involves utilizing time-delay coordinate embedding. These are used to construct, from the time-series, a point-cloud in some high-dimensional Euclidean space. Then the dynamics on the point-cloud is used to detect tipping points. For example, this approach has been used to study the 2000 dot-com bubble, the 2008 subprime mortgage crisis, and the 2018 cryptocurrency crash. However, most of the evidence on the ability of TDA to detect financial bubbles has been empirical. It is still an open problem to understand which features of a time series yield significant changes in the shape of the associated point cloud.

In this paper, we propose a heuristic argument based on the Log-Periodic Power Law Singularity (LPPLS) model. This model characterizes financial bubbles as super-exponential growth (or decay) of an asset price superimposed with oscillations increasing in frequency and decreasing in amplitude when approaching a critical transition (tipping point). From a topological view-point, the time-delay embedding of the LPPLS oscillatory signal yields loops (surrounding holes) in the point-clouds. Changes in the frequency and/or amplitude of the oscillations yield changes in the structure of the holes in the point-cloud, which can be measured via persistent homology. When approaching the tipping point of a bubble, there is a significant change in the features of the oscillatory signal and consequently in the TDA output.

The upshot of our work is that whenever the LPPLS model for the detection of a positive (negative) bubble applies to some data set, the TDA method on that data set provides early warning signals of critical transition. Moreover, even when the LPPLS model fits poorly with some of the data, the TDA method can show relatively strong signal prior to the tipping point. While the TDA method is robust, the LPPLS appears to be sensitive to noise.

As an illustration, we apply both the LPPLS model and TDA to real data (Bitcoin price).

Akingbade, S. W., Gidea, M., Manzi, M., & Nateghi, V. (2024). Why topological data analysis detects financial bubbles?. Communications in Nonlinear Science and Numerical Simulation, 128, 107665.

[MS33] Topology and Geometry of Random Cubical Complexes

Erika Roldan

Max Planck Institute for Mathematics in the Sciences

Monday, 17:15, 1B309

In this talk, we explore the expected topology (measured via homology) and local geometry of two different models of random subcomplexes of the regular cubical grid: percolation clusters, and the Eden Cell Growth model. We will also compare the expected topology that these average structures exhibit with the topology of the extremal structures that it is possible to obtain in the entire set of these cubical complexes. You can look at some of these random structures here (https://skfb.ly/6VINC) and start making some guesses about their topological behavior.

[MS33] Wavelet-Based Density Estimation for Persistent Homology

Konstantin Häberle
ETH Zurich

Monday, 17:45, 1B309

This is a joint work with Barbara Bravi and Anthea Monod (Imperial College London, UK).

Persistent homology is a central methodology in topological data analysis that has been successfully implemented in many fields and is becoming increasingly popular and relevant. The output of persistent homology is a persistence diagram—a multiset of points supported on the upper half plane—that is often used as a statistical summary of the topological features of data. In this talk, we discuss the random nature of persistent homology and estimate the density of expected persistence diagrams from observations using wavelets; we show that our wavelet-based estimator is optimal. Furthermore, we propose an estimator that offers a sparse representation of the expected persistence diagram that achieves near-optimality. We demonstrate the utility of our contributions in a machine learning task in the context of dynamical systems.

[MS34] The time evolution of electromagnetic knots

Benjamin Bode
Universidad Politécnica de Madrid

Monday, 13:45, 1B364

A knot is a simple closed loop in 3-dimensional space. A link is a collection of several knots, possibly intertwined in a non-trivial way. Two links are considered equivalent if they are ambient isotopic, that is, if one can be smoothly deformed into the other without passing through itself.

In earlier work [1] I showed that for every link L in $\mathbb{R}^3$ there exists an electromagnetic field that satisfies Maxwell's equations in vacuum and that for all time has a set of closed (electric and/or magnetic) field lines that is ambient isotopic to L. The time evolution of the field lines is a smooth isotopy.

However, the proof of the existence of such electromagnetic knots and links is not constructive, meaning that explicit examples of such fields are hard to come by and the precise time evolution of the knots remains largely mysterious. In this talk I will present progress on constructions of electromagnetic fields with stable knotted field lines and new results on the time evolution of the resulting knots. We show for example that no bounded region can contain an electromagnetic (Bateman) knot for all time.

[1] B. Bode, *Stable knots and links in electromagnetic fields* Comm. Math. Phys. **387** (2021), 1757–1770.

[MS34] Geometry of Plasma Knots

Oliver Gross

TU Berlin

Monday, 14:15, 1B364

The magnetic relaxation problem studies the self-organization of magnetic field lines in a perfectly conducting fluid to a steady state [9]. We propose a novel interpretation of such a relaxation process in terms of conformal geometry. A key insight is the conformal equivalence between force-free magnetic fields, *i.e.*, divergence-free vector fields which satisfy $(\operatorname{curl} B) \times B = 0$ and so-called *geodesible* vector fields [1,2,8]. It states that the field lines of are geodesics with respect to a conformally changed metric [1].

Inspired by Faraday's idea of electromagnetic fields as "lines of force," we will discuss a computational method for magnetic relaxation which is driven only by geometry optimization. It is based on variational principles and a discrete differential geometric model for pressure confined regions of ideal plasma with free boundary conditions, which is represented in terms of a collection of curves with thickness which interact with one another [5]. This novel approach allows us to describe the dynamics of the field lines, which are governed by the laws of electromagnetism, in terms of mechanical interactions. The resulting algorithm can be used to study the energy of plasma knots and links, for which it is geometrically less rigid than previous state of the art methods [3,4,6,7].

This is joint work with Albert Chern (UC San Diego, CA, USA), Ulrich Pinkall (TU Berlin, Germany) and Peter Schröder (Caltech, Pasadena, CA USA).

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[2] H. Gluck. *Dynamical behavior of geodesic fields.* In: Global Theory of Dynamical Systems. Ed. by Z. Nitecki and C. Robinson. Vol. 819. Lecture Notes in Mathematics. Springer, 2006, pp. 190–215.

[3] O. Gross, U. Pinkall, and P. Schröder. *Plasma Knots.* In: Phys. Lett. A **480** (2023), p. 128986.

[4] H. K. Moffatt. *The energy spectrum of knots and links.* In: Nature **347.6291** (1990), pp. 367–369.

[5] M. Padilla, O. Gross, F. Knöppel, A. Chern, U. Pinkall, and P. Schröder. *Filament Based Plasma.* In: ACM Trans. Graph. **41.4** (2022), 153:1–153:14.

[6] R. L. Ricca and F. Maggioni. *On the groundstate energy spectrum of magnetic knots and links.* In: J. Phys. A: Math. Theor. **47.20** (2014), p. 205501.

[7] C. B. Smiet, S. Candelaresi, and D. Bouwmeester. *Ideal relaxation of the Hopf fibration.* In: Phys. Plasmas. **24.7** (2017), p. 072110.

[8] D. Sullivan. *A foliation of geodesics is characterized by having no "tangent homologies".* In: J. Pure Appl. Algebra **13.1** (1978), pp. 101–104.

[9] A. R. Yeates. *Magnetohydrodynamic Relaxation Theory.* In: Topics in Magnetohydrodynamic Topology, Reconnection and Stability Theory. Ed. by D. MacTaggart and A. Hillier. Springer, 2020, pp. 117–143

[MS34] Measures of topological structure in non-symmetric volumes

Mitchell Berger

University of Exeter

Tuesday, 15:15, 1D341

In many circumstances we need continuous measures of topological structure. For example, one might look at the winding number between two molecular chains rather than linking number. Here we examine curves confined to a sub-volume of space bounded by a surface which may not be symmetric. Measures such as winding, twist, and writhe will be shown to depend on the shape of the bounding surface, including the distribution of curvature via the Gauss-Bonnet theorem. An application to solar wind simulations will be discussed.

[MS34] Particle-relabeling as variational and divergence symmetries for 3D ideal fluid dynamics and magnetohydrodynamics

Yasuhide Fukumoto

Kyushu University

Tuesday, 15:45, 1D341

This is a joint work with Dr. Rong Zou (Hawaii Pacific University, Honolulu, HI 96813, USA).

The helicity is a topological invariant of ideal Euler flows in three dimensions. This is no longer the case if the baroclinic effect and, for a conducting fluid, the Lorentz force are called into play. By appealing to Noether's theorem, we show that the cross-helicity is the integral invariant associated with the particle-relabeling symmetry of the action for ideal magnetohydrodynamics [1]. Employing the Lagrange label function, as the independent variable in the variational framework, facilitates implementation of the relabeling transformation [2]. The Casimirs in the form of integrals including Lagrangian invariants like the Ertel invariant are found to be variants of the cross-helicity [3]. By incorporating the divergence symmetry, other known topological invariants are put on the same ground of Noether's theorem [1].

[1] Y. Fukumoto, R. Zou, *Nambu bracket for 3D ideal fluid dynamics and magnetohydrodynamics.* Prog. Theor. Exp. Phys. **2024** (2024), ptae025.

[2] Y. Fukumoto, *A unified view of topological invariants of fluid flows.* Topologica **1** (2008), 003.

[3] Y. Fukumoto, H. Sakuma, *A unified view of topological invariants of barotropic and baroclinic fluids and their application to formal stability analysis of three-dimensional ideal gas flows.* Procedia IUTAM **7** (2013), 213-222.

[MS34] Topological flow data analysis and its application to vortex flows inside a heart

Takashi Sakajo

Kyoto University

Tuesday, 16:15, 1D341

We have developed a mathematical theory classifying topological structures of particle orbits generated by two-dimensional vector fields [1]. Based on the theory, we propose a method of topological data analysis for flow patterns, called Topological Flow Data Analysis (TFDA). By TFDA, the topological structure of a given flow pattern is uniquely converted into a labeled plane tree and its associated string expression. Each character of the string expression and each node's label of the tree describes a local topological flow structure that exists in the flow pattern. Through the trees and the symbolic expressions, we can extract qualitative and/or quantitative information from flow data obtained from numerical simulations and laboratory experiments. In addition, the continuous long-time evolution of flow patterns is reduced to a discrete dynamical system among topologically equivalent flow patterns. In the present talk, we introduce the TFDA theory and its application to flows in the heart's left ventricle [2]. Complex vortex patterns of blood flow in the heart play an important role in an efficient blood flow supply from the heart to the organs. Recent progress in medical imaging and computational technology such as echocardiography and cardiac MRI has recently yielded blood flow visualization tools. Nevertheless, there are still few mathematical theories to clearly define the vortex flow structures such as size and location, or change over time in the main chamber in the heart, since the evolution of vortex structures in blood flow is highly unstable and complex. With our TFDA theory, we can identify well-organized vortex flow structures as topological vortex structures. This provides a new topological method, tracking vortex dynamics from the flow patterns in healthy and diseased hearts.

[1] T. Sakajo and T. Yokoyama, *Discrete representations of orbit structures of flows for topological data analysis*, Discrete Mathematics, Algorithms and Applications (2022), 2250143. [arXiv:2010.13434]

[2] T. Sakajo and K. Itatani, *Topological identification of vortical flow structures in the left ventricle of the heart*, SIAM J. Imag. Sci. **16** No.3 (2023), 1491–1519. Open Access.

[MS34] Topology of configuration space for a particle in a lattice

Jean-Luc Thiffeault

University of Wisconsin - Madison

Tuesday, 16:45, 1D341

This is a joint work with Sophia Wiedmann and Sanchita Chakraborty.

The transport of particles immersed in fluids has been a longstanding subject of study, going back to Einstein and many others. Recent applications involve active matter – particles that have internal degrees of freedom. Many examples of active matter come from biological systems, where swimming organisms navigate a complex environment.

A simple model of a complex environment is a two-dimensional lattice of point obstacles. A particle of arbitrary shape interacts with the obstacles and is thus constrained in its motion. A necessary first step in modeling such a system using PDEs is a characterization of the *configuration space* for a particle – the range of allowable values for its positional and rotational degrees of freedom.

As the shape of the particle is varied, its configuration space undergoes dramatic bifurcations in connectivity. For instance, a particle may need to undergo an intricate 'parallel parking' motion to be able to turn around. In this work we investigate the topology of the space by a combination of numerical and analytical approaches. We use computational geometry software to calculate and visualize the configuration space, and classify some bifurcations for simple shapes, such as elliptical particles.

[MS34] Construction, evolution, and helicity dynamics of knotted vortex tubes

Yue Yang

Peking University

Wednesday, 15:15, 1A305

We develop a general method for constructing knotted vortex tubes with the finite thickness, arbitrary shape, and tunable twist [1]. The central axis of the knotted tubes is determined by a given smooth parametric equation. The helicity of the knotted tubes can be explicitly decomposed into the writhe and twist. We construct a series of complex vortex tubes with various geometry and topology – from knotted vortex knots [2] to turbulent fields consisting of intertwined vortex tubes [3]. We investigate their helicity dynamics in the viscous evolution using direct numerical simulation. The results reveal independent evolution routes of the vortex knots with their helicity conversion mechanisms, including merging, reconnection, bursting, and transition [2, 4].

[1] S. Xiong, Y. Yang, Effects of twist on the evolution of knotted magnetic flux tubes, J. Fluid Mech. 895 A28, 2020.

[2] W. Shen, J. Yao, F. Hussain, Y. Yang, Topological transition and helicity conversion of vortex knots and links, J. Fluid Mech. 943 A41, 2022.

[3] W. Shen, J. Yao, Y. Yang, Weaving classical turbulence with quantum skeleton, arXiv:2401.11149, 2024.

[4] W. Shen, J. Yao, F. Hussain, Y. Yang, Role of internal structures within a vortex in helicity dynamics, J. Fluid Mech. 970 A26, 2023.

[MS34] Vortex cascades as geodesics in knot polynomial space

Renzo Ricca

University of Milano-Bicocca

Wednesday, 15:45, 1A305

This is a joint work with Xin Liu (Beijing University of Technology, P.R. China) and Xin-Fei Li (Guangxi U. Science & Technology, P.R. China).

It is well known that during evolution topologically complex tangles of superfluid vortices tend to decay through a topological cascade to form a system of unlinked vortex rings [1, 2]. Here we propose a geometric interpretation of this topological cascade in terms of geodesic flows in a knot polynomial space [3]. By taking advantage of the derivation of the Jones polynomial to its adapted version in terms of helicity contributions [4] we introduce a discrete metric space, whose points represent different knot types, and show that optimal unlinking paths can describe many features of the observed vortex cascade.

[1] C.F. Barenghi, R.L. Ricca, D.C. Samuels, *How tangled is a tangle?*, Physica D **157** (2001), 197–206.

[2] D. Kleckner, L.H. Kauffman, W.T.M. Irvine, *How superfluid vortex knots untie*, Nature Physics **12** (2016), 650–655.

[3] X. Liu, R.L. Ricca, X.-F. Li, *Minimal unlinking pathways as geodesics in knot polynomial space*, Nature Comm. Physics **3** (2020), 136.

[4] X. Liu, R.L. Ricca, *The Jones polynomial for fluid knots from helicity*, J. Phys. A: Math. & Theor. **45** (2012), 205501.

[MS34] On matrix hydrodynamics and the long-time behavior of 2-D fluids

Klas Modin

Chalmers and University of Gothenburg

Wednesday, 16:15, 1A305

This is a joint work with Milo Viviani (Scuola Normale Superiore, Pisa, Italy).

In 1991, Zeitlin showed how quantization theory naturally leads to a spatial discretization of the 2-D incompressible Euler equations. This finite dimensional model takes the form of an isospectral flow of matrices. It is the only known discretization that preserves the *Lie–Poisson structure*, i.e., the Hamiltonian structure associated with geodesic flows on Lie groups equipped with a right invariant Riemannian metric—the 1966 discovery of Arnold.

Despite over three decades in presence, Zeitlin's model is surprisingly underexplored. In the last couple of years, however, this beautiful approach has seen something of a renaissance, with several new developments, and applications to geophysical flows (quasi-geostrophic equations) as well as incompressible 2-D magnetohydrodynamics. In this talk, I wish to present some insights gained during these developments, particularly those related to the long-time behavior of 2-D Euler—the long-standing problem described as the "jewel of 2-D turbulence".

[MS35] On evolution of vortex filament

Francisco Gancedo

University of Seville

Monday, 15:15, 1D328

In this talk we show two new results of vortex filament evolution for incompressible Navier-Stokes and Euler equations. For Navier-Stokes, we prove global-in-time regularity for initial helical vortex filament. For Euler, we give existence of weak dissipative solutions with initial vorticity concentrated in a circle.

[MS35] Traveling wave solutions to the free boundary incompressible Navier-Stokes equations

Junichi Koganemaru
Carnegie Mellon University
Monday, 15:45, 1D328

Consider a single layer of viscous fluid in a horizontally infinite strip-like domain that is bounded below by a flat rigid surface and above by a moving free surface, subject to the effects of gravity and surface tension. Additionally, the fluid is acted upon by a bulk force and a surface stress that are both stationary in a coordinate system moving parallel to the fluid bottom. In this talk we show that traveling wave solutions can be constructed for this model under a variety of physical considerations, including periodization and inclination of the fluid domain, and incorporating more general Navier-slip type boundary conditions at the rigid bottom. An essential component of our analysis involves the study of a new scale of anisotropic Sobolev spaces. This is joint work with Ian Tice.

[MS35] Approximation of Classical Two-Phase Flows by a Navier-Stokes/Allen-Cahn System

Maximilian Moser
IST Austria
Monday, 16:15, 1D328

This is a joint work with Prof Abels (University of Regensburg, Germany) and Prof Fischer (ISTA, Klosterneuburg, Austria).

In this talk we consider the sharp interface limit of a Navier-Stokes/Allen Cahn system in a bounded smooth domain in two or three space dimensions, for the case of vanishing mobility $m_\varepsilon = m_0\varepsilon^\alpha$, where $m_0 > 0$ and $\alpha \in (0, 2)$, when the small parameter $\varepsilon > 0$ related to the thickness of the diffuse interface is sent to zero. The limit problem is given by the classical two-phase Navier-Stokes system with surface tension and we show convergence for well-prepared initial data and for small times such that a strong solution to the limit problem exists. The approach is via the relative entropy method, i.e. one shows Gronwall-type estimates for suitable energy functionals that control the error between the solution to the diffuse and sharp interface systems in a suitable way.

In Hensel, Liu [2] the same method was applied for constant mobility, i.e. $\alpha = 0$. In this case the limit problem is given by a two-phase Navier-Stokes system coupled to mean curvature flow. We use similar error functionals, but in a first step we compare to the solution of a perturbed version of the two-phase flow problem, where in the equation for the normal velocity the curvature term $m_\varepsilon H$ is added. In contrast to [2] we have to deal with problematic remainder terms in the Gronwall estimates which we estimate in a novel way.

[1] H. Abels, J. Fischer and M. Moser, *Approximation of Classical Two-Phase Flows of Viscous Incompressible Fluids by a Navier-Stokes/Allen-Cahn System.* Preprint arXiv:2311.02997 (2023).

[2] S. Hensel and Y. Liu, *The Sharp Interface Limit of a Navier–Stokes/Allen–Cahn System with Constant Mobility: Convergence Rates by a Relative Energy Approach.* SIAM J. Math. Anal. **55**(5) (2023), 4751–4787.

[MS35] Well-posedness of the stationary and slowly traveling wave problems for the free boundary incompressible Navier-Stokes equations

Noah Stevenson

Princeton University

Monday, 16:45, 1D328

This is a joint work with Professor Ian Tice of Carnegie Mellon University.

We establish that solitary stationary waves in three dimensional viscous incompressible fluids are a general phenomenon and that every such solution is a vanishing wave-speed limit along a one parameter family of traveling waves. The setting of our result is a horizontally-infinite fluid of finite depth with a flat, rigid bottom and a free boundary top. A constant gravitational field acts normal to bottom, and the free boundary experiences surface tension. In addition to these gravity-capillary effects, we allow for applied stress tensors to act on the free surface region and applied forces to act in the bulk. These are posited to be in either stationary or traveling form. In the absence of any applied stress or force, the system reverts to a quiescent equilibrium;

in contrast, when such sources of stress or force are present, stationary or traveling waves are generated. We develop a small data well-posedness theory for this problem by proving that there exists a neighborhood of the origin in stress, force, and wave speed data-space in which we obtain the existence and uniqueness of stationary and traveling wave solutions that depend continuously on the stress-force data, wave speed, and other physical parameters. To the best of our knowledge, this is the first proof of well-posedness of the solitary stationary wave problem and the first continuous embedding of the stationary wave problem into the traveling wave problem. Our techniques are based on vector-valued harmonic analysis, a novel method of indirect symbol calculus, and the implicit function theorem.

[MS35] Asymmetric capillary–gravity water waves in the steady periodic setting

Mats Ehrnström

Norwegian University of Science and Technology

Wednesday, 8:45, 1D226

We discuss ongoing work on asymmetric capillary–gravity surface waves in the Euler equations. It has been known for a long time that the setting of weak surface tension allows for higher-dimensional bifurcation from still water, giving rise to multimodal waves with more than one crest in a period. These waves have, however, all been symmetric, although numerical calculations indicate the presence of truly asymmetric waves in the steady periodic setting. Recently, Mæhlen and Svensson Seth extended earlier bifurcation results for the gravity–capillary Whitham equation, showing that asymmetric solutions exist as natural extensions of bimodal waves. In this work, which is joint with Douglas Svensson Seth (NTNU) and Boris Buffoni (EPFL), we investigate the existence of such asymmetric solutions in the Euler equations.

[MS35] Three-dimensional gravity-capillary solitary waves on Beltrami flows

Mark Groves

Universität des Saarlandes

Wednesday, 9:15, 1D226

In this talk I consider steady gravity-capillary surface waves ‘riding’ a perfect fluid in Beltrami flow (a three-dimensional flow with parallel velocity and vorticity fields). I will first demonstrate how the hydrodynamic problem can be formulated as two equations for two scalar functions of the horizontal spatial coordinates, namely the elevation of the free surface and the potential defining the gradient part (in the sense of the Hodge–Weyl decomposition) of the horizontal component of the tangential fluid velocity there. The formulation is nonlocal, has a variational structure and generalises the Zakharov–Craig–Sulem formulation for the classical water-wave problem, reducing to it in the irrotational limit.

Starting from the above formulation, one can derive the Kadomtsev-Petviashvili-I (KP-I) equation (strong surface tension) or the Davey-Stewartson (DS) system (weak surface tension) for such Beltrami flows using formal weakly nonlinear theory. These equations have 'lump' solutions and thus predict the existence of fully localised solitary water waves for the full problem. I will show how to rigorously reduce the full problem to a perturbation of the KP-I or DS equations and thus construct an existence proof for fully localised solitary waves 'riding' Beltrami flows.

[MS35] On steady waves with a large adverse vorticity

Evgeniy Lokharu

Lund University

Wednesday, 9:45, 1D226

This is a joint work with Dr. Miles Wheeler from Bath University, UK.

We will present several new results on two-dimensional water waves with a constant adverse vorticity, including various bounds for the velocity field, amplitude and the Froude number. As an application, we will show that extreme waves do not appear for a sufficiently large adverse vorticity and global bifurcation curves of unidirectional waves always lead to the formation of stagnation points at the bottom, right below the crest.

[MS35] Gravity wave-borne vortices

Samuel Walsh

University of Missouri

Wednesday, 10:15, 1D226

In this talk, we'll present some recent work on traveling waves in water that carry vortices in their bulk. We show that for any supercritical Froude number (non-dimensionalized wave speed), there exists a continuous one-parameter family of solitary waves with a submerged point vortex in equilibrium. This family bifurcates from an irrotational laminar flow, and, at least for large Froude numbers, it extends up to the development of a surface singularity. These are the first rigorously constructed gravity wave-borne point vortices without surface tension, and notably our formulation allows the free surface to be overhanging. Through a separate numerical study, we find strong evidence that many of the waves do indeed have an overturned air—water interfaces. Finally, we prove that generically one can perform a desingularization procedure to obtain a solitary wave with a submerged hollow vortex. Physically, these can be thought of as traveling waves carrying spinning bubbles of air in their bulk.

This is joint work with Ming Chen, Kristoffer Varholm, and Miles Wheeler.

Contributed Talks

[CT1] Generalizing dynamical properties for systems defined by ill-posed problems

Tomoharu Suda

Tokyo University of Science

Monday, 13:45, 1D222

A large portion of the study of the dynamics of an ordinary differential equation consists of considering properties such as invariance or invariant measures. However, in the application, not every 'continuous-time dynamics' originates from an ODE. Therefore, it is of interest to consider a generalization of the classical notions to the situation where the dynamics are derived from an 'ill-posed' problem, such as a differential inclusion. In this talk, we will introduce the axiomatic theory of ODE as proposed by J.A. Yorke [2] and show that, under suitable definitions, much of the classical concepts and results of dynamical systems theory can be generalized to a considerable extent within this framework. For example, we have a generalization of the Poincaré recurrence theorem [1]. We will also indicate some applications to the study of a parametrized family of ordinary differential equations.

[1] T. Suda, *Dynamical properties in the axiomatic theory of ordinary differential equations* J. Differential Equations **391** (2024), 1–24.

[2] J. A. Yorke, *Spaces of solutions,* Mathematical Systems Theory and Economics I/II (1969), 383–403.

[CT1] Generalization of Lyapunov center theorem for Hamiltonian systems

Anna Gołębiewska

Nicolaus Copernicus University in Toruń

Monday, 14:15, 1D222

The aim of the talk is to show the new results concerning the existence of branches of trajectories of nonstationary periodic solutions of autonomous Hamiltonian systems. We consider the system

$$\dot{x}(t) = JH'(x(t)),$$

where J is the standard symplectic matrix, $\Omega \subset \mathbb{R}^{2N}$ is an open set containing the origin and $H \in C^2(\Omega, \mathbb{R})$ satisfy $H'(0) = 0$. Our goal is to study the existence of connected sets of closed trajectories of solutions of such system, emanating from the origin.

This result is obtained with the use of the global bifurcation theory for autonomous Hamiltonian systems and the theory of normal forms for Hamiltonian matrices. It generalizes the Lyapunov center theorem as well as some other known results. In particular, our result applies not only in the nondegenerate and nonresonant context, but provides a global answer also in some degenerate cases.

Moreover, we are going to show the application of the abstract result to some problems from Celestial Mechanics. For Hamiltonian systems obtained in such problems we can prove the existence of branches of periodic orbits emanating from the stationary points.

[CT2] A variational perspective on auxetic metamaterials of checkerboard-type

Wolf-Patrick Düll

Stuttgart

Monday, 13:45, 1D226

This is a joint work with Dominik Engl (Katholische Universität Eichstätt-Ingolstadt, Eichstätt, Germany) and Carolin Kreisbeck (Katholische Universität Eichstätt-Ingolstadt, Eichstätt, Germany).

We present a homogenization theorem via variational convergence for elastic materials with stiff checkerboard-type heterogeneities under the assumptions of physical growth and non-self-interpenetration. While the obtained energy estimates are rather standard, determining the effective deformation behavior, or in other words, characterizing the weak Sobolev limits of deformation maps whose gradients are locally close to rotations on the stiff components, is the challenging part. To this end, we establish an asymptotic rigidity result, showing that, under suitable scaling assumptions, the attainable macroscopic deformations are affine conformal contractions. This identifies the composite as a mechanical metamaterial with a negative Poisson's ratio. Our proof strategy is to tackle first an idealized model with full rigidity on the stiff tiles to acquire insight into the mechanics of the model and then transfer the findings and methodology to the model with diverging elastic constants. The latter requires, in particular, a new quantitative geometric rigidity estimate for non-connected squares touching each other at their vertices and a tailored Poincaré type inequality for checkerboard structures.

[1] W.-P. Düll, D. Engl, C. Kreisbeck, *A variational perspective on auxetic metamaterials of checkerboard-type.* Arch. Ration. Mech. Anal., accepted 2024, arXiv:2303.16159.

[CT2] Structural stability of the Brinkman–Forchheimer equations for flow in porous media with variable porosity

Evangelos Petridis

Université catholique de Louvain

Monday, 14:15, 1D226

This is a joint work with Prof. Militadis Papalexandris (UCLouvain, Belgium).

In [1] Payne and Straughan established structural stability of the Brinkman–Forchheimer equations for porous media of constant porosity. The main difference with the constant-porosity case, besides the introduction of the porosity as a time-independent field variable, is that the term related to the shear viscous stresses does not reduce to a Laplacian but involves the full deviatoric deformation tensor. Further, there is an additional term due to normal viscous stresses. Herein we establish continuous dependence of solutions with respect to the physical parameters entering the equations, namely, the shear and bulk viscosities and the coefficients of the linear (Darcy) and quadratic (Forchheimer) terms for the interfacial drag. More specifically, we first prove that the shear viscous term is coercive and then establish continuous dependence in the weighted L^2-norm, with the porosity being the weight.

[1] L. E. Payne, B. Straughan *Convergence and Continuous Dependence for the Brinkman–Forchheimer Equations*. Stud. Appl. Math. **102** (1999), 419–439.

[CT3] Polynomial reconstruction for the discontinuous Galerkin method on curved domains

Milene Santos

University of Coimbra

Monday, 13:45, 1D236

The ever-increasing complexity of real-word applications has raised important challenges in the quest for accurate and efficient numerical methods for solving partial differential equations. In particular, we are interested in solving boundary value problems in curved boundary domains considering the Discontinuous Galerkin (DG) method. The question that arises concerns the reduction of the order of convergence of numerical methods when considering the approximation of the domain by a polygonal mesh. In this talk, we present a strategy called DG-ROD (Reconstruction for Off-site Data) method, which is based on a polynomial reconstruction of the boundary condition imposed on the computational domain [1]. The main advantages are simplicity, since the PDE solver only considers polygonal domains, and versatility, since any type of boundary condition can be imposed. The developed DG–ROD method consists in splitting the boundary conditions treatment and the leading discrete equations from a classical DG formulation into two independent solvers coupled in a simple and efficient iterative procedure. Numerical tests with Dirichlet and Neumann boundary conditions prescribed on curved boundaries show that the optimal convergence order is effectively achieved.

Co-authors: Adérito Araújo (1), Sílvia Barbeiro (1), Stéphane Clain (1), Ricardo Costa (2), Gaspar J. Machado (3).

(1) Centre for Mathematics, University of Coimbra, Portugal.

(2) Institute for Polymers and Composites, University of Minho, Azurém *Campus*, Guimarães, Portugal. Department of Polymer Engineering, University of Minho, Azurém *Campus*, Guimarães, Portugal.

(3) Physics Center of Minho and Porto Universities, University of Minho, Azurém *Campus*, Guimarães, Portugal. Department of Mathematics, University of Minho, Azurém *Campus*, Guimarães, Portugal.

[1] M. Santos, A. Araújo, S. Barbeiro, S. Clain, R. Costa, G.J. Machado. *Very high-order accurate discontinuous Galerkin method for curved boundaries with polygonal meshes*. Manuscript submitted for publication, 2023.

[CT3] On the solvability of some boundary value problems for the equation of oscillation string in rectangular domains

Otar Jokhadze

Tbilisi State University

Monday, 14:15, 1D236

This is a joint work with S. Kharibegashvili.

For the equation of oscillation string in a rectangular domain the problems of the Dirichlet and Robin type are considered. The correctness of the Robin problem is proved and in the case of the Dirichlet problem, necessary and sufficient conditions for its solvability are found.

[CT4] Material distribution topology optimization for boundary effect dominated problems

Eddie Wadbro

Karlstad University

Monday, 13:45, 1D237

To solve design optimization, the material distribution method (density-based topology optimization) uses a material indicator function that represents the presence or absence of material within the domain. To use this approach for boundary-effect-dominated problems, we need to identify the boundary of the design at each iteration; this talk discusses two methods to achieve this. The first is to use a boundary strip indicator function defined on the elements of the computations mesh. The second is to use a boundary indicator function defined on the mesh faces (edges in 2D and facets in 3D). The presentation covers the main ideas behind both approaches and showcases results from two applications, one suitable for each approach. More precisely, as a first test case, we use a face-based material indicator for the design of an acoustic resonator. This optimization targets a specific frequency range. The effectiveness of the design is evaluated based on the

absorption coefficient, which represents the proportion of acoustic power from incoming waves that is absorbed due to visco-thermal losses in the boundary layer. As a second test case, we utilize the element-based boundary indicator function to design coated structures. The objective is to minimize the thermal compliance of a coated structure. To achieve this, we introduce a coating layer to the conventional minimal heat compliance problem. The conductivity of the coating, as well as its cost (considered in a material usage constraint), are a constant times the conductivity and cost of the base material.
Acknowledgments: For the work on boundary indicators leading up to this presentation, I thank Martin Berggren (Umeå University, Sweden), Linus Hägg (Umeå University, Sweden), Abbas Mousavi (KTH Royal Institute of Technology, Sweden), Bin Niu (Dalian University of Technology, China), Khanh Nguyen (Karlstad University, Sweden), and Mario Setta (Karlstad University, Sweden) for our fruitful collaborations and joint work over the last years.

[CT4] Probability of early infection extinction depends linearly on the virus clearance rate

Nóra Juhász

University of Szeged

Monday, 14:15, 1D237

We provide an *in silico* study of stochastic infection extinction from a pharmacokinetical viewpoint. Our work considers a non-specific antiviral drug that increases the virus clearance rate, and we investigate the effect of this drug on early infection extinction. Infection extinction data is generated by a hybrid multiscale framework that applies both partial differential equations and discrete mathematical approaches. The central result of our paper is the observation, analysis and explanation of a linear relationship between the virus clearance rate and the probability of early infection extinction. The derivation behind this simple relationship is given by merging different mathematical toolboxes. *Joint work with Ferenc Bartha, Sadegh Marzban, Renji Han and Gergely Röst.*

[CT5] Stability Analysis of Abstract Thermoelastic Systems following Cattaneo's Law

Chenxi Deng

Delft University of Technology

Monday, 15:15, 1D340

This is a joint work with Zhong-Jie Han (Tianjin University, Tianjin, China), Zhaobin Kuang (Stanford University, Stanford, U.S.A.) and Qiong Zhang (Beijing Institute of Technology, Beijing, China)

We will discuss the stability of abstract thermoelastic systems where the heat conduction law follows Cattaneo's law. This law describes the finite speed of heat propagation in a medium, addressing the paradox of infinite speed of heat transfer according to Fourier's law.The study encompasses two cases: systems with and without inertial terms. For systems incorporating inertial terms, by analyzing the spectral properties of the system's generator, we partition the parameter space involving coupling, thermal dissipation, and the inertial term into four parts. We provide polynomial decay rates for each region, demonstrating that the decay rates for three of these regions are optimal. In the case without inertial terms, we divide the parameter space into three parts and present the optimal polynomial decay rates for each part.

[CT5] Analytic verification of stable periodic orbits: a study on Mackey-Glass type equations

Gábor Benedek

University of Szeged, Bolyai Institute

Monday, 15:45, 1D340

This is a joint work with T. Krisztin (University of Szeged, Szeged, Hungary) and R. Szczelina (Jagiellonian University, Kraków, Poland).

We study Mackey-Glass type equations of the form

$$y'(t) = -ay(t) + bf(y(t-1)) \tag{38}$$

with positive parameters a, b and a hump-shaped nonlinear $f : [0, \infty) \to [0, \infty)$: a class of nonlinear delay differential equations renowned for their rich dynamical behavior. For the nonlinearity f we assume that, in some sense, it is close to a discontinuous function $g : [0, \infty) \to [0, 1]$ satisfying $g(\xi) = 0$ for $\xi > 1$. The fact that $g(\xi) = 0$ for $\xi > 1$ enables the construction of a stable periodic orbit for the equation $x'(t) = -cx(t) + dg(x(t-1))$ for some constants d and c with $d > c > 0$. This is an important step toward to the proof that Equation (38) also exhibits a stable periodic orbit, provided that the parameters a, b, and the function f are close to c, d, and g, respectively. In this talk, we give an analytic proof for the existence of a stable periodic orbit for equation $x'(t) = -cx(t) + dg(x(t-1))$ with given $c > 0$ and a discontinuous g, provided $d > 0$ is sufficiently large.

The main example is $f(\xi) = \frac{\xi^k}{1+\xi^n}$, $k > 0$. For large n, function f is close to the function $g : [0, \infty) \to [0, 1]$, where $g(\xi) = 0$ for $\xi > 1$, and $g(\xi) = \xi^k$ for $\xi \in [0, 1)$.

[CT5] Travelling waves of a scalar conservation law perturbed with a saturating diffusion and linear dispersion

Gnord Maypaokha

Université de Picardie Jules Verne

Monday, 16:15, 1D340

We consider a scalar hyperbolic conservation law with convex flux perturbed by a generalised Rosenau-type diffusion [1] and linear dispersion,

$$u_t + f(u)_x = \epsilon\Big(\frac{u_x}{(1+u_x^2)^{\alpha}}\Big)_x + \delta u_{xxx} \tag{39}$$

where $\alpha \geq 0$, and ϵ, δ are positive parameters. The case of linear diffusion ($\alpha = 0$) called KdV Burgers' equation was treated by Schonbek [1]. In the general case $\alpha \geq 0$, the existence and certain qualitative properties of travelling wave solutions of (39) are established. We first examine the existence of regular travelling waves depending on α when $\delta = 0$, and the limiting behaviour of this case when ε tends to zero is proven. Then, we discuss the conditions for the existence of monotone traveling waves for (39) when $\epsilon, \delta > 0$, which hinge on the regimes $\alpha < 1/2$ and $\alpha \geq 1/2$ (saturating diffusion). Additionally, we present numerical simulations to validate the theoretical findings. Finally, we offer conjectures regarding the convergence (or not) of the general solutions to the entropy solution of the hyperbolic equation.

[1] A. Kurganov and P. Rosenau, *Effects of a saturating dissipation in Burgers-type equations.* Comm. Pure Appl. Math. **50** (1997), 753–771.

[2] M. E. Schonbek, *Convergence of solutions to nonlinear dispersive equations.* Comm. PDEs **7** (1982), 959–1000.

[CT5] Smooth linearization of contractive random dynamical systems

Iryna Vasylieva

University of Klagenfurt

Monday, 16:45, 1D340

This talk focuses on the linearization problem of random dynamical systems, which enables us to deduce local information about the dynamics of nonlinear systems from their linear parts.

Specifically, we consider dynamical systems generated by random semilinear Carathéodory differential equations and stochastic differential equations with a uniformly exponentially stable linear part, where the growth rates are not assumed to be constant. We present sufficient conditions for strong topological equivalence between the given systems and their linearizations in arbitrary Banach spaces. Additionally, we establish criteria for the conjugating map to be a C^k diffeomorphism between the aforementioned systems.

This research was funded by the Austrian Science Fund (FWF) [10.55776/DOC78].

[CT6] Groundwater flow through porous media layered over sloping bedrock

Lukáš Kotrla

University of West Bohemia

Monday, 15:15, 1D341

This is a joint work with Prof Petr Girg (University of West Bohemia, Pilsen, Czechia).

We study a new mathematical model of fluid flow through porous media layered over an inclined impermeable bedrock. While a classical approach is based on linear Darcy's law as a constitutive law, we use the nonlinear power law as was proposed in the case of horizontal flow by O. Smreker in 19$^{\text{th}}$ century. This choice of the constitutive law leads to the model based on p-Laplacian type parabolic PDE. As a result of the nonlinearity, it is not possible to separate diffusion and convection term as in the case of the classical approach.

We will analyse qualitative properties of the stationary solutions of our model in one dimension. In particular, we will study regularity of weak solutions, validity of weak and strong maximum principle for the following boundary value problem

$$\begin{aligned}
&-\frac{\mathrm{d}}{\mathrm{d}x}\left[(u(x)+H)\left|\frac{\mathrm{d}u}{\mathrm{d}x}(x)\cos(\varphi)+\sin(\varphi)\right|^{p-2}\left(\frac{\mathrm{d}u}{\mathrm{d}x}(x)\cos(\varphi)+\sin(\varphi)\right)\right]\\
&=f(x)\,, \qquad\qquad\qquad x\in(-1,1)\,,\\
&u(-1)=u(1)=0\,,
\end{aligned}$$

where $p > 2$, $H > 0$, $\varphi \in (0, \pi/2)$, $f \geq 0$, $f \in L^1(-1,1)$. We use methods based on the linearization of the p-Laplacian-type problems in the vicinity of known solution, error estimates, and analysis of the Green function of the linearized problem.

Acknowledgments: This work was supported by the Grant Agency of the Czech Republic, Grant No. 22-18261S.

[CT6] Flow of thermomicropolar fluid through a thin channel

Marko Radulovic

University of Zagreb, Faculty of Science, Department of Mathematics

Monday, 15:45, 1D341

This is a joint work with Professor G. Łukaszewicz (University of Warsaw, Poland), I. Pažanin (University of Zagreb, Croatia) and B. Rukavina (University of Zagreb, Croatia).

In this talk, we consider a steady–state flow of the thermomicropolar fluid through a thin straight channel. The flow is governed by the prescribed pressure drop between channel's ends. The heat exchange between the fluid inside the channel and the exterior medium is allowed through the upper wall, whereas the lower wall is insulated.

We will employ asymptotic analysis with respect to the thickness of the channel and derive a first–order asymptotic solution acknowledging the effects of the fluid's microstructure. A rigorous justification of the proposed effective model will be provided by proving the error estimates in suitable norms.

These results were published in [1] and [2].

[1] G. Łukaszewicz, I. Pažanin, M. Radulović, *Asymptotic analysis of the thermomicropolar fludi flow through a thin channel with cooling*, Appl. Anal. **101** (9) (2022), 3141–3169.

[2] I. Pažanin, M. Radulović, B. Rukavina, *Rigorous derivation of the asymptotic model describing a steady thermomicropolar fluid flow through a curvilinear channel*, Z. Angew. Math. Phys. **73**, 195 (2022), pp. 1–25.

[CT6] Global well-posedness of the one-dimensional hydrodynamic Gross-Pitaevskii equations without vacuum

Robert Wegner

Karlsruhe Institute of Technology

Monday, 16:15, 1D341

We consider the Gross-Pitaevskii equation

$$i\partial_t q + \partial_{xx} q - 2(|q|^2 - 1)q = 0\,, \tag{40}$$

where $q(t,x) : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{C}$ represents an unknown wave function, subject to the boundary condition at infinity $\lim_{|x|\to\infty} |q(t,x)| = 1$. It can be put into a hydrodynamic form via the Madelung transform

$$\mathcal{M}(q) = (\rho, v) = \Big(|q|^2, \mathrm{Im}\Big[\frac{\partial_x q}{q}\Big]\Big)\,. \tag{41}$$

We establish global-in-time well-posedness of the one-dimensional hydrodynamic Gross-Pitaevskii equations in the absence of vacuum in $(1 + H^s) \times H^{s-1}$ with $s \geq 1$. Our core result is a local bilipschitz equivalence of the relevant function spaces for $s > \frac{1}{2}$. This enables the transfer of results between the two equations. The stated result is then a consequence of the previous global well-posedness result for the Gross-Pitaevskii equation in [2, 3].

The obtained results can be found in [1].

[1] R. Wegner *Global-in-time well-posedness of the one-dimensional hydrodynamic Gross–Pitaevskii equations without vacuum.* Z. Angew. Math. Phys. **74** (2023), no.5, Paper No. 194, 29 pp.

[2] H. Koch, X. Liao, *Conserved quantities for the Gross-Pitaevskii equation.* Adv. Math. **377** (2021), Paper No. 107467, 83 pp.

[3] H. Koch, X. Liao, *Conserved quantities for the Gross-Pitaevskii equation: low regularity case.* Adv. Math. **420** (2023), Paper No. 108996, 61 pp.

[CT6] On a compressible fluid-structure interaction problem with slip boundary conditions

Yadong Liu

Nanjing Normal University

Monday, 16:45, 1D341

This is a joint work with Sourav Mitra (Indian Institute of Technology Indore, India) and Šárka Nečasová (Czech Academy of Sciences, Prague, Czech Republic).

In this talk, I will present a recent project concerning a system describing the compressible barotropic fluids interacting with (visco) elastic solid shell/plate. In particular, the elastic structure is part of the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, via a modified relative entropy method in time-dependent domains, we prove the weak-strong uniqueness property of weak solutions. Finally, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data.

[CT6] Continua of periodic solutions of autonomous Hamiltonian systems

Igor Białecki

Nicolaus Copernicus University

Monday, 17:15, 1D341

The aim of the talk is to show the application of the global bifurcation theorem proved by Dancer and Rybicki in article [1]. We use this theorem to study the existence of unbounded sets of periodic solutions of Hamiltonian systems, emanating from equilibria. We consider the autonomus Hamiltonian system of the form

$$\dot{x}(t) = JH'(x(t)),$$

describing the movement of Saturn rings. Therefore, the results we obtain can motivate the work of physicists and astronomers. Depending on the movement of Saturn and the objects in its ring, we can distinguish two cases: alternating and nonalternating.

Moreover, using elements of the invariant bifurcation theory, we will discuss how to search periodic solutions of Hamiltonian system with any period.

This is a joint work with prof. Sławomir Rybicki.

[1] Dancer, E.N.; Rybicki, S., *A note on periodic solutions of autonomous Hamiltonian systems emanating from degenerate stationary solutions.* Differential Integral Equations 12 (1999), no. 2, 147–160.

[CT7] Mathematical Modeling of Fluid Flows Through Pipes: Coherence and Control

Andrea Corli

University of Ferrara

Monday, 15:15, 21A342

This is a joint work with Prof. Ulrich Razafison (Laboratoire de Mathématiques de Besançon, Université Bourgogne Franche-Comté, Besançon, France) and Massimiliano D. Rosini (Department of Business Administration, University of Chieti-Pescara, Chieti, Italy and Uniwersytet Marii Curie-Skłodowskiej, 20-031 Lublin, Poland).

We consider an isothermal flow through two pipes. At the junction, the flow is possibly modified by some devices, such as valves, compressors, and so on, or by the geometry of the junction; coupling conditions between the traces of the flow must be given.

We first provide a general framework to model this situation by means of constrained Riemann problems, and provide some theoretical results. A key issue for both the validity of a coupling model and the robustness of numerical schemes to find solutions is whether the coupling Riemann solver is coherent. This property implies that applying the coupling Riemann solver to the traces at the junction of a coupling solution results in finding the same solution locally. We also give theoretical results for coherence.

Then, we consider several couplings; we discuss the uniqueness of the corresponding solvers and, in particular, their coherence. Surprisingly, some solvers of wide use are proven not to be uniquely defined, and others are not coherent. We present numerical examples to illustrate this property.

The content of the talk is taken from [1,2].

[1] A. Corli, U. Razafison, and M. D. Rosini. *Coherence and flow-maximization of a one-way valve.* ESAIM Math. Model. Numer. Anal., **56** (2022), 1715–1739.

[2] A. Corli, U. Razafison, and M. D. Rosini. *Coherence of coupling conditions for the isothermal Euler system.* Submitted, (2024).

[CT7] Energy preserving methods for a strongly coupled nonlinear Schrödinger equation

Canan AKKOYUNLU

ISTANBUL KULTUR UNIVERSITY

Monday, 15:45, 21A342

In this work, we consider the integration of a strongly coupled nonlinear Schrödinger equation, e.g. [1], [2]. The equation arises as a system of partial differential equations in many problems of mathematical physics, nonlinear optics, solid and fluid mechanics, as well as biological structures. The equation has two standard conserved quantities, namely mass and energy. We discretize the system in time by using the average vector field method and a linearly implicit mass conserving scheme. Both methods are second order. The schemes are quite accurate in preserving the conserved quantities of the system. Numerical experiments are carried out to show the efficiency and reliability of proposed schemes in long term integration.

[1] A. Aydin, B. Karasözen, *Lobatto IIA-IIB discretization of the strongly coupled nonlinear Schrödinger equation.* Journal of Computational and Applied Mathematics **235** (2011), 4770–4779.

[2] J. Cai, *Multisymplectic schemes for strongly coupled Schrödinger system.* Applied Mathematics and Computation **216** (2010), 2417–2429.

[CT7] First-order scheme for jump-diffusion SDE with Markovian switching

Divyanshu Vashistha

Indian Institute of Technology Roorkee

Monday, 16:15, 21A342

This is a joint work with Dr. Chaman Kumar (Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee, Uttarakhand, India).

We establish an Itô's formula for a Stochastic Differential Equation with Lévy noise and Markovian switching, which is then used to derive a first-order Milstein scheme. The challenges raised due to intertangling of the cádlág solution process with the discontinuous dynamics of the Markov chain are resolved. Under the assumption that the coefficients and their derivatives satisfy a Lipschitz continuity condition, we analyze the moment stability of the Milstein scheme and establish a rate of convergence of 1.0. We conclude by discussing the practical implementation of our scheme.

[1] S.L. Nguyen, T.A. Hoang, D.T. Nguyen and G. Yin (2017). Milstein-type procedures for numerical solutions of stochastic differential equations with Markovian switching. *SIAM J. Numer. Anal.*, 55(2) 953–979.

[2] F. Xi (2008). On the stability of jump-diffusions with Markovian switching. *J. Math. Anal. Appl.*, 341 588–600.

[3] D. Vashistha and C. Kumar (2024). Milstein scheme for stochastic differential equation with Markovian switching and Lévy noise. *J. Math. Anal. Appl.*, 536(1).

[CT7] Not quite standard instability and spatial patterns

Jan Eisner

University of South Bohemia

Monday, 16:45, 21A342

This is a joint work with Prof. M. Kučera (Mathematical Institute, Academy of Sciences, Prague, Czechia), V. Klika (Czech Technical University, Prague, Czechia) and M. Fencl (University of West Bohemia, Pilsen, Czechia).

We introduce non-standard notions of instability and spatial patterns and discuss their robustness. In fact, these notions correspond to what is really usually done in numerical computations or in a laboratory. We describe situations when our patterns evolve due to the newly introduced instability of the basic homogeneous steady state even if it is stable and even if heterogeneous stationary solutions do not exist.

[1] M. Kučera, V. Klika, M. Fencl and J. Eisner, *A new concept of instability and spatial patterns.* Submitted.

[CT8] Convex monotone semigroups on spaces of continuous functions

Jonas Blessing

ETH Zurich

Monday, 15:15, 1D227

This is a joint work with Robert Denk, Michael Kupper (both University of Konstanz, Konstanz, Germany) and Max Nendel (Bielefeld University, Bielefeld, Germany).

Motivated by model uncertainty and stochastic control problems, we develop a systematic theory for convex monotone semigroups on spaces of continuous functions. The present approach is self-contained and does, in particular, not rely on the theory of viscosity solutions. Instead, we provide a comparison principle for semigroups related to HJB-type equations which uniquely determines the semigroup by its infinitesimal generator evaluated at smooth functions. While the statement itself resembles the classical analogue for linear semigroups, the proof requires the introduction of several novel analytical concepts such as the Lipschitz set and the Γ-generator. Furthermore, starting with a generating family $(I(t))_{t\geq 0}$ of operators, we show that the limit $S(t)f := \lim_{n\to\infty} I(\frac{t}{n})^n f$ defines a semigroup which is uniquely determined by the time derivative $I'(0)f$ for smooth functions f. We identify explicit conditions for the generating family that are transferred to the semigroup and can easily be verified in applications. The abstract results are illustrated by emphasizing the structural link between approximation schemes for semigroups and LLN and CLT-type results for convex expectations. Furthermore, the limit can be represented as a stochastic control problem.

[CT8] A dynamic approach to heterogeneous elastic wires

Leonie Langer

Ulm University

Monday, 15:45, 1D227

The elastic energy of a bending-resistant interface depends both on its geometry and its material composition. We consider such a heterogeneous interface in the plane, modeled by a curve equipped with an additional density function. The resulting energy, which depends on material parameters, captures the complex interplay between curvature and density effects and resembles the Canham–Helfrich functional.

We study a family of planar curves with density evolving in time according to the steepest descent associated to this energy. Describing the curves by their inclination angle, the L^2-gradient flow is a nonlocal coupled parabolic system of second order. We shortly discuss local well-posedness via maximal regularity theory on time-dependent little Hölder spaces. Once global existence is established, convergence of solutions follows with a suitable constrained Łojasiewicz–Simon gradient inequality. We show that the (non)preservation of quantities such as convexity and positivity of the density depends delicately on the choice of material parameters. The same applies for the asymptotic behavior of the system.

This is based on joint work with Anna Dall'Acqua (Ulm University), Fabian Rupp (University of Vienna) and Gaspard Jankowiak (University of Graz).

[1] A. Dall'Acqua, L. Langer, and F. Rupp, *A dynamic approach to heterogeneous elastic wires*, J. Differential Equations **392** (2024), 1–42.

[2] A. Dall'Acqua, G. Jankowiak, L. Langer, and F. Rupp, *Conservation, convergence, and computation for evolving heterogeneous elastic wires*, arXiv preprint (2023), accepted for publication in SIAM J. Math. Anal.

[CT8] Infinitely Many Commuting Nonlocal Symmetries for Modified Martínez Alonso–Shabat Equation

Hynek Baran

Silesian University in Opava

Monday, 16:45, 1D227

We study the modified Martínez Alonso–Shabat equation

$$u_y u_{xz} + \alpha u_x u_{ty} - (u_z + \alpha u_t) u_{xy} = 0$$

and present its recursion operator and an infinite commuting hierarchy of full-fledged nonlocal symmetries. To date, such hierarchies have been found only for very few integrable systems in more than three independent variables. We will also briefly discuss how we used the computer algebra package Jets [2] on our way to the result.

[1] H. Baran, *Infinitely Many Commuting Nonlocal Symmetries for Modified Martínez Alonso–Shabat Equation,* Commun. Nonlinear Sci. Numer. Simul. **96** (2021) 105692.

[2] H. Baran, M. Marvan, *Jets. A software for differential calculus on jet spaces and diffieties,* http://jets.math.slu.cz.

[CT9] Existence of extremal solutions for differential equations with singular $p(t)$-Laplacian

Kodai Fujimoto

Shimane University

Tuesday, 8:45, 1D340

This is a joint work with Prof. Zuzana Došlá (Masaryk University, Brno, Czech Republic) [1]. We consider the second-order nonlinear differential equation

$$(a(t)|x'|^{p(t)-2}x')' + b(t)|x|^{\lambda-2}x = 0, \quad t \geq t_0 \in \mathbb{R}, \tag{42}$$

where $a(t)$ and $b(t)$ are positive continuous functions, $p(t) > 1$ is a continuously differentiable function, and $\lambda > 1$ is a parameter. In addition, we assume $a(t) \to \alpha > 0$ as $t \to \infty$ and $\int_{t_0}^{\infty} b(t)\,dt < \infty$. In this talk, we focus on the classification of positive increasing solution $x(t)$ such that $\lim_{t\to\infty} x'(t)$ exists, and we obtain the sufficient conditions for the existence of extremal solutions ($x'(t) \to \infty$ as $t \to \infty$) and weakly increasing solutions ($x'(t) \to 0$ as $t \to \infty$).

[1] Z. Došlá and K. Fujimoto, *Asymptotic properties for solutions of differential equations with singular $p(t)$-Laplacian*, Monatshefte für Mathematik **201** (2023), 65–78.

[CT9] Synchronized motion of bubbles interacting through the emission of pressure waves

Masashi Ohnawa

Tokyo University of Marine Science and Technology

Tuesday, 9:15, 1D340

We develop a model system of ODEs describing motions of bubbles interacting through the emission of sound waves of finite speed. The model has its basis on the Keller-Miksis model, which governs oscillation of a single spherical bubble under the influence of vapor pressure, surface tension, compressibility of surrounding liquid, and the presence of inactive gas inside the bubble. In the most fundamental case of two bubbles, numerical computation indicates that they fall into a state of synchronization, where the limit phase difference is either 0 or π depending on the distance of the bubbles. We clarify the mechanism of synchronization by analyzing the phase coupling function, and try to relate it to energy production rate due to interaction. If time permits, we apply our method to general cases of multiple bubbles.

[CT9] D-stability of the model of the Stieltjes string

Natalia Dilna

Institute of Mathematics Slovak Academy of Sciences

Tuesday, 9:45, 1D340

We consider in [1] the regular model of the Stieltjes string with a nonlocal initial value condition

$$p(t)u'(t) = p(0)\beta + \int_0^t \Big((gu)(s) + \sum_{i=1}^{m}(\xi_i(s)u(\nu_i(s)) - \psi_i(s)u(\mu_i(s)))\Big)dQ(s) - F(t),$$

$$u(0) = \alpha,$$

where $g : C([0,l],\mathbb{R}^n) \to L([0,l],\mathbb{R}^n)$ is nonlinear operator, $\{\xi_i, \psi_i\} \in L([0,l],\mathbb{R}^n)$, $i = 1, 2, \dots, m$, μ_i, ν_i, $i = 1, 2, \dots, m$, are measurable functions, and $\{\beta, \alpha\} \in \mathbb{R}^n$. The integral on the right-hand side is the Lebesgue-Stieltjes integral with respect to a nondecreasing function $Q : [0,l] \to \mathbb{R}^n$, $F \in L([0,l],\mathbb{R}^n)$. Function $p(t) > 0$ on $[0,l]$, $p(0)$ is continuous and $p, p^{-1} \in L^\infty([0,l],\mathbb{R})$, where $L^\infty([0,l],\mathbb{R})$ is the space of measurable and essentially bounded functions on $[0,l]$, moreover, under our assumption function p could be discontinuous on $[0,l]$.

D-stability conditions of the unique solution of the mentioned problem are established in [2,3].

[1] N. Dilna and M. Feckan. *The Stieltjes string model with external load.* Appl. Math. Comput., **337** (2018), pp. 350-359. https://doi.org/10.1016/j.amc.2018.05.026

[2] N. Dilna. *D-stability of the model of the Stieltjes string related to the functional differential equations.* Ex. Countex., **2**, (2022), 100092. https://doi.org/10.1016/j.exco.2022.100092

[3] N. Dilna, S. Leshchuk, *D-stability of the model of the Stieltjes string.* Appl. Anal., **102**(18), (2023), pp. 5157–5169. https://doi.org/10.1080/00036811.2023.2168654

[CT9] Modelling of Differential Equations from Noisy Data using Physics-informed Deep Learning Approach

Sanjeev Kumar

IIT Roorkee

Tuesday, 10:15, 1D340

This is a joint work with Prof Jan Mares (University of Chemistry and Technology, Prague, Czech Republic) and Sumit Vishwakarma (IIT Roorkee, India)

Modelling of differential equations serve as valuable tools for studying the intricate dynamics exhibited by various physical and biological systems. The parameters within these models possess scientific significance, yet their values remain unknown and require estimation using noisy observed data from the system. This study aims to perform parameter estimation for the differential equation models (ODE as well as PDE) that describes the movement of mimetic muscles in individuals with facial paresis resulting from surgical complications. We followed a two-step approach; the first step involves smoothing the observed noisy data using the basis function, i.e., putting it into functional form by representing as linear combination of B-spline basis functions. Second step involves the parameter estimation of the model using a recently developed scientific machine-learning method called Physics-informed neural networks (PINNs). PINNs combine the strengths of physics-based modeling and deep learning to provide accurate and efficient solutions. The model parameters are estimated as a trainable parameter of the neural network. The results show that the obtained model parameters, which minimizes the model-based loss function, converges during the training of the network and fairly fits some of the differential equations model.

[1] G. E. Karniadakis, I. G. Kevrekidis, L. Lu, P. Perdikaris, S. Wang and L. Yang, *P*hysics informed machine learning, Nature Reviews Physics 3, 422–440 (2021)

[2] M. Shayestegan, J. Kohout, K. Štícha, J. Mareš, *A*dvanced Analysis of 3D Kinect Data: Supervised Classification of Facial Nerve Function via Parallel Convolutional Neural Networks. Appl. Sci. 12:5902 (2022).

[CT10] A characterization of integrability of nearly integrable systems using the subharmonic Melnikov function and dissipativity

Shoya Motonaga

Ritsumeikan University

Tuesday, 8:45, 1D341

For a continuous dynamical system, the system is said to be integrable (in the sense of Bogoyavlenskij) if there exists a sufficient number of first integrals (conservative quantities) and commutative vector fields (continuous symmetries), which is a generalization of the classical integrability in the sense of Liouville for Hamiltonian systems. In this talk, we consider autonomous and non-autonomous perturbations of integrable systems and characterize their non-integrability by using the subharmonic Melnikov function. Here, the subharmonic Melnikov function is known as a function which gives a sufficient condition for the existence of periodic orbits for perturbed systems. In particular, we show that if many of the subharmonic Melnicov functions corresponding to periodic orbits in unperturbed systems are not identically zero, then the perturbed system is not real-analytically integrable such that the first integrals and the commutative vector fields depend on the perturbation parameter analytically. Moreover, in some cases, we see that dissipative perturbations prevent real-analytic integrability of these system. This implies that the perturbed systems can be real-analytically non-integrable even if there is no homoclinic or heteroclinic orbit in the unperturbed systems.

[CT10] Pushed and bistable waves in Belousov-Zhabotinsky reaction

Karel Hasík

Silesian University in Opava

Tuesday, 9:15, 1D341

This is a joint work with Jana Kopfová, Petra Nábělková (Silesian University at Opava, Czech Republicy), Olena Trofymchuk (Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine), Sergei Trofimchuk (Universidad de Talca, Chile)

We prove the existence minimal speed of propagation $c_*(r,b,K) \in [2\sqrt{1-r}, 2]$ for wavefronts in the Belousov-Zhabotinsky system with a spatiotemporal interaction defined by the convolution with (possibly, "fat-tailed") kernel K.

$$
\begin{aligned}
u_t(t,x) &= \Delta u(t,x) + u(t,x)(1 - u(t,x) - r(K * v)(t,x)), \\
v_t(t,x) &= \Delta v(t,x) - bu(t,x)v(t,x),\ u, v \geq 0,\ \ x \in \mathbb{R}^n,
\end{aligned}
\tag{43}
$$

The model is assumed to be monostable non-degenerate, i.e. $r \in (0,1)$. The slowest wavefront is termed pushed or non-linearly determined if its velocity $c_*(r,b,K) > 2\sqrt{1-r}$. We show that $c_*(r,b,K)$ is close to 2 if i) positive system's parameter b is sufficiently large or ii) if K is spatially asymmetric to one side (e.g. to the left: in such a case, the influence of the right side concentration of the bromide ion on the dynamics is more significant than the influence of the left side). Consequently, this reveals two reasons for the appearance of pushed wavefronts in the Belousov-Zhabotinsky reaction.

Next we study the Murray adaptation of the Noyes-Field five-step model of the Belousov-Zhabotinsky (BZ) reaction

$$
\begin{aligned}
u_t(t,x) &= D_u \Delta u(t,x) + u(t,x)(1 - u(t,x) - w(t-h,x)), \\
w_t(t,x) &= D_w \Delta w(t,x) - bu(t,x)w(t,x),\ \ u, w \geq 0,\ \ x \in \mathbb{R}^2,
\end{aligned}
\tag{44}
$$

in the case when a tuning parameter r, which determines the level of the bromide ion far ahead of the propagating wave, is bigger than 1 and when the delay in generation of the bromous acid is taken into account. The existence of wavefronts in the delayed BZ system was previously established only in the monostable situation with $r \in (0,1]$, the physically relevant bistable situation where $r > 1$ (in real experiments r varies between 5 and 50) was left open. We complete the study by showing that the BZ system with $r > 1$ admits monotone traveling fronts. Note that one of the stable equilibria of the BZ model is not isolated. This circumstance does not allow the direct application of the topological or analytical methods previously elaborated for the analysis of the existence of bistable waves.

[CT10] Analytic invariants of parabolic germs from their orbits

Maja Resman
University of Zagreb

Tuesday, 9:45, 1D341

This is a joint work with Martin Klimeš, Pavao Mardešić (University of Burgundy, France) and Goran Radunović (University of Zagreb, Croatia), based on [1].

The moduli of analytic classification of parabolic germs of diffeomorphisms, germified at a parabolic fixed point, are given by a finite number of diffeomorphisms, called the *Horn- maps* (Écalle, Voronin). We read the analytic invariants by fractal analysis of one orbit (i.e. one realization) of a diffeomorphism. The object that we study is the so-called theta function of one orbit, which, in the case of real orbits considered as *fractal strings* (introduced by Lapidus), is closely related to their fractal theta function. The fractal theta function of a fractal string is inspired by and generalizes the geometric zeta function of a fractal string (Lapidus). Standardly, fractal zeta functions talk about the geometry of a fractal string, its first singularity being the box dimension of the string. We show how to read the analytic class analysing the singularities of the theta function of one orbit in the integral plane.

[1] Klimes, M., Mardesic, P., Radunovic, G., Resman, M., Analytic invariants of a parabolic diffeomorphism from its orbit, accepted for publication in Annali della Scuola Normale Superiore di Pisa, Classe di Scienze (2023), https://arxiv.org/pdf/2112.14324.pdf

[CT10] Traveling waves in reaction-diffusion-convection equations with p-Laplacian-type diffusion

Michaela Zahradníková
University of South Bohemia

Tuesday, 10:15, 1D341

This is a joint work with Prof. Pavel Drábek (University of West Bohemia, Pilsen, Czech Republic).

We study the existence and properties of traveling wave solutions to the reaction-diffusion-convection equation

$$v_t = \Big[D(v)|v_x|^{p-2}v_x\Big]_x + h(v)v_x + g(v), \quad x \in \mathbb{R},\, t \geq 0, \tag{45}$$

with $p > 1$. We consider a possibly degenerate or singular diffusion coefficient $D \in C^1(0,1)$ and an ignition-type reaction $g \in C[0,1]$ which is Lipschitz continuous on $[\theta, 1)$ where $\theta \in (0,1)$ denotes the ignition threshold.

We present sufficient conditions for the existence and non-existence of traveling waves to (45), which extend and generalize the results established for $p = 2$ to the case $p > 1$. Our approach is based on the investigation of an equivalent non-Lipschitz first order problem. We also examine the shape of the wave profiles near 0 and 1 by assuming power-type behavior of the reaction and diffusion terms.

[CT11] Global-in-Time Vortex Configurations for the 2D Euler Equations

Shrish Parmeshwar

University of Warwick

Tuesday, 8:45, 21A258

This is joint work with Prof. Juan Dávila, Prof. Manuel del Pino, Prof. Monica Musso (University of Bath, Bath, United Kingdom).

A long-standing topic of interest is to understand solutions of the incompressible 2D Euler equations, stream-vorticity form given by

$$\partial_t \omega + \nabla^\perp \psi \cdot \nabla \omega = 0, \; -\Delta\psi = \omega, \tag{46}$$

where the vorticity ω of the solution stays highly concentrated around a finite number of points $(\xi_1(t), \ldots, \xi_n(t))$ on some interval of time, approximating the behaviour of solutions to the N point vortex ODE system

$$\dot{\xi}_j(t) = -\sum_{\ell \neq j} \frac{m_\ell}{2\pi} \frac{(\xi_j(t) - \xi_\ell(t))^\perp}{|\xi_j(t) - \xi_\ell(t)|^2}, \quad j = 1, \ldots, N, \tag{47}$$

in the sense that

$$\omega_\varepsilon(x,t) \rightharpoonup \sum_{j=1}^{N} m_j \delta(x - \xi_j(t)),\ \varepsilon \to 0. \tag{48}$$

There are a large class of steady states that satisfy this behaviour, and also solutions that exhibit this behaviour dynamically on finite time intervals. We exhibit solutions of 2D Euler that are genuinely dynamic, and also retain this concentration of vorticity around points for all time: a configuration approximating two vortex pairs separating at linear speed, and a configuration approximating three vortices separating like a self-similar spiral at sublinear speed [1–3].

[1] J. Dávila, M. del Pino, M. Musso, S. Parmeshwar *Global in Time Vortex Configurations for the 2D Euler Equations.* Preprint (2023), https://arxiv.org/abs/2310.07238.

[2] J. Dávila, M. del Pino, M. Musso, S. Parmeshwar *Asymptotic properties of vortex-pair solutions for incompressible Euler equations in R2.* Preprint (2023), https://arxiv.org/abs/2311.12039.

[3] J. Dávila, M. del Pino, M. Musso, S. Parmeshwar *An Expanding Self-Similar Vortex Configuration for the 2D Euler Equations.* In preparation (2024).

[CT11] On the full dispersion Kadomtsev–Petviashvili equations for dispersive elastic waves

Husnu Ata Erbay

Ozyegin University

Tuesday, 9:15, 21A258

This is a joint work with Prof Saadet Erbay (Ozyegin University, Istanbul, Turkey) and Prof Albert Erkip (Sabanci University, Istanbul, Turkey).

Full dispersive models of water waves, such as the Whitham equation [1] and the full dispersion Kadomtsev–Petviashvili (KP) equation [2], are interesting from both the physical and mathematical points of view. We introduce analogous full dispersive KP models of nonlinear elastic waves propagating in a nonlocal elastic medium. In particular we consider anti-plane shear elastic waves which are assumed to be small-amplitude long waves. We propose two different full dispersive extensions of the KP equation in the case of cubic nonlinearity and "negative dispersion". One of them is the Whitham-type full dispersion KP equation

$$w_{xt} + L(D_x, D_y)w_{xx} + \mu(w_x)^2 w_{xx} = 0 \tag{49}$$

and the other one is the BBM-type full dispersion KP equation

$$[1 + M(D_x, D_y)]^{1/2} w_{xt} + (1 + \frac{D_y^2}{D_x^2})^{1/2} w_{xx} + \mu(w_x)^2 w_{xx} = 0. \tag{50}$$

Most of the existing KP-type equations in the literature are particular cases of (49) and (50). We also introduce the simplified models of the new proposed full dispersion KP equations by approximating the operators L and M. We show that the line solitary wave solution of a simplifed form of (49) is linearly unstable to long-wavelength transverse disturbances if the propagation speed of the line solitary wave is greater than a certain value. A similar analysis for a simplifed form of (50) does not provide a linear instability assessment. (See [3] for more detailed results.)

[1] G. B. Whitham, Linear and nonlinear waves. John Wiley, New York, 1974.

[2] D. Lannes, The water waves problem: Mathematical analysis and asymptotics. Amer. Math. Soc., Providence, RI, 2013.

[3] H.A. Erbay, S. Erbay and A. Erkip, *On the full dispersion Kadomtsev–Petviashvili equations for dispersive elastic waves*, Wave Motion **114** (2022), Article number:103015.

[CT11] Existence and stability of spatially localized Lugiato-Lefever waves

Lukas Bengel

Karlsruhe Institute of Technology

Tuesday, 9:45, 21A258

This is a joint work with Björn de Rijk (Karlsruhe Institute of Technology).

We consider optical Kerr frequency combs generated in a nonlinear microresonator. Frequency combs are very promising devices with a broad range of applications including optical communications and frequency metrology. Mathematically, they are modeled by stationary spatially localized solutions of the Lugiato-Lefever equation (LLE)

$$\mathrm{i}u_t = -du_{xx} + (\zeta - \mathrm{i} - |u|^2)u + \mathrm{i}f,$$

which is a damped, detuned, and driven nonlinear Schrödinger equation. We show by using the Lyapunov-Schmidt reduction method that localized pulse solutions of LLE bifurcate from the bright NLS soliton. We then analyze their spectral and dynamical stability against localized perturbations. Our stability analysis exploits the instability index count for linear Hamiltonian systems and relies on high-frequency resolvent estimates. Numerical simulations with pde2path corroborate our analytical findings.

[CT11] Maximal regularity of Stokes problem with dynamic boundary condition

Tomas Barta

Charles University

Tuesday, 10:15, 21A258

T. Bárta, P. Davis, P. Kaplický

For the evolutionary Stokes problem with dynamic boundary conditions we show the maximal regularity of weak solutions in time. Due to the characteriation of R-sectorial operators on Hilbert spaces, the proof reduces to finding the correct functional analytic setting and proving that the corresponding operator is sectorial, i.e. generates an analytic semigroup. Exponential decay of solutions also follows. The results can be found in arXiv:2308.01616.

[CT12] Noncoercive anisotropic operator with a singular convection term

Giuseppina di Blasio

Universita della Campania

Tuesday, 15:15, 1B309

This is a joint work with F. Feo (Università degli Studi di Napoli "Parthenope", Naples, Italy) and G. Zecca (Università degli Studi di Napoli Federico II, Naples, Italy).

The aim of this seminar is to outline some results obtained in recent years concerning a class of anisotropic elliptic equations with the coefficients of a convection term belonging to some suitable Marcinkiewicz spaces. More precisely, considering Ω a bounded domain of $\mathbb{R}^{\mathbb{N}}$, our model equation is

$$-\sum_{i=1}^{N}\partial_{x_i}\left[|\partial_{x_i}u|^{p_i-2}\partial_{x_i}u+\kappa_i(x)|u|^{\frac{\bar{p}}{p_i'}-1}u\right]+\tilde{\mu}|u|^{\gamma-1}u=\mathcal{F} \qquad \text{in } \Omega, \tag{51}$$

where $p_i > 1$ such that $\bar{p} < N$, where $\bar{p}$ is the harmonic mean of $\vec{p} = (p_1, \cdots, p_N)$, *i.e.*

$$\frac{1}{\bar{p}}=\frac{1}{N}\sum_{i=1}^{N}\frac{1}{p_i},$$

$\tilde{\mu} \geq 0, 1 \leq \gamma < p_\infty - 1, \kappa_i$ belongs to the Marcinkiewicz space $L^{\frac{N p_i'}{\bar{p}},\infty}(\Omega)$ and $\mathcal{F}$ belongs to the dual space. We stress that when $\bar{p}^* \geq \max_i p_i$ using the Sobolev embedding in the Lorentz space $L^{\bar{p}^*,\bar{p}}(\Omega)$ the summability assumption on κ_i is optimal to assure $|\kappa_i|^{p_i'}|u|^{\bar{p}} \in L^1(\Omega)$.

[1] G. di Blasio, F. Feo, G. Zecca, *Regularity results for local solutions to some anisotropic elliptic equations*, Isr. J. Math. (2023), https://doi.org/10.1007/s11856-023-2564-y.

[2] G. di Blasio, F. Feo, G. Zecca, *Existence and uniqueness of solutions to some anisotropic elliptic equations with a singular convection term*, arXiv:2307.13564.

[CT12] Hysteretic ARZ Model, the Effect of Different Driving Behaviors on Traffic Jams

Haitao Fan

Georgetown University

Tuesday, 15:45, 1B309

The ARZ traffic model has undergone extensive study, accommodating various driving behaviors. The hysteretic LWR model was shown to exhibit stop-and-go waves, also called phantom jams. Naturally, one would inquire about the impact of different hysteretic driving habits on the emergence and dissipation of phantom jams. In pursuit of this, in this talk, hysteresis is incorporated into the ARZ traffic model, resulting in the HARZ model. The presentation covers the discovery of basic viscous waves and Riemann solutions. A unique Riemann solution with monotone flux is identified. The string stability of a vehicle platoon with drivers having different hysteretic driving habits is studied via various examples of initial-boundary value problems, simulating vehicle platoons subject to speed oscillations introduced by the leading vehicles. Through this study, the emergence, persistence or damping of stop-and-go speed oscillations are found to be determined by the chord-over-scanning-curve condition.

[CT12] Mass-Conserving Solutions to Collision-Induced Breakage Equations with Non-Integrable Daughter Distribution Function

RAM GOPAL JAISWAL

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

Tuesday, 16:15, 1B309

This is a joint work with Prof. Ankik Kumar Giri (Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee-247667, Uttarakhand, India) and Prof. Philippe Laurençot (Laboratoire de Mathématiques (LAMA), UMR 5127, Université Savoie Mont Blanc, CNRS, F–73000 Chambéry, France) .

Breakage is a fundamental process that arises in numerous scientific and technical disciplines such as chemistry, cellular biology, astronomy, and atmospheric science. This phenomenon describes the process of particle separation, which is classified into two primary types: linear breakage and collision-induced (nonlinear) breakage. Linear breakage occurs either spontaneously or due to external forces acting upon the particles, while collision-induced breakage arises from the collision of two particles. Specifically, if we denote the size distribution function of particles of size $x \in (0,\infty)$ at time $t \geq 0$ as $f = f(t,x) \geq 0$, the dynamics of f is described by the collision-induced breakage equation

$$\begin{aligned}\partial_t f(t,x) = &- \int_0^\infty K(x,y)(t,x)f(t,y), dy \\ &+ \frac{1}{2}\int_x^\infty \int_0^y B(x,y-z,z)K(y-z,z)f(t,y-z)f(t,z), dz, dy, \quad (t,x) \in (0,\infty)^2 \quad (52)\end{aligned}$$

In this talk, we will discuss the existence, uniqueness, and non-existence of mass-conserving weak solutions to (52) when an infinite number of daughter particles are produced after each collision, corresponding to a non-integrable daughter distribution function $B(x,y,z)$, along with the collision kernel given by $K(x,y) = x^a y^b + y^a x^b,\ \ell_0 \le a \le b \le 1$, for a fixed $\ell_0 \in (0,1)$, and for all $(x,y) \in (0,\infty)^2$.

[1] J. Banasiak, W. Lamb and Ph. Laurençot, Analytic Methods for Coagulation-Fragmentation Models, Chapman and Hall/CRC, 2019.

[2] A. K. Giri and Ph. Laurençot, *Existence and non-existence for the collision-induced breakage equation.* SIAM J. Math. Anal. **53** (2021), 4605–4636.

[3] Ph. Laurençot, *Mass-conserving solutions to coagulation-fragmentation equations with nonintegrable fragment distribution function.* Quart. Appl. Math. **76** (2018), 767–785.

[CT12] Global existence and boundedness in a degenerate parabolic–elliptic–elliptic attraction-repulsion chemotaxis system

Yutaro Chiyo

Tokyo University of Science

Tuesday, 16:45, 1B309

In this talk we consider the degenerate parabolic–elliptic–elliptic attraction-repulsion chemotaxis system

$$\begin{cases} u_t = \nabla \cdot (u^{m-1}\nabla u - \chi u^{p-1}\nabla v + \xi u^{q-1}\nabla w), & x \in \Omega,\ t > 0, \\ 0 = \Delta v + \alpha u - \beta v, & x \in \Omega,\ t > 0, \\ 0 = \Delta w + \gamma u - \delta w, & x \in \Omega,\ t > 0, \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ $(n \in \mathbb{N})$ is a smoothly bounded domain, $m \ge 1$, $p, q \ge 2$, $\chi, \xi, \alpha, \beta, \gamma, \delta > 0$ are constants. In the case that $\chi\alpha - \xi\gamma < 0$, global existence and boundedness in the nondegenerate version of the above system were obtained in [1]. In this talk we derive global existence and boundedness in the above degenerate system when $\chi\alpha - \xi\gamma < 0$, and we present recent observations on stabilization.

[1] Y. Chiyo, T. Yokota, *Boundedness and finite-time blow-up in a quasilinear parabolic–elliptic–elliptic attraction-repulsion chemotaxis system*, Z. Angew. Math. Phys. **73** (2022), Paper No. 61, 27 pages.

[CT13] Stability criteria for differential equations with discrete and distributed delays

MINGZHU QU

Osaka Prefecture University

Tuesday, 15:15, 1D327

This is a joint work with Hideaki Matsunaga (Osaka Metropolitan University).
We will summarize some recent results on stability criteria for a linear delay differential equation

$$x'(t) = -ax(t) - bx(t-\tau) - c\int_{t-\tau}^{t} x(s)ds, \quad t \geq 0, \tag{53}$$

where a, b, $c \in \mathbb{R}$ and $\tau > 0$. We present new necessary and sufficient conditions guaranteeing the asymptotic stability of the zero solution of (53) under $bc \neq 0$ those are composed of explicit delay-dependent criteria. The proofs of our results are obtained using the root analysis of the associated characteristic equation.

[1] M. Funakubo, T. Hara, S. Sakata, *On the uniform asymptotic stability for a linear integro–differential equation of Volterra type.* J. Math. Anal. Appl. **324** (2006), 1036–1049.

[2] Mingzhu Qu, H. Matsunaga, *Exact stability criteria for linear differential equations with discrete and distributed delays.* submitted for publication.

[3] S. Sakata, T. Hara, *Stability regions for linear differential equations with two kinds of time lags.* Funkcialaj Ekvacioj. **47** (2004), 129–144.

[CT13] Hopf Bifurcations of Neural Fields on the Sphere with Diffusion and Distributed Delays

Mónika Polner

University of Szeged

Tuesday, 15:45, 1D327

This is a joint work with L. Spek, S. A. van Gils, Yu. A. Kuznetsov, (University of Twente, Enschede, The Netherlands).

We study a neural field model that consists of two populations, incorporating synaptic connections with transmission delays and gap junctions. The model is formulated on the sphere, which is suitable for describing the cortical activity of the brain without boundaries. We have investigated how the addition of gap junctions, modeled as a diffusion term, affects delay-induced oscillations in neural fields.

In [1] we derived analytical conditions for bifurcations of the trivial equilibrium and presented a systematic method to obtain analytical formulas for the normal form coefficients of Hopf bifurcations with spherical symmetry in terms of the model parameters. Moreover, expanding the state of the system in spherical harmonics also makes it possible to study the stability of the bifurcating branches. Numerical simulations complement our analytical findings, illustrating the emergence of various spatio-temporal patterns, including rotating and standing waves.

The novelty of this work lies in the systematic analysis of the interplay between the different mechanisms of pattern formation. Our investigations demonstrate that there is a non-trivial interaction between the delay-induced oscillations and diffusion. Specifically, when the diffusion coefficient between excitatory neurons is significantly larger than that between inhibitory neurons, periodic orbits with higher-order harmonics become stable compared to the cases where the diffusion coefficients are equal or vanish. This is in contrast to earlier findings in less detailed models where a large diffusion coefficient destabilizes all spatially non-uniform patterns.

These contributions are specific for neural fields, but can be applied to other models with spherical symmetry.

[1] L. Spek, S. A. van Gils, Yu. A. Kuznetsov, M. Polner, *Hopf Bifurcations of Two Population Neural Fields on the Sphere with Diffusion and Distributed Delays*, SIAM J. Appl. Dyn. Syst. (to appear)

[CT13] Propagation reversal for bistable differential equations on trees

Petr Stehlik

University of West Bohemia

Tuesday, 16:15, 1D327

This is a joint work with Hermen Jan Hupkes, Mia Jukić (University of Leiden) and Vladimír Švígler (Uniersity of West Bohemia).

In this talk we study travelling wave solutions to bistable differential equations on infinite k-ary trees.These graphs generalize the notion of classical square infinite lattices and our results complement those for bistable lattice equations on . Using comparison principles and explicit lower and upper solutions, we show that wave-solutions are pinned for small diffusion parameters. Upon increasing the diffusion, the wave starts to travel with non-zero speed, in a direction that depends on the detuning parameter. However, once the diffusion is sufficiently strong, the wave propagates in a single direction up the tree irrespective of the detuning parameter. In particular, our results imply that changes to the diffusion parameter can lead to a reversal of the propagation direction.

[1] H. J. Hupkes, M. Jukić, P. Stehlík, V. Švígler, *Propagation Reversal for Bistable Differential Equations on Trees* SIAM J. Appl. Dyn. Syst. **22** (2023), 1906–1944.

[CT13] Bifurcations of periodic solutions to Newtonian systems with symmetries

Piotr Stefaniak

Nicolaus Copernicus University in Toruń

Tuesday, 16:45, 1D327

This is a joint work with Anna Gołębiewska and Sławomir Rybicki (Nicolaus Copernicus University in Toruń, Poland).

The aim of the talk is to study existence of non-stationary 2π-periodic solutions of families of autonomous Newtonian systems of the form

$$\ddot{u}(t) = -\lambda^2 U'(u(t)). \tag{54}$$

We consider the system with an S^1-symmetric potential U satisfying some standard technical assumptions. Our main tools are equivariant degree and equivariant bifurcation theories. In particular, we consider an S^1-index of a critical point of U. We show that its non-triviality, combined with the other assumptions on the right-hand site of (54), implies an occurrence of a global bifurcation phenomenon. This means the existence of a connected family of non-stationary solutions of the system (54), emanating from the family of stationary ones.

We emphasize that the non-triviality of the S^1-index follows when the critical point is non-degenerate or that its Brouwer index is non-trivial, although our results are not limited only to such cases.

[CT14] A probabilistic interpretation of solutions of first-order conservation PDE, with application to blow-ups

Jochem Hoogendijk
Utrecht University
Tuesday, 15:15, 1D227

This is a joint work with Dr. I.V. Kryven (Utrecht University, The Netherlands).

From the point of view of statistical physics, it is natural to expect that some PDEs should have a probabilistic interpretation. The most famous example of this connection is the solution of heat equation being formally expressed through random walks. In this talk, we will consider conservation laws of the form $u_t = F(u)_x$, where F is a power series, and show how their solutions can be represented using a random trees. Furthermore, this probabilistic interpretation reveals new properties of the PDEs that could not be computed otherwise. For instance, it indicates an explicit expression for the blow-up time of the PDE and a possible numerical method to approximate its solutions.

[CT14] A Numerical Scheme Using Projection for the Perfect Plasticity Model with Time-Dependent Thresholds

Kazunori Matsui
Tokyo University of Marine Science and Technology
Tuesday, 15:45, 1D227

This is a joint work with Lecturer Yoshiho Akagawa (National Institute of Technology (KOSEN), Gifu College, Motosu-shi, Japan).

Elastoplastic problems, which analyze the behavior of materials such as metals and concrete, involve nonlinear stress-strain relations described by nonlinear evolution equations. One of the simplified models frequently used in engineering for these relations is the perfect plasticity model. This model has been studied not only from an engineering perspective but also from a mathematical point of view, particularly in connection with the Moreau sweeping process. We consider the problem proposed in [1] for the perfect plasticity model with a time-dependent threshold.

Let $T > 0$, and consider a bounded Lipschitz domain Ω in $\mathbb{R}^d$ ($2 \leq d \in \mathbb{N}$). We seek the displacement velocity $v : [0,T] \times \Omega \to \mathbb{R}^d$ and the stress $\sigma : [0,T] \times \Omega \to \mathcal{S}_d$ satisfying:

$$\frac{\partial v}{\partial t} = \nu \mathrm{div} \mathcal{E}(v) + \mathrm{div}\sigma + f, \quad \frac{\partial \sigma}{\partial t} \in \mathcal{E}(v) + h - \partial I_K(\sigma) \quad \text{in } (0,T) \times \Omega, \tag{55}$$

with suitable boundary and initial conditions. Here, $\mathcal{S}_d$ is the space of symmetric matrices of order d, $\nu > 0$, $f : (0,T) \times \Omega \to \mathbb{R}^d$, $h : (0,T) \times \Omega \to \mathcal{S}_d$ are given, $K = K(t,x) \subset \mathcal{S}_d$ is a given closed convex set called the constraint set, and ∂I_K is the subdifferential of the indicator function I_K on K. Also, $\mathcal{E}(v) = (\nabla v + (\nabla v)^{\mathrm{T}})/2$. We consider the von Mises yield criterion:

$$K(t,x) = \tilde{K}(t,x) - p(t,x), \quad \tilde{K}(t,x) = \left\{ \tau \in \mathcal{S}_d : |\tau^D| \leq g(t,x) \right\},$$

where $\tau^D = \tau - ((\mathrm{tr}\, \tau)/d) E_d$ is the deviatoric part of τ, $|\cdot|$ denotes the Frobenius norm for matrices, E_d is the identity matrix of size d, and $p : (0,T) \times \Omega \to \mathcal{S}_d$ and $g : (0,T) \times \Omega \to \mathbb{R}$ are given.

We consider the following numerical scheme for (55), using the following projection $\mathcal{P}_R : \mathcal{S}_d \to \mathcal{S}_d$: Find $v_n : \Omega \to \mathbb{R}^d$ and $\sigma_n^*, \sigma_n : \Omega \to \mathcal{S}_d$ such that

$$\begin{cases} \dfrac{v_n - v_{n-1}}{\Delta t} = \nu \mathrm{div} \mathcal{E}(v_n) + \mathrm{div}\sigma_n^* + f_n & \text{in } \Omega, \\ \dfrac{\sigma_n^* - \sigma_{n-1}}{\Delta t} = \mathcal{E}(v_n) + h_n & \text{in } \Omega, \\ \quad \sigma_n + p_n = \mathcal{P}_{g_n}(\sigma_n^* + p_n) & \text{in } \Omega, \end{cases} \qquad \mathcal{P}_R(A) = \begin{cases} \dfrac{\mathrm{tr} A}{d} E_d + R \dfrac{A^D}{|A^D|} & \text{if } |A^D| > R, \\ A & \text{if } |A^D| \leq R. \end{cases}$$

where $f_n = \frac{1}{\Delta t} \int_{t_{n-1}}^{t_n} f dt,\ h_n = \frac{1}{\Delta t} \int_{t_{n-1}}^{t_n} h dt,\ p_n = p(t_n),\ g_n = g(t_n)$ for all $n = 1, 2, \ldots, N$.

The proposed scheme projects the stress within constraints after solving the linear elliptic PDE at each step, avoiding the need to solve nonlinear problems. We demonstrate the stability of the proposed scheme independent of the time step size Δt. Moreover, we derive the existence of an exact solution to the original problem (55) using the obtained stability. We also prove that the solution of the proposed scheme strongly converges to the exact solution under suitable norms.

[1] Y. Akagawa, T. Fukao, and R. Kano, Time-dependence of the threshold function in the perfect plasticity model, Adv. Math. Sci. Appl., 32 (2023), 371–398.

[CT14] Structure-preserving schemes for the two-dimensional Cahn–Hilliard equation with dynamic boundary conditions and its applications

Makoto Okumura

Konan University

Tuesday, 16:15, 1D227

Recently, Cahn–Hilliard models with dynamic boundary conditions have been proposed by Goldstein et al. [1] and Liu and Wu [2]. These models have characteristic conservation laws, and we have constructed structure-preserving schemes for these models in the two-dimensional spatial case that retain such conservation laws and the total energy dissipation law in a discrete setting [3]. Through numerical computations with the schemes, we have obtained interesting results on the difference in the behavior of numerical solutions due to different boundary conditions between these models. In this talk, we will introduce the numerical examples and, if possible, talk about the solvability of the proposed schemes.

[1] G. R. Goldstein, A. Miranville, and G. Schimperna, A Cahn–Hilliard model in a domain with non-permeable walls, Physica D, **240** (2011), 754–766.

[2] C. Liu and H. Wu, An energetic variational approach for the Cahn–Hilliard equation with dynamic boundary condition: model derivation and mathematical analysis, Arch. Rational Mech. Anal., **233** (2019), 167–247.

[3] M. Okumura and T. Fukao, Structure-preserving schemes for Cahn–Hilliard equations with dynamic boundary conditions, Discrete Contin. Dyn. Syst. Ser. S, **17** (2024), 362–394.

[CT14] Convergence of a second-order scheme for non-local conservation laws

Nikhil Manoj

Indian Institute of Science Education and Research

Tuesday, 16:45, 1D227

This is a joint work with Dr. Sudarshan Kumar K. (IISER Thiruvananthapuram, Thiruvananthapuram, India) and Prof. G. D. Veerappa Gowda (TIFR-CAM, Bangalore, India).
In this talk, we discuss the convergence analysis of a second-order numerical scheme for traffic flow models that incorporate non-local conservation laws to capture the interaction between drivers and the surrounding density of vehicles. We combine a MUSCL-type spatial reconstruction with strong stability preserving Runge-Kutta time-stepping to devise a fully discrete second-order scheme. The resulting scheme is shown to converge to a weak solution by establishing the maximum principle, bounded variation estimates and L^1- Lipschitz continuity in time. Further, using a space-step dependent slope limiter, we prove its convergence to the entropy solution. We also propose a MUSCL-Hancock type second-order scheme which requires only one intermediate stage unlike the Runge-Kutta schemes and is easier to implement. The performance of the proposed second-order schemes in comparison to a first-order scheme is demonstrated through several numerical experiments.

[1] S. Blandin and P. Goatin, Well-posedness of a conservation law with non-local flux arising in traffic flow modeling, Numer. Math. **132** (2016), 217–241.

[2] F.A. Chiarello and P. Goatin, Global entropy weak solutions for general non-local traffic flow models with anisotropic kernel, ESAIM Math. Model. Numer. Anal. **52** (2018), 163–180.

[3] C. Chalons, P. Goatin and L.M. Villada, High-order numerical schemes for one-dimensional nonlocal conservation laws, SIAM J. Sci. Comput. **40** (2018), A288–A305.

[4] J. Friedrich and O. Kolb, Maximum principle satisfying CWENO schemes for nonlocal conservation laws, SIAM J. Sci. Comput. **41** (2019), A973–A988.

[CT15] Minkowski dimension of degenerate spiral trajectories of a class of ordinary differential equations

Domagoj Vlah

University of Zagreb

Wednesday, 8:45, 1D340

This is a joint work [1] with Prof Renato Huzak (Hasselt University, Hasselt, Belgium), Prof Vesna Županović (University of Zagreb, Zagreb, Croatia), and Prof Darko Žubrinić (University of Zagreb, Zagreb, Croatia).

We study the Minkowski dimension, also called the box dimension, of degenerate spiral trajectories of a class of ordinary differential equations. Specifically, a class of singularities of focus type with two zero eigenvalues (nilpotent or more degenerate) has been studied. We compute the box dimension of a polynomial degenerate focus of type (n, n) by exploiting the well-known fractal results for α-power spirals. In the general focus type (m, n) case, we formulate a conjecture about the box dimension of a degenerate focus backed up with numerical experiments.

Additionally, we construct a three-dimensional vector field that contains a degenerate spiral, called an elliptical power spiral, as a trajectory.

[1] R. Huzak, D. Vlah, D. Žubrinić, V. Županović, *Fractal analysis of degenerate spiral trajectories of a class of ordinary differential equations* Appl. Math. Comput. **438** (2023); 127569.

[CT15] ODE-systems with boundary conditions containing higher order derivatives

Olena Atlasiuk

University of Helsinki

Wednesday, 9:15, 1D340

This is a joint work with Prof Volodymyr Mikhailets (Institute of Mathematics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine).

We develop a general theory of solvability of linear inhomogeneous boundary-value problems for systems of ordinary differential equations of arbitrary order in Sobolev spaces. Boundary conditions are allowed to be overdetermined or underdetermined. They may contain derivatives, of the unknown vector-valued function, whose integer or fractional orders exceed the order of the differential equation. Similar problems arise naturally in various applications. The theory introduces the notion of a rectangular number characteristic matrix of the problem. The index and Fredholm numbers of this matrix coincide respectively with the index and Fredholm numbers of the inhomogeneous boundary-value problem. Unlike the index, the Fredholm numbers (i.e. the dimensions of the problem kernel and co-kernel) are unstable even for small (in the norm) finite-dimensional perturbations. We give examples in which the characteristic matrix can be explicitly found. We also prove a limit theorem for a sequence of characteristic matrices.

Specifically, it follows from this theorem that the Fredholm numbers of the problems under investigation are semicontinuous in the strong operator topology. Such a property ceases to be valid in the general case.

The research is financially supported by the Academy of Finland, grant no. 359642.

[1] V. Mikhailets, O. Atlasiuk *The solvability of inhomogeneous boundary-value problems in Sobolev spaces* Banach J. Math. Anal. 18(2), 12 (2024).

[CT15] A differential equation with a state-dependent queueing delay

István Balázs

University of Szeged

Wednesday, 9:45, 1D340

This is a joint work with Prof. Tibor Krisztin (University of Szeged, Hungary).

We consider a differential equation with a state-dependent delay motivated by a queueing process. The time delay is determined by an algebraic equation involving the length of the queue for which a discontinuous differential equation holds. The new type of state-dependent delay raises some problems that are studied in this talk. We formulate an appropriate framework to handle the system, and show that the solutions define a Lipschitz continuous semiflow in the phase space. The second main result guarantees the existence of slowly oscillating periodic solutions.

[1] I. Balázs, T. Krisztin, *A differential equation with a state-dependent queueing delay.* SIAM J. Math. Anal. **52** (2020), 3697–3737.

[CT15] Analysis of the Geometric Structure of Neural Networks and Neural ODEs via Morse Functions

Sara-Viola Kuntz

Technical University of Munich

Wednesday, 10:15, 1D340

This is a joint work with Professor Christian Kuehn (Department Mathematics, Technical

University of Munich, Germany and Munich Data Science Institute, Garching, Germany).

Besides classical feed-forward neural networks, also neural ordinary differential equations (neural ODEs) gained particular interest in the last years. Neural ODEs can be interpreted as an infinite depth limit of feed-forward or residual neural networks. In this presentation, we study the input-output dynamics of finite and infinite depth neural networks with scalar output. In the finite depth case, the input is a state associated to a finite number of nodes, which maps under multiple non-linear transformations to the state of one output node. In analogy, a neural ODE maps a linear transformation of the input to a linear transformation of its time-T map. We show that depending on the specific structure of the network, the input-output map has different properties regarding the existence and regularity of critical points. These properties can be characterized via Morse functions, which are scalar functions, where every critical point is non-degenerate. We prove that critical points cannot exist, if the dimension of the hidden layer is monotonically decreasing or the dimension of the phase space is smaller or equal to the input dimension. In the case that critical points exist, we classify their regularity depending on the specific architecture of the network. We show that each critical point is non-degenerate, if for finite depth neural networks the underlying graph has no bottleneck, and if for neural ODEs, the linear transformations used have full rank. For each type of architecture, the proven properties are comparable in the finite and in the infinite depth case. The established theorems allow us to formulate results on universal embedding, i.e. on the exact representation of maps by neural networks and neural ODEs. Our dynamical systems viewpoint on the geometric structure of the input-output map provides a fundamental understanding, why certain architectures perform better than others.

[CT16] To Study Approximate Controllability of Semilinear Hemivariational Inequalities with Nonlocal Conditions

Dabas Jaydev

Indian Institute of Technology Roorkee

Wednesday, 8:45, 1D341

This is a joint work with Mr. Akhilesh Verma (Department of Applied Mathematics and Scientific Computing, Indian Institute of Technology Roorkee, India).

The objective of this study is to establish criteria for the existence of mild solutions and approximate controllability for semilinear impulsive hemivariational inequalities with nonlocal conditions. We demonstrate the equivalence between the original problem and a corresponding inclusion problem, introduce the notion of mild solutions, and provide a sufficient condition ensuring approximate controllability. Additionally, we outline conditions under which the nonlinear system achieves approximate controllability and demonstrate that our system adheres to a finite number of constraints, supporting our main finding.

[1] R. E. Kalman, *Mathematical description of linear dynamical systems.* Journal of the Society for Industrial and Applied Mathematics, Series A: Control, (1963), 152–192.

[2] N.I. Mahmudov, *Approximate controllability of evolution systems with nonlocal conditions.* Nonlinear Analysis: Theory, Methods & Applications, **68(3)**, (2008) 536–546.

[CT16] Green's function for solving initial-boundary value problem of evolutionary partial differential equations

Hung-Wen Kuo

National Cheng Kung University

Wednesday, 9:15, 1D341

We propose a new method to solve the initial-boundary value problem for hyperbolic-dissipative partial differential equations based on the spirit of LY algorithm [1]. The new method can handle more general domains than that of LYs'. We convert the evolutionary PDEs into the elliptic PDEs by the Laplace transformation. Using the Laplace transformation of the fundamental solutions of the evolutionary PDEs and the image method, we can construct Green's functions for the corresponding elliptic PDEs. Finally, we obtain Green's functions for the evolutionary PDEs by inverting the Laplace transformation. As a consequence, we establish Green's functions for some basic PDEs such as the heat equation, the wave equation and the damped wave equation, in a half space and a quarter plane with various boundary conditions. On the other hand, the structure of hyperbolic-dissipative PDEs means its fundamental solution is non-symmetric and hence the image method does not work. We utilize the idea of Laplace wave train introduced by Liu and Yu in [2] to generalize the image method. Combining this with the notions of Rayleigh surface wave operators introduced in [3], we are able to obtain the complete representations of Green's functions for the convection-diffusion equation and the drifted wave equation in a half space with various boundary conditions.

[1] T.-P. Liu and S.-H. Yu, *Dirichlet-Neumann kernel for hyperbolic-dissipative system in half-space.* Bull. Inst. Math. Acad. Sin. **7** (2012), 477–543.

[2] T.-P. Liu and S.-H. Yu, *Navier-Stokes equations in gas dynamics: Greens function, singularity and well-posedness.* Comm. Pure Appl. Math. **75** (2022), 223–348.

[3] S. J. Deng, W. K. Wang and S.-H. Yu, *Green's functions of wave equations in* $\mathbb{R}^n_+ \times \mathbb{R}_+$. Arch. Ration. Mech. Anal. **216** (2015) 881-903.

[CT16] Sobolev Stability for the 2D MHD Equations in the Non-Resistive Limit

Niklas Knobel

Karlsruhe Institute of Technology

Wednesday, 9:45, 1D341

The magnetohydrodynamic (MHD) equations model an incompressible and electrically conducting fluid. We are interested in the effects of mixing and quantitative stability of the MHD equations. Therefore, we consider perturbations of a combination of an affine shear flow and a constant magnetic field in a periodic channel. For the partial dissipation regime, when magnetic resistivity is smaller than fluid viscosity $\nu \geq \kappa > 0$, our main result [1] is the nonlinear stability of perturbations that are small enough in Sobolev spaces. Furthermore, if $\kappa \leq \nu^3$ the system exhibits norm inflation of the Sobolev norm by $\nu\kappa^{-\frac{1}{3}}$.

[1] N. Knobel, *Sobolev Stability for the 2D MHD Equations in the Non-Resistive Limit* arXiv preprint arXiv:2401.12548, 2024.

[CT16] Asymptotic analysis of transmission problem and dynamic boundary condition

Takeshi Fukao

Ryukoku University

Wednesday, 10:15, 1D341

In 2023, Giga–Łasica–Rybka [2] studied the relationship between a heat equation with dynamic boundary condition and a transmission problem, in which the main domain is surrounded by the thin subdomain, its width δ converges to 0. This kind of problem is called "concentrated capacity" and originally studied by Colli–Rodrigues [1] in some weak sense. In this talk, we discuss the asymptotics from the transmission problem to the nonlinear parabolic equation with various boundary conditions including the dynamic boundary condition, and the regularity of solutions, respectively.

Let $N \in \mathbb{N}$, $\delta \in (0,1]$, $\sigma \in [1,+\infty)$. We set up the transmission problem of the nonlinear parabolic equation of the following form:

$$\partial_t u - \lambda \Delta u + f(u) = h \quad \text{in } (0,T) \times \mathbb{R}^N_+, \quad \mathbb{R}^N_+ := \{x \in \mathbb{R}^N : x_N > 0\}, \tag{56}$$

$$\sigma \partial_t v - \Delta v + f(v) = h \quad \text{in } (0,T) \times \Omega_\delta, \quad \Omega_\delta := \{x \in \mathbb{R}^N : -\delta < x_N < 0\}, \tag{57}$$

$$u = v, \quad \lambda \partial_{x_N^-} u + \partial_{x_N^+} v = 0 \quad \text{on } (0,T) \times \Gamma, \quad \Gamma := \{x \in \mathbb{R}^N : x_N = 0\} = \mathbb{R}^{N-1} \times \{0\}, \tag{58}$$

$$\partial_{x_N^-} v = g \quad \text{on } (0,T) \times \Gamma_\delta, \quad \Gamma_\delta := \{x \in \mathbb{R}^N : x_N = -\delta\}, \tag{59}$$

$$u(0) = \phi \quad \text{in } \mathbb{R}^N_+, \tag{60}$$

$$v(0) = \phi \quad \text{in } \Omega_\delta, \tag{61}$$

where $0 < T < \infty$, $\lambda > 0$ are fixed constants, $f : \mathbb{R} \to \mathbb{R}$, $h : \mathbb{R}^N_{>-\delta} \to \mathbb{R}$, $g : \mathbb{R}^{N-1} \to \mathbb{R}$, $\phi : \mathbb{R}^N_{>-\delta} \to \mathbb{R}$ are give functions, here we used the notation $\mathbb{R}^N_{>-\delta} := \{x \in \mathbb{R}^N : x_N > -\delta\}$. Moreover, $\partial_{x_N^+}$ (resp. $\partial_{x_N^-}$) stand for the upward (resp. downward) normal derivatives on Γ or Γ_δ. As a remark, the conditions in (58) are the Dirichlet and Neumann boundary conditions for (56), it seems over determinant. However, v is also unknown which is given as a solution of (57) and then we can prove the well-posedness of (56)–(61). This kind of problem is called "transmission problem".

There are two parameters of asymptotics, the width δ of thin domain and the heat capacity $\sigma := \sigma(\delta)$ in front of the time derivative in (57). The asymptotic analysis as $\delta \to 0$ of the transmission problem depends on the limit of its product $\delta\sigma$. In this talk, to clarify the essence of the dynamic boundary condition, we discuss firstly the existence and uniqueness of the weak solution for (56)–(61), secondly we discuss the asymptotics depending on the limit of $\delta\sigma$, more precisely if we put $\alpha := \lim_{\sigma\to 0} \delta\sigma$ then we see that $\alpha = 0$ corresponds to the problem with Neumann boundary condition $\lambda\partial_{x_N^-} u = g$ on Γ, $\alpha = +\infty$ with the Dirichlet boundary condition $u = \phi$ on Γ, and $\alpha \in (0, +\infty)$ to the dynamic boundary condition $\alpha\partial_t u + \lambda\partial_{x_N^-} u = g$ on Γ, respectively. Finally we obtain the standard regularity results for each target problems from the fundamental way.

[1] P. Colli and J.-F. Rodrigues, Diffusion through thin layers with high specific heat Asymptotic Anal., **3** (1990), 249–263.

[2] Y. Giga, M. Łasica, and P. Rybka, The heat equation with the dynamic boundary condition as a singular limit of problems degenerating at the boundary, Asymptotic Anal., **135** (2023), 303–341.

[CT17] Stationary fronts of lattice Nagumo equation and a functional equation

Jakub Hesoun

University of West Bohemia

Wednesday, 8:45, 1D227

This is a joint work with Petr Stehlík and Jonáš Volek.

We study stationary solutions of a semidiscrete version of reaction diffusion equation with bistable dynamics (Nagumo equation). First, we use a functional form of the mirroring technique to fully characterize equivalence classes of unbounded stationary solutions. We show that solutions which connect a stable fixed point of the nonlinearity with infinity can be characterized by a single parameter from a bounded interval. Additionally, we reveal a natural relationship of lattice equations with an interesting functional equation which involves an unknown function and its inverse. Finally, we show that solutions of such functional equation can describe bounded fronts connecting two stable (not necessarily different) homogeneous equilibria.

[1] J. Hesoun, P. Stehlík, and J. Volek. *Unbounded asymmetric stationary solutions of lattice Nagumo equation.* Qual. Theory Dyn. Syst., 23(2): Paper No. 50, 1-14, 2024.

[2] J. Hesoun, P. Stehlík, and J. Volek. *Mirroring in lattice equations and an interesting functional equation.* Manuscript submitted for publication.

[CT17] A series Evans function approach to locate the point spectrum of traveling wavefronts of reaction-diffusion systems

Tzi-Sheng Yang

Tunghai University

Wednesday, 9:15, 1D227

This is a joint work with Prof Chueh-Hsin Chang (National Chung Cheng University, Chia-Yi, Taiwan) and Prof Cheng-Hsiung Hsu (National Central University, Taoyuan, Taiwan).

It was known that the spectrum of a traveling wave of a reaction-diffusion system is consisted the essential spectrum and the point spectrum. The essential spectrum can be located by the Fredholm borders [5], while the point spectrum is rather difficult to locate. This work focuses on locating the point spectrum of a wavefront. We transform the linearized eigenvalue problem for a wavefront $\psi(\xi)$,

$$\mathcal{L}(\xi)\phi = \lambda\phi, \lambda \in \mathbb{C},$$

into an ODE system with two finite regular singular points. Its fundamental solutions subject to one-sided boundary conditions are derived as infinite series in terms of powers of ξ, whose exponents and leading coefficients correspond to the constant multiples of matrix eigenvalues and the associated eigenvectors for the asymptotic eigenvalue problem, respectively. We prove that the *series* Evans function, which is defined to be the determinant of the above series fundamental solutions evaluate at $\xi = 0$, is analytic on the natural domain excluding the exceptional points. As the zeros of the Evans function are exactly the point spectrum, counting multiplicities [1–4], the number of the point spectrum lying outside the small disks centred at the exceptional points, can be numerically identified by counting the winding numbers of the images of suitable contours mapped by the *series* Evans function. In the mean while, using the perturbation method, the number of the point spectrum lying inside the small disks centred at the exceptional points, can also be numerically identified in a similar way. We apply the *series* Evans function to the two-species Lotka-Volterra competition system, of which several of types wavefronts with exact forms have been derived in [6, 7]. We numerically conclude the spectral stability for some of these wavefronts, with respect to specific asymptotically exponential weighted spaces.

[1] J. W. Evans, Nerve axon equations, I: Linear approximations, *Indiana Univ. Math. J.*, **21** (1972), 877–955.

[2] J. W. Evans, Nerve axon equations, II: Stability at rest, *Indiana Univ. Math. J.*, **22** (1972), 75–90.

[3] J. W. Evans, Nerve axon equations, III: Stability of the nerve impulse, *Indiana Univ. Math. J.*, **22** (1972), 577–593.

[4] J. W. Evans, Nerve axon equations, IV: The stable and the unstable impulse, *Indiana Univ. Math. J.*, **24** (1975), 1169–1190.

[5] D. Henry, *Geometric theory of semilinear parabolic equations*, Lecture Notes in Mathematics, Vol. 840, Springer-Verlag, New York Berlin, 1981.

[6] L.-C. Hung, Exact traveling wave solutions for diffusive Lotka-Volterra systems of two competing species, *Japan J. Indust. Appl. Math.*, **29** (2012), 237–251.

[7] Rodrigo, M. and Mimura, M., Exact solutions of a competition-diffusion system, *Hiroshima Math J.*, **30** (2000), 257-270.

[CT17] Bifurcations of limit cycles in multi-parameter dynamical systems

Valery Gaiko

National Academy of Sciences of Belarus

Wednesday, 9:45, 1D227

Multi-parameter polynomial dynamical systems are considered. To control global bifurcations of limit cycles in planar systems, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the development of new bifurcation-geometric methods based on the Wintner–Perko termination principle [1]. Using these methods, we present, e. g., a solution of Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the Kukles cubic-linear system, the general Liénard polynomial system with an arbitrary number of singular points, the Euler–Lagrange–Liénard mechanical system, Leslie–Gower systems which model the population dynamics in real ecological or biomedical systems, and a reduced planar quartic Topp system which models the dynamics of diabetes. Applying a similar approach, we study also three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenarios in Lorenz type systems connecting globally the Andronov, Shilnikov, homoclinic, period-doubling, period-halving and other bifurcations of limit cycles [2].

[1] V. A. Gaiko, Global bifurcation theory and Hilbert's sixteenth problem. Kluwer, Boston, 2003.

[2] V. A. Gaiko, *Limit cycles of multi-parameter polynomial dynamical systems.* J. Math. Sci. **260** (2022), 662–677.

[CT17] Transverse instability of line periodic waves to the KP-I equation

Wei Lian

Lund University

Wednesday, 10:15, 1D227

This is a joint work with Prof. Erik Wahlén (Lund University, Lund, Sweden).

The passage from linear instability to nonlinear instability has been shown for 1D solitary waves under 2D perturbations. Although transverse instability of periodic waves to the KdV equation under the KP-I flow has been expected to be true from spectral instability for a long time, it has not been clear how to adapt the general instability theory for solitary waves to periodic waves until now. In this talk, we present how such an adaptation works with the aid of exponential trichotomies and multivariable Puiseux series.

[CT18] Fine Boundary Regularity For The Fractional (p,q)-Laplacian

Ritabrata Jana

IISER Thiruvananthapuram

Wednesday, 15:15, 1D236

This is a joint work with Dr. R. Dhanya (IISER Thiruvananthapuram, Thiruvananthapuram, India), Dr. Uttam Kumar (IIT Kanpur, Kanpur, India), Dr. Sweta Tiwari (IIT Guwahati, Guwahati, India).

In this talk, we deal with the fine boundary regularity, a weighted Holder regularity of weak solutions up to the boundary, to the problem involving the fractional (p, q)-Laplacian denoted by

$$\begin{aligned} (-\Delta)_p^s u + (-\Delta)_p^s u &= f(x) \quad \text{in } \Omega \\ u &= 0 \qquad\; \text{in } \mathbb{R}^N \setminus \Omega; \end{aligned}$$

where Ω is a $C^{1,1}$ domain, and $2 \leq p \leq q < \infty$. For $0 < s < 1$, and the bounded non-negative data $f \in L^\infty(\Omega)$, $f \geq 0$, we employ the nonlocal analogue of the boundary Harnack method to establish that $u/d_\Omega^s \in C^\alpha(\bar{\Omega})$ for some $\alpha \in (0, 1)$ and d_Ω is the distance from the boundary. A novel barrier construction allows us to analyse the regularity theory even in the absence of the scaling or the homogeneity properties of the operator. Additionally, we extend our discussion on the fine boundary regularity for bounded data $f \in L^\infty(\Omega)$, specifically when $s \in (0, 1/p)$.

[CT18] Fractional Dynamics: Existence, Inequalities and Applications in Difference Equations

Nick Fewster-Young

University of South Australia (Adelaide University)

Wednesday, 15:45, 1D236

This talk explores fractional dynamical systems in the discrete setting, and presents very specific results to some proposed adapted fractional versions of some Painlevé and Ricker Equations. The works build on previous works with Prof Martin Bohner (Missouri University, Rolla, USA) and Jagan Jonnaalagadda (BITS Pilani, Hyperbad, India). The motivation for studying these well-studied dynamical systems stems from both the applications and recent heighten interest in the community. The applications of these equations are in geometric groups and walks in lattice structures, nonlinear waves, quantum field theory, general relativity, soliton theory, and Ricker equations being well studied in understanding stock and population models in fisheries.

The aim is to further understand the fractional versions of these equations, and more broadly, the classical problems and the class of these types of equations. With these goals at the focus, the existence of solutions and the bounded nature of the solutions are the main interest initially. Since, once the existence and bounds of the possible solutions are established then further qualitative analysis can be performed and numerical analysis as well, which will motivate others to explore these particular problems in their fields. The methods implemented here are based on nonlinear topological methods, proposed inequalities and initial behaviour of the discrete equations. The topological methods are well known in the community such as Brouwer fixed point theorem and an adaptation of a classical result (inequality) presented firstly by Hartman (1964) and adapted by Fewster-Young. Briefly, in conclusion, the analogous results in the continuous deterministic will be presented for comparison and future works.

[1] Bohner, M., Fewster-Young, N. *Discrete Fractional Boundary Value Problems and Inequalities.* Fract Calc Appl Anal **24**, (2021), 1777–1796.

[2] Fewster-Young, N, Tisdell C.C, *Existence of solutions to derivative-dependent, nonlinear singular boundary value problems.* Applied Mathematics Letters, **28**, (2014) 42-46.

[3] Fewster-Young, N. *Existence results for Caputo Fractional Boundary value problems with Unrestricted growth conditions.* Differential Equations and Applications, **15**, no 2 (2023), 135-146.

[4] P. Hartman, Ordinary differential equations. John Wiley, New York, 1964.

[CT19] Boundary layers of the Boltzmann equation for polyatomic gases

Niclas Bernhoff

Karlstad University

Wednesday, 15:15, 1D237

Half-space problems in the kinetic theory of gases are of great importance in the study of the asymptotic behavior of solutions of boundary value problems for the Boltzmann equation for small Knudsen numbers. They provide the boundary conditions for the fluid-dynamic-type equations and Knudsen-layer corrections to the solution of the fluid-dynamic-type equations in a neighborhood of the boundary. These problems are well-studied for monatomic species, especially for single, but to some extent also for multicomponent, gases. It is well-known that the number of additional conditions needed to be imposed depends on different regimes for the Mach number (corresponding to subsonic/supersonic evaporation/condensation). The case of polyatomic species is not as well studied in the literature.

In this talk, we will discuss some extensions of the results for linearized half-space problems for monatomic gases to the case of polyatomic gases.

[CT19] One dimensional inelastic Boltzmann equation - self-similarity for moderately hard potentials

Sebastian Throm
Umeå University

Wednesday, 15:45, 1D237

Inelastic interactions in granular systems appear frequently in natural processes. They are characterised by a loss of kinetic energy during collision. Mathematically, such systems can often be described by a modification of Boltzmann's equation. In this talk, we consider the one-dimensional version of this model for moderately hard potentials, i.e. for collision kernels $|\cdot|^{\gamma}$ with $\gamma \ll 1$. More precisely, we study the behaviour of self-similar solutions for $\gamma \to 0$. In particular, we discuss questions like regularity and uniqueness of self-similar profiles. Our analysis relies on a spectral gap for the linearised equation and a perturbation argument from the case of Maxwell molecules.

This is joint work with Ricardo Alonso (Texas A&M University at Qatar, Doha, Qatar), Véronique Bagland (Université Clermont Auvergne, Aubière Cedex, France), José Cañizo (Universidad de Granada, Granada, Spain) and Bertrand Lods (Università degli Studi di Torino & Collegio Carlo Alberto, Torino, Italy)

[CT21] On the singular limit problem for a class of first order nonlocal lane-changing traffic flow models

Felisia Angela Chiarello
University of L'Aquila

Wednesday, 15:15, 1D327

In this talk, we will present a nonlocal-to-local convergence result with applications to traffic models. In particular, we will consider a system of two (or more) nonlocal balance laws characterized by lane-changing functions on the right-hand sides and exponential kernels in the nonlocal terms of the flux functions, i.e.

$$\begin{aligned} \partial_t \boldsymbol{\rho}^1(t,x) + \partial_x\Big(V_1(\mathcal{W}[\boldsymbol{\rho}^1](t,x))\boldsymbol{\rho}^1(t,x)\Big) &= S\big(\boldsymbol{\rho}(t,x), \mathcal{W}[\boldsymbol{\rho}](t,x), x\big) && (t,x) \in (0,T)\times\mathbb{R} \\ \partial_t \boldsymbol{\rho}^2(t,x) + \partial_x\Big(V_2(\mathcal{W}[\boldsymbol{\rho}^2](t,x))\boldsymbol{\rho}^2(t,x)\Big) &= -S\big(\boldsymbol{\rho}(t,x), \mathcal{W}[\boldsymbol{\rho}](t,x), x\big) && (t,x) \in (0,T)\times\mathbb{R} \\ \boldsymbol{\rho}(0,x) &= \boldsymbol{\rho}_0(x) && x \in \mathbb{R} \\ \mathcal{W}[\rho](t,x) &= \tfrac{1}{\eta}\int_x^{\infty} \exp\left(\tfrac{x-y}{\eta}\right)\rho(t,y)\,\mathrm{d}y && (t,x) \in (0,T)\times\mathbb{R} \end{aligned}$$

with the nonlocal operator $\mathcal{W}$ defined for $\rho \in C([0,T]; L^1_{\text{loc}}(\mathbb{R})) \cap L^\infty((0,T); L^\infty(\mathbb{R}))$,

$$\mathcal{W}[\boldsymbol{\rho}](t,x) = (\mathcal{W}[\boldsymbol{\rho}^1], \mathcal{W}[\boldsymbol{\rho}^2])(t,x)$$

and $\boldsymbol{\rho} = (\boldsymbol{\rho}^1, \boldsymbol{\rho}^2)$. Here, the exponential kernels are approximations of the Dirac distribution and the coupling between the equations of the system is only through the right-hand sides. We will analytically prove that the solution of the nonlocal system converges to the solution of the corresponding local one when the kernels of the nonlocal terms approach the Dirac delta. Numerical illustrations supporting the main results will be also shown.

Acknowledgments: F.A.C. is member of Gruppo Nazionale per l'Analisi Matematica, la Probabilit'a e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM). F.A.C. was partially supported by 'INdAM-GNAMPA Project', code CUP_E53C22001930001.

[1] F. A. Chiarello and A. Keimer, On the singular limit problem for a class of first order nonlocal lane-changing traffic flow models. *Preprint.* (2023).

[CT21] Traffic and Pedestrian Flow Models: Micro and Macro Approaches

Massimiliano Daniele Rosini

d'Annunzio University of Chieti-Pescara

Wednesday, 15:45, 1D327

This is a joint work with B. Andreianov (Université de Tours, Université d'Orléans, Tours, France,

and Peoples' Friendship University of Russia (RUDN University), Moscow, Russian Federation), M. Di Francesco (University of L'Aquila, L'Aquila, Italy), S. Fagioli (University of L'Aquila, L'Aquila, Italy), G. Russo (University of Catania, Catania, Italy), and G. Stivaletta (University of L'Aquila, L'Aquila, Italy).

This presentation examines the relation between deterministic particle systems and 1D conservation laws, focusing on their applications in traffic flow and pedestrian modeling. We show how systems of ordinary differential equations representing follow-the-leader dynamics can be used to approximate conservation laws.

We start with the Cauchy problem for the Lighthill-Whitham-Richards (LWR) model for traffic flow. We highlight then how this model has been extended to initial-boundary value problems, demonstrating its flexibility and relevance for various traffic scenarios.

Further, we explore additional models such as the Aw-Rascle-Zhang (ARZ) model for traffic and the Hughes model for pedestrian dynamics. The ARZ model is interpreted as a generalisation of the LWR model to a multi-population traffic, while the Hughes model addresses of complexities of choosing between two exits.

The aim of this talk is to present an overview of how deterministic particle systems can provide valuable insights into the macroscopic modelling of traffic and pedestrian flows.

[1] B. Andreianov, M.D. Rosini, G. Stivaletta, On existence, stability and many-particle approximation of solutions of 1D Hughes model with linear costs, J. Differential Equations, 369, 253-298, 2023

[2] M. Di Francesco, S. Fagioli, M.D. Rosini, Many particle approximation of the Aw-Rascle-Zhang second order model for vehicular traffic, Math. Biosci. Eng., 14(1), 127-141 2017

[3] M. Di Francesco, S. Fagioli, M.D. Rosini, G. Russo, Follow-the-leader approximations of macroscopic models for vehicular and pedestrian flows, Active Particles, Volume 1, Springer International Publishing, Cham, 333-378 2017

[4] M. Di Francesco, S. Fagioli, M.D. Rosini, G. Russo, Deterministic particle approximation o the Hughes model in one space dimension, Kinet. Relat. Models, 10(1), 215-237, 2017

[5] M. Di Francesco, M.D. Rosini, Rigorous derivation of nonlinear scalar conservation laws from follow-the-leader type models via many particle limit, Arch. Ration. Mech. Anal., 217(3), 831-871 2015

[CT20] Homogenization of electro-diffusive transport for multiple species in a heterogeneous medium

Apratim Bhattacharya

Umeå University

Thursday, 8:45, 1B306

Electro-diffusion is a transport phenomenon of charged particles due to diffusion and the electric force induced by the charges of the particles. This talk is devoted to the homogenization of the electro-diffusion process in a domain with microscopic heterogeneities. Such events take place naturally in the physical world and engineering sciences, such as ion transport through biological ion channels of the cell membrane, transport in rocks and water purification. Our main contribution is that we consider the transport of multiple species. It is known that there exists a nonnegative energy functional which decreases in time along the solutions of the system. This fact serves as a starting point towards obtaining the estimates for the solutions uniformly with respect to the microscopic scale of the domain. Finally, the homogenized equations have been derived using the two-scale convergence method.

[1] A. Bhattacharya, M. Gahn and M. Neuss-Radu, *Homogenization of a nonlinear drift-diffusion system for multiple charged species in a porous medium* Nonlinear Anal. Real World Appl. **68** (2022), 103651, 28 pp.

[CT20] A homogenization approach to the effects of surfactant concentration and interfacial slip on the flow past viscous drops

Hari Shankar Mahato

IIT Kharagpur

Thursday, 9:15, 1B306

This is a joint work with Prof. G. P. Raja Sekhar (Department of Mathematics, IIT Kharagpur, 721302, India).

We are interested in the case of arbitrary Stokes flow past viscous drops. We are motivated by the motion of a viscous spherical drop whose interface is covered with a stagnant layer of surfactant in an arbitrary unsteady Stokes flow. The existing literature covers only the case of axisymmetric steady Stokes flow past a drop for high Péclet number when the arbitrary cap angle is considered (see [1]). The arbitrary Stokes flow case is considered in [2], but they restrict the flow to be steady, and assumed the surfactant coating the whole interface. Moreover, in the above studies, the authors have not considered the interfacial slip effect. In our present case, we attempt a more generalized problem of an arbitrary transient Stokes flow past drops for low surface Péclet number. We assume that the drop is covered with the surfactant layer, so that an arbitrary cap angle is considered. Also, we take into account the effect of interfacial slip on the flow. The objective of our present paper is to analyze the behavior of the flow on drops when the interfacial slip effect and the surfactant concentration effect occur for low surface Péclet numbers. We use slip boundary conditions to see the effect of interfacial slip on the flow behavior. Since the drops are present in a fluid flow which obeys the Stoke's law, we assume it to be periodically arranged in such a way that for any scale parameter $0 < \varepsilon \ll 1$, the drops are $\varepsilon-$distance apart. This gives the flexibility to view the problem in the micro-macro scale setting. The microscopic description will give us precisely an insight into the processes that are happening with the drops where as their macroscopic description will help us to obtain an averaged behavior of the model and other physcial parameters involved in the process. The modeling of the physical processes leads to a system of parabolic PDEs with interfacial boundary conditions. In this work, we study the existence of solution and homogenisation for such a system.

[1] O.S. Pak, J. Feng, and H.A. Stone, Viscous Marangoni migration of a drop in a poiseuille flow at low surface Peclet numbers. J. Fluid Mech., **753** (2014), 535–552.

[2] S.S. Sadhal and R.E. Johnson, *Stokes flow past bubbles and drops partially coated with thin films. part 1. stagnant cap of surfactant film-exact solution.* J. Fluid Mech., **126** (1983), 237–250.

[CT20] Model Reduction of Complex Systems

Hong Duong

University of Birmingham

Thursday, 9:45, 1B306

Complex systems in nature and in applications (such as molecular systems, crowd dynamics, swarming, opinion formation, just to name a few) are often described by systems of stochastic differential equations (SDEs) and partial differential equations (PDEs). It is often analytically impossible or computationally prohibitively expensive to deal with the full models due to their high dimensionality (degrees of freedom, number of involved parameters, etc.). It is thus of great importance to approximate such large and complex systems by simpler and lower dimensional ones, while still preserving the essential information from the original model. This procedure is referred to as model reduction or coarse-graining in the literature. In this talk, I will present methods for qualitative and quantitative coarse-graining of several SDEs and PDEs, in the presence or absence of a scale-separation.

This talk is based on my collaborative works [1]- [4].

[1] M. H. Duong, A. Lamacz, M. A. Peletier, A. Schlichting, and U. Sharma. *Quantification of coarse-graining error in Langevin and overdamped Langevin dynamics.* Nonlinearity, 31(10), pp. 4517-4566, 2018.

[2] M. Colangeli, M. H. Duong, A. Muntean. *A reduction scheme for coupled Brownian harmonic oscillators.* Journal of Physics A: Mathematical and Theoretical, volume 55, number 50, pages 505002, 2022.

[3] M. Colangeli, M. H. Duong, and A. Muntean. *Model reduction of Brownian oscillators: quantification of errors and long-time behaviour.* Journal of Physics A: Mathematical and Theoretical, 56 345003, 2023.

[4] M. H. Duong and H. D. Nguyen. *Asymptotic analysis for the generalized Langevin equation with singular potentials.* arXiv:2305.03637, 2023.

[CT20] From nonlocal to local phase field system with inertial term

Shunsuke Kurima

Tokyo University of Science

Thursday, 10:15, 1B306

We consider the local problem

$$\begin{cases} \theta_t + \varphi_t - \Delta\theta = f & \text{in } \Omega \times (0,T), \\ \varphi_{tt} + \varphi_t - \Delta\varphi + \beta(\varphi) + \pi(\varphi) = \theta & \text{in } \Omega \times (0,T), \\ \partial_\nu \theta = \partial_\nu \varphi = 0 & \text{on } \partial\Omega \times (0,T), \\ \theta(0) = \theta_0,\ \varphi(0) = \varphi_0,\ \varphi_t(0) = v_0 & \text{in } \Omega, \end{cases} \tag{P}$$

and the corresponding family of nonlocal problems

$$\begin{cases} (\theta_\varepsilon)_t + (\varphi_\varepsilon)_t - \Delta\theta_\varepsilon = f & \text{in } \Omega \times (0,T), \\ (\varphi_\varepsilon)_{tt} + (\varphi_\varepsilon)_t + a_\varepsilon\varphi_\varepsilon - J_\varepsilon * \varphi_\varepsilon + \beta(\varphi_\varepsilon) + \pi(\varphi_\varepsilon) = \theta_\varepsilon & \text{in } \Omega \times (0,T), \\ \partial_\nu \theta_\varepsilon = 0 & \text{on } \partial\Omega \times (0,T), \\ \theta_\varepsilon(0) = \theta_{0,\varepsilon},\ \varphi_\varepsilon(0) = \varphi_{0,\varepsilon},\ (\varphi_\varepsilon)_t(0) = v_{0,\varepsilon} & \text{in } \Omega, \end{cases} \tag{P$_\varepsilon$}$$

where Ω is a bounded domain in $\mathbb{R}^d$ ($d \in \{1,2,3\}$) with smooth boundary $\partial\Omega$, $T > 0$, $\varepsilon \in (0,1)$, J_ε is a kernel function, $a_\varepsilon(\cdot) := \int_\Omega J_\varepsilon(\cdot - y)\,dy$, $(J_\varepsilon * \varphi_\varepsilon)(\cdot) := \int_\Omega J_\varepsilon(\cdot - y)\varphi_\varepsilon(y)\,dy$, β is a maximal monotone function, π is a Lipschitz continuous function, f, θ_0, φ_0, v_0, $\theta_{0,\varepsilon}$, $\varphi_{0,\varepsilon}$, $v_{0,\varepsilon}$ are given functions. There exists a unique solution of $(\mathrm{P})_\varepsilon$ ([4]).

Davoli–Scarpa–Trussardi [2,3] and Abels–Terasawa [1] have studied nonlocal-to-local convergence of Cahn–Hilliard equations. On the other hand, regarding phase field systems, in the case of a conserved phase field system related to entropy balance, nonlocal-to-local convergence has already been confirmed ([5]).

In this talk we verify that we can prove existence of weak solutions to 69 by passing to the limit in $(\mathrm{P})_\varepsilon$ as $\varepsilon \searrow 0$. This is a joint work with Professors Pierluigi Colli (University of Pavia) and Luca Scarpa (Polytechnic University of Milan).

[1] H. Abels, Y. Terasawa, *Convergence of a nonlocal to a local diffuse interface model for two-phase flow with unmatched densities.* Discrete Contin. Dyn. Syst. Ser. S **15** (2022), 1871–1881.

[2] E. Davoli, L. Scarpa, L. Trussardi, *Nonlocal-to-local convergence of Cahn-Hilliard equations: Neumann boundary conditions and viscosity terms.* Arch. Ration. Mech. Anal. **239** (2021), 117–149.

[3] E. Davoli, L. Scarpa, L. Trussardi, *Local asymptotics for nonlocal convective Cahn-Hilliard equations with $W^{1,1}$ kernel and singular potential.* J. Differential Equations **289** (2021), 35–58.

[4] S. Kurima, *Time discretization of a nonlocal phase-field system with inertial term.* Matematiche (Catania) **77** (2022), 47–66.

[5] S. Kurima, *Nonlocal to local convergence of singular phase field systems of conserved type.* Adv. Math. Sci. Appl. **31** (2022), 481–500.

[CT22] A self-similar solution to time-fractional Stefan problem

Adam Kubica

Warsaw University of Technology

Thursday, 8:45, 1D227

This is a joint work with Dr Katarzyna Ryszewska (Warsaw University of Technology).

We derive the fractional version of one-phase one-dimensional Stefan model, where we assume that the diffusive flux q^* is given by the time-fractional Riemann-Liouville derivative, i.e.

$$q^*(x,t) = -\partial^{1-\alpha} u_x(x,t), \text{ where } \alpha \in (0,1).$$

Next, we construct a self-similar solutions of this problem and prove its convergence as $\alpha \to 1$ to the self-similar solution of the classical Stefan problem.

[CT22] Inverse problems for semilinear elliptic PDE with a general nonlinearity $a(x,u)$

David Johansson

University of Jyväskylä

Thursday, 9:15, 1D227

This is a joint work with Janne Nurminen (University of Jyväskylä, Jyväskylä, Finland) and Professor Mikko Salo (University of Jyväskylä, Jyväskylä, Finland).

In this talk I present results on the inverse boundary value problem of determining the function $a(x,u)$ in the semilinear equation $\Delta u + a(x,u(x)) = 0$ from the Cauchy data of the solutions.

The inverse problem is solved by a method based on reduction to an inverse problem for the linearized equation. The reduced problem is solved by standard results on the Calderón problem. The full nonlinearity $a(x,u)$ is then identified by a Runge approximation argument. This improves on existing results primarily by removing the well-posedness assumption in the first linearization method, and by avoiding the real-analytic smoothness required for the so-called higher linearization method.

[1] D. Johansson, J. Nurminen, and M. Salo, *Inverse problems for semilinear elliptic PDE with a general nonlinearity* $a(x,u)$. arXiv preprint arXiv:2312.12196.

[CT22] On a two-scale PDE system with a free boundary

Kota Kumazaki

Kyoto University of Education

Thursday, 9:45, 1D227

This is a joint work with Prof. Adrian Muntean (Karlstad University, Sweden).
In this talk, we consider a two-scale PDE system with a free boundary. This system consists of a parabolic equation in a domain of $\mathbb{R}^3$ and a free boundary problem in a one dimensional halfline which is studied in [1, 2]. This system is given as a mathematical model describing swelling phenomena in porous materials, and coupled a diffusion of the relative humidity in the whole materials with the water-swelling process in each microscopic pocket inside of the materials. In this talk, we discuss the existence and uniqueness of a solution to the system and investigate the large-time behavior of a solution as time goes to infinity.

[1] Kota Kumazaki, Adrian Muntean, *Local weak solvability of a moving boundary problem describing swelling along a halfline*, vol. 14 (2019), no.3, 445-469.

[2] Kota Kumazaki, Adrian Muntean, *Global weak solvability, continuous dependence on data, large time growth of swelling moving interfaces*, Interfaces and Free boundaries, vol. 22 (2020), no.1, 27-49.

[CT22] Complete Classification of Local Conservation Laws for Generalized Cahn–Hilliard–Kuramoto–Sivashinsky Equation

Pavel Holba

Silesian University in Opava

Thursday, 10:15, 1D227

In this talk we consider the following PDE in $n+1$ independent variables $t, x_1 \ldots, x_n$ and one dependent variable u,

$$u_t = a\Delta^2 u + b(u)\Delta u + f(u)|\nabla u|^2 + g(u), \tag{62}$$

to which we refer as to the generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation, as it is a natural generalization of nonlinear multidimensional Cahn–Hilliard and Kuramoto–Sivashinsky equations that have many important applications in physics, chemistry, and biology. Here b, f, g are arbitrary smooth functions of the dependent variable u, a is a nonzero constant, n is a natural number, $\Delta = \sum_{i=1}^{n} \partial^2/\partial x_i^2$ is the Laplace operator and $|\nabla u|^2 = \sum_{i=1}^{n} (\partial u/\partial x_i)^2$.

For an arbitrary natural number n of spatial independent variables we present a complete list of cases when equation (62) admits nontrivial local conservation laws of any order, and for each of those cases we give an explicit form of all the local conservation laws of all orders modulo trivial ones admitted by the equation under study.

As a consequence of the above general result, we show that the Kuramoto–Sivashinsky equation,

$$u_t = -(\Delta^2 u + \Delta u + |\nabla u|^2/2),$$

admits no nontrivial local conservation laws, and list all of the inequivalent nontrivial local conservation laws for the Cahn–Hilliard equation

$$u_t = c_1 \Delta(u^3 - u + c_2 \Delta u),$$

where c_1 and c_2 are arbitrary nonzero constants.

[1] P. Holba, *Complete classification of local conservation laws for generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation.* Stud. Appl. Math. **151** (2023), 171–182.

[CT23] Positive solutions for logistic type elliptic equation with a non Lipschitz boundary condition arising in coastal fishery harvesting

Kenichiro Umezu

Ibaraki University

Thursday, 15:15, 1D341

In this talk, for bifurcation analysis we study the positive solution set for a semilinear elliptic equation of the logistic type, equipped with a sublinear boundary condition modeling coastal fishery harvesting ([1]). Hopf's boundary point lemma is violated in the presence of the sublinear boundary condition with non Lipschitz nonlinearity. Because this nonlinear boundary condition is not right–differentiable at zero, we exploit a sort of non standard bifurcation technique with *regularization* at zero to evaluate the positive solution set depending on a parameter. First, certain results for a *non resonant* case are obtained, including the existence, uniqueness, and multiplicity of positive solutions ([2,3]). Next, the resonant case is analyzed on the basis of the orthogonal decomposition of positive solutions by a positive eigenfunction of an associated Dirichlet eigenvalue problem.

[1] D. Grass, H. Uecker, T. Upmann, Optimal fishery with coastal catch, Nat. Resour. Model. **32** (2019), e12235, 32 pp.

[2] K. Umezu, Logistic elliptic equation with a nonlinear boundary condition arising from coastal fishery harvesting, Nonlinear Anal. Real World Appl. **70** (2023), No. 103788.

[3] K. Umezu, Logistic elliptic equation with a nonlinear boundary condition arising from coastal fishery harvesting II, J. Math. Anal. Appl. **534** (2024), No. 128134.

[CT23] Parabolic scaling on time evolution of the incompressible Navier–Stokes flow

Masakazu Yamamoto

Niigata University

Thursday, 15:45, 1D341

In this talk, the asymptotic expansion with high-order of incompressible Navier–Stokes flow in whole space is discussed. This expansion draws large-time behavior of solutions. For this theme, Carpio [1] and Fujigaki and Miyakawa [2] developed the basic expansion. It is expected that the renormalization yields higher-order expansions. Here an approximation with double precision is illustrated. Especially any terms on this expansion are arranged according to the parabolic scaling. Namely the velocity u is approximated by $u(t) \sim \sum_{m=1}^{2n} U_m(t) + \sum_{m=n+1}^{2n} K_m(t) \log t$ as $t \to +\infty$ for some smooth functions U_m and K_m which satisfy $\lambda^{n+m}(U_m, K_m)(\lambda^2 t, \lambda x) = (U_m, K_m)(t, x)$ for $\lambda > 0$, where n is the space-dimension. The constituents with $m \geq n+1$ are newly added and the logarithmic evolutions describe the characteristic nonlinearity of Navier–Stokes flow. To introduce the concrete expansion, the idea via Kukavica and Reis [3] is employed.

Acknowledgments: This research was funded by JSPS KAKENHI Grand Number : 19K03560.

[1] Carpio, A., SIAM J. Math. Anal. **27** (1996), 449–475.

[2] Fujigaki, Y., Miyakawa, T., SIAM J. Math. Anal. **33** (2001), 523–544.

[3] Kukavica, I., Reis, E., J. Differential Equations **250** (2011), 607–622.

[CT23] Periodic solutions to second order nonlinear quasi-variational evolution inclusions

Noriaki Yamazaki

Kanagawa University

Thursday, 16:15, 1D341

This is a joint work with Prof. Ken Shirakawa (Chiba University, Japan).

In this take we consider a time-periodicity problem for a class of second order nonlinear quasi-variational evolution inclusions in Banach spaces of the form

$$\begin{cases} u''(t) + \partial_* \psi^t(u'(t)) + \partial_* \varphi^t(\theta; u(t)) \ni f(t) \text{ in } V^*, \ 0 < t < T_0, \quad \theta = \Lambda u, \\ u(0) = u_0, \quad u'(0) = u'(T_0), \end{cases} \tag{63}$$

where V is a Banach space, V^* is the dual space of V, and H is a (real) Hilbert space, such that V is dense and compactly embedded in H; in this case we have the usual triplet $V \subset H \subset V^*$. In addition, $T_0 > 0$ is a fixed time-period, $u' = \frac{du}{dt}$ in V, f is a given V^*-valued function, and

$u_0 \in V$ is a given initial datum.

In the formulation (63), we introduce a space Θ of parameters θ, which is a complete metric space with metric $d_\Theta(\cdot,\cdot)$, and a family $\{\varphi^t(\theta; z)\}_{t\in[0,T_0]}$ of proper, l.s.c., and convex functions with respect to z on V is associated with each $\theta \in \Theta$. Similarly, $\{\psi^t(z)\}_{t\in[0,T_0]}$ denotes a family of proper, l.s.c., and convex functions on V. In addition, we denote by $\partial_*\psi^t(z)$ and $\partial_*\varphi^t(\theta; z)$ the subdifferentials of $\psi^t(z)$ and $\varphi^t(\theta; z)$ from V into V^* with respect to z, respectively, and Λ is a mapping from $D(\Lambda) \subset L^2(0, T; V)$ into Θ. Note that (63) is a quasi-variational evolution inclusions, since $\varphi^t(\theta; \cdot)$ is a unknown-dependent convex function because of $\theta = \Lambda u$.

In this talk, assuming the appropriate T_0-periodicity of assumptions on functions $\psi^t(\cdot)$ and $\varphi^t(\theta; \cdot)$, we shall give an abstract existence result of T_0-periodic solutions to (63) by using the fixed-point arguments and the known result obtained in [2, Proposition 4.1]. Moreover, we provide application examples for quasi-variational inequality with time-periodic constraints.

[1] N. Kenmochi, *Periodic quasi-variational inequalities of parabolic type and applications*, Adv. Math. Sci. Appl., 33 (2024), 283–301.

[2] N. Kenmochi, K. Shirakawa, and N. Yamazaki, *Second order doubly nonlinear evolution inclusions –quasi-variational approach–*, Discrete Contin. Dyn. Syst. Ser. S, 17 (2024), 304–325.

[CT23] A comparison of solutions of two convolution-type unidirectional wave equations

Saadet Erbay

Ozyegin University

Thursday, 16:45, 1D341

This is a joint work with Prof Husnu Ata Erbay (Ozyegin University, Istanbul, Turkey) and Prof Albert Erkip (Sabanci University, Istanbul, Turkey).

In this work, we prove a comparison result for a general class of nonlinear dispersive unidirectional wave equations:

$$u_t + \alpha * \left(u + u^{p+1}\right)_x = 0, \tag{64}$$

where $u = u(x,t)$ is a real-valued function, p is a positive integer, $\alpha(x)$ is a general kernel function and the symbol $*$ denotes convolution in the x-variable. The dispersive nature of the linear part of (64) is due to the convolution integral in space. For two specific choices of the kernel function, the Benjamin-Bona-Mahony equation [1] and the Rosenau equation [2] that are particularly suitable to model water waves and elastic waves, respectively, are two members of the class. We first prove an energy estimate for the Cauchy problem of the nonlocal unidirectional wave equation. Then, for the same initial data, we consider two distinct solutions corresponding to two different kernel functions. Our main result is that the difference between the solutions remains small in a suitable Sobolev norm if the two kernel functions have similar dispersive characteristics in the long-wave limit. As a sample case of this comparison result, we provide the approximations to the hyperbolic conservation law. (See [3] for more detailed results.)

[1] T. B. Benjamin, J. L. Bona and J. J. Mahony, *Model equations for long waves in nonlinear dispersive systems*, Philos. Trans. Roy. Soc. A **272** (1972), 47–78.

[2] P. Rosenau, *Dynamics of dense discrete systems: High order effects*, Prog. Theor. Phys. **79** (1988), 1028–1042.

[3] H.A. Erbay, S. Erbay and A. Erkip, *A comparison of solutions of two convolution-type unidirectional wave equations*, Appl. Anal. **114** (2022), 4422–4431.

[CT24] Existence of weak solutions to a Baer–Nunziato type system

Martin Kalousek

Institute of Mathematics, Czech Academy of Sciences

Thursday, 15:15, 1B306

This is a joint work with Šárka Nečasová (Institute of Mathematics, Czech Academy of Sciences). In this talk, a dissipative version of a compressible one velocity Baer–Nunziato type system for a mixture of two compressible heat conducting gases is considered. The complete existence proof for weak solutions to this system was addressed as an open problem in [2, Section 5]. The purpose of the talk is the presentation of most essential elements of the proof of the global in time existence of weak solutions to the one velocity Baer–Nunziato type system for arbitrary large initial data. Namely, the attention will be focused on the following three steps:

1. Transformation of the given system into a new one which possesses the Navier-Stokes-Fourier structure;
2. Showing the existence of weak solutions of the new system by an adaptation of the approach used in the existence theory for the compressible Navier–Stokes–Fourier equations which is presented in [1];
3. Showing the existence of a weak solution to the original one velocity Baer–Nunziato system using the almost uniqueness property of renormalized solutions to pure transport equations.

[1] E. Feireisl, A. Novotný: Singular limits in thermodynamics of viscous fluids. Advances in Mathematical Fluid Mechanics. Birkhäuser Verlag, Basel, (2009).

[2] Y.-S. Kwon, A. Novotný, C.H. Arthur Cheng: *On weak solutions to a dissipative Baer–Nunziato–type system for a mixture of two compressible heat conducting gases*, Math. Models Methods Appl. Sci. **30** (2020) no. 8, 1517–1553.

[CT24] Validity of the nonlinear Schrödinger approximation for a general class of dispersive quasilinear systems with quadratic nonlinearities

Wolf-Patrick Düll

Stuttgart

Thursday, 15:45, 1B306

This is a joint work with Franz Schewe (Universität Stuttgart, Stuttgart, Germany).

In [1], the validity of the nonlinear Schrödinger approximation was proven for the two-dimensional water wave problem with and without surface tension by error estimates being valid over a physically relevant timespan and uniform with respect to the strength of the surface tension, as the surface tension goes to zero. As a central step towards establishing a corresponding approximation result for the three-dimensional water wave problem we present the justification of the nonlinear Schrödinger approximation for a large class of dispersive systems with quadratic nonlinearities. Our proof relies on a new and systematic procedure of constructing energies for the error estimates taking advantage of the Hamiltonian structure of the quasilinear terms of the systems.

[1] W.-P. Düll. *V*alidity of the nonlinear Schrödinger approximation for the two-dimensional water wave problem with and without surface tension in the arc length formulation. Arch. Ration. Mech. Anal. **239** (2021), no. 2, 831–914.

[2] W.-P. Düll, F. Schewe. *V*alidity of the nonlinear Schrödinger approximation for a general class of dispersive quasilinear systems with quadratic nonlinearities. Preprint, 2024.

[CT24] Steady states for the Gierer-Meinhardt systems in unbounded domains

Marius Ghergu

Univerity College Dublin

Thursday, 16:15, 1B306

The Gierer-Meinhardt system was introduced in 1972 as a mathematical model for pattern formation. It assumes the existence of two antagonist chemicals: a short-range autocatalytic activator on one hand, and a long-range, cross-catalytic inhibitor on the other hand. Mathematically, the model is described by two coupled reaction-diffusion equations as follows:

$$\begin{cases} u_t - \Delta u = \dfrac{u^p}{v^q} + \rho(x)\,, u > 0 & \text{in } \Omega, \\ v_t - \Delta v = \dfrac{u^m}{v^s}\,, v > 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \tag{65}$$

where the unknowns u and v represent the densities of the two chemicals introduced above. In the present talk, various results related to existence, nonexistence, asymptotic behaviour of solutions to (65) in case of unbounded domains Ω are presented. These were recently obtained in [1, 2] and rely on integral representation formulae, properties of Riesz and other integral operators, fixed point theorems.

[1] M. Ghergu, *Steady state solutions for the Gierer-Meinhardt system in the whole space.* J. Differential Equations **363** (2023), 518–545.

[2] M. Ghergu, *The stationary Gierer-Meinhardt system in the upper half-space: existence, nonexistence and asymptotics.* Math. Ann, in press 2024.

[CT24] Stability of equilibria to generalized Navier-Stokes-Fourier system

Petr Kaplicky

Charles University

Thursday, 16:45, 1B306

This is a joint work with Prof Anna Abbatiello (University of L'Aquila, L'Aquila, Italy) and Prof Miroslav Bulíček (Charles University, Prague, Czech Republic).

In a given domain we consider a generalized Newtonian incompressible heat conducting fluid with prescribed nonuniform temperature on the boundary of the domain and with the no-slip boundary conditions for the velocity. We study stability of equilibria if no external body forces are applied to the fluid. In dependence on the growth of the constitutively determined part of the Cauchy stress we identify different classes of proper solutions that converge to the equilibrium exponentially in a suitable metric. Consequently, the equilibrium is nonlinearly stable and attracts all weak solutions from these classes. We also show that these classes of solutions are nonempty.

[1] A. Abbatiello, M. Bulíček and P. Kaplický, *On the exponential decay in time of solutions to a generalized Navier-Stokes-Fourier system* J. Differential Equations **379** (2024), 762–793.

[2] A. Abbatiello, M. Bulíček and P. Kaplický, *On solutions for a generalized Navier-Stokes-Fourier system fulfilling the entropy equality* Philos. Trans. Roy. Soc. A **380** (2022).

[CT25] Possible points of blow-up in a chemotaxis system with spatially heterogeneous logistic term

Masaaki Mizukami

Kyoto University of Education

Thursday, 15:15, 1D327

This is a joint work with Dr. Mario Fuest (Leibniz Universität Hannover, Hannover, Germany) and Professor Johannes Lankeit (Leibniz Universität Hannover, Hannover, Germany).

This talk discusses the influence of possible spatial inhomogeneities in the coefficients of logistic source terms in chemotaxis-growth systems

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v) + \kappa(x) u - \mu(x) u^2, & x \in \Omega,\ t > 0, \\ \tau v_t = \Delta v - v + u, & x \in \Omega,\ t > 0 \end{cases} \tag{66}$$

in two-dimensional smoothly bounded domains Ω under the Neumann boundary conditions and initial conditions. Here $\tau = 0, 1,\ \kappa, \mu \in C^0(\overline{\Omega})$ with $\mu \geq 0$. About this problem a property of possible blow-up points is recently studied; in [1], in the case that $\tau = 0$, it is shown that finite-time blow-up of the classical solution can only occur in points where μ is zero. In this talk we give a recent development of a result about a property of blow-up points in the system (66) in the case that $\tau = 1$.

[1] T. Black, M. Fuest, J. Lankeit, M. Mizukami, *Possible points of blow-up in chemotaxis systems with spatially heterogeneous logistic source.* Nonlinear Analysis: Real World Applications **73** (2023), 103868.

[CT25] Stability of constant equilibria in a flux-limited Keller–Segel system with nonlinear production

Shohei Kohatsu

Tokyo University of Science

Thursday, 15:45, 1D327

We consider the flux-limited Keller–Segel system with nonlinear production,

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u |\nabla v|^{p-2} \nabla v), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u^\theta, & x \in \Omega,\ t > 0 \end{cases} \tag{67}$$

in a smoothly bounded domain $\Omega \subset \mathbb{R}^n$ $(n \in \mathbb{N})$, complemented with the homogeneous Neumann boundary condition and suitable initial conditions, where $p \in (1, \infty)$ and $\theta > 0$. In the case that $\theta = 1$ and p satisfies $p \in (1, \infty)$ if $n = 1$, and $p \in (1, \frac{n}{n-1})$ if $n \geq 2$, stability of constant equilibria in (67) was recently established in [1]. In this talk, we give a result regarding this stability including observations about the case that $\theta \neq 1$.

[1] S. Kohatsu, T. Yokota, *Stability of constant equilibria in a Keller–Segel system with gradient dependent chemotactic sensitivity.* Matematiche (Catania) **78** (2023), 213–237.

[CT25] Global existence and boundedness in a degenerate chemotaxis system for tumor invasion

Tomomi Yokota

Tokyo University of Science

Thursday, 16:15, 1D327

This is a joint work with Professor Sachiko Ishida (Chiba University, Chiba, Japan).
In this talk we give a new result for the a degenerate chemotaxis system for tumor invasion,

$$\begin{cases} u_t = \nabla \cdot (f(u,w)\nabla u - g(u)\nabla v), & x \in \Omega,\ t > 0, \\ v_t = \Delta v + wz, & x \in \Omega,\ t > 0, \\ w_t = -wz, & x \in \Omega,\ t > 0, \\ z_t = \Delta z - z + u, & x \in \Omega,\ t > 0 \end{cases} \tag{68}$$

in a smoothly bounded domain $\Omega \subset \mathbb{R}^N$ under the no-flux boundary condition with nonnegative initial data $u_0 \in L^\infty(\Omega)$ and $v_0, w_0, z_0 \in W^{1,\infty}(\Omega)$. Here $f, g \geq 0$ satisfy $f(u,w) \geq u^{m-1}$ $(m > 1)$, $g(u) \leq u^\alpha$ $(\alpha \geq 1)$. In the case that $N \geq 2$ and $\alpha + 1 < m + \frac{4}{N}$, global existence and boundedness of weak solutions to (69) were recently established in [1]. In this talk we present recent developments of the result including observations on the critical and super-critical cases that $\alpha + 1 \geq m + \frac{4}{N}$.

[1] S. Ishida, T. Yokota, *Boundedness and weak stabilization in a degenerate chemotaxis model arising from tumor invasion.* J. Differential Equations **371** (2023), 450–480.

[CT25] Regularizing effect for L^p-initial data in a Keller–Segel system with density-dependent sensitivity

Yuya Tanaka

Kwansei Gakuin University

Thursday, 16:45, 1D327

In this talk we consider the Keller–Segel system with density-dependent sensitivity,

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u(u+1)^{\alpha-1} \nabla v) & \text{in } \Omega \times (0,\infty), \\ v_t = \Delta v - v + u & \text{in } \Omega \times (0,\infty), \\ \nabla u \cdot \nu = \nabla v \cdot \nu = 0 & \text{on } \partial\Omega \times (0,\infty), \\ u(\cdot,0) = u_0,\ v(\cdot,0) = v_0 & \text{in } \Omega, \end{cases} \tag{69}$$

where $\Omega \subset \mathbb{R}^N$ $(N \in \mathbb{N})$ is a bounded domain with smooth boundary $\partial\Omega$, $\chi, \alpha \in \mathbb{R}$ are constants, ν is the outward normal vector to $\partial\Omega$, and $u_0, v_0 \geq 0$ in Ω. The previous work [1] established a result on regularizing effect; for more details, global classical solutions were constructed for initial data $u_0 \in L^2(\Omega)$ and $v_0 \in W^{1,2}(\Omega) \cap W^{2,q}(\Omega)$ with some $q > 1$ in the case that $\chi \in \mathbb{R}$ and $\alpha < \frac{2}{N}$ $(N \in \mathbb{N})$. In this talk we present recent development of the result on regularizing effect for L^p-initial data $(p > 1)$ in (69).

[1] Y. Sugawara, Y. Tanaka and T. Yokota, *Regularizing effect in a Keller–Segel system with density-dependent sensitivity for L^2-initial data of cell density.* submitted.

[CT26] On recursion operators for full-fledged nonlocal symmetries of the reduced quasi-classical self-dual Yang-Mills equation

Petr Vojčák

Silesian University in Opava

Friday, 8:45, 1D328

This is a joint work with Jiřina Jahnová (Mathematical Institute of the Silesian University in

Opava, Czech Republic).

We introduce the idea of constructing recursion operators for full-fledged nonlocal symmetries and apply it to the reduced quasi-classical self-dual Yang-Mills equation

$$u_{yz} = u_{tx} - u_z u_{xx} + u_x u_{xz}. \tag{70}$$

It turns out that the discovered recursion operators can be interpreted as infinite-dimensional matrices of differential functions which act on the generating vector-functions of the nonlocal symmetries simply by matrix multiplication. To the best of our knowledge, there are no other examples of such recursion operators in the literature so far, so our approach is completely innovative. Further, we investigate the algebraic properties of the discovered operators and discuss the $\mathbb{R}$-algebra structure on the set of all recursion operators for full-fledged nonlocal symmetries of the equation in question. Finally, we illustrate the action of the obtained recursion operators on particularly chosen full-fledged symmetries and emphasize their advantages compared to the action of traditionally used recursion operators for shadows.

[1] J. Jahnová, P. Vojčák, *On recursion operators for full-fledged nonlocal symmetries of the reduced quasi-classical self-dual Yang-Mills equation,* arXiv:2310.11194, accepted in Ann. Henri Poincaré (2024).

[CT26] Existence and Approximation of Time Harmonic Nonlinear Surface Plasmons

Tomas Dohnal

Martin Luther University Halle-Wittenberg

Friday, 9:15, 1D328

This is a joint work with Maximilian Hanisch and Runan He (Martin Luther University Halle-Wittenberg) as with Giulio Romani (Insubria University).

We consider cubically nonlinear Maxwell equations for an interface of two generally dispersive, i.e. frequency dependent, materials, and study the existence of solutions localized at the interface and propagating in the form of a plane wave in a direction tangential to the interface.

The main physical application of this set-up is to surface plasmon polaritons (SPPs) in a metal/dielectric setting. In the time harmonic case the linear Maxwell equations reduce to an operator pencil problem which is non-self-adjoint in the presence of metals.

$$\nabla \times \nabla \times E - \omega^2 \epsilon(x_1, \omega)E = 0, \quad \epsilon : \mathbb{R} \times \mathbb{C} \to \mathbb{C},$$
$$E(x) = e^{ikx_2} u(x_1), k \in \mathbb{R}, u : \mathbb{R} \to \mathbb{C}^3.$$

The frequency ω plays the role of a spectral parameter. The interface is at $x_1 = 0$. Both the transverse electric case $u(x_1) = (0, 0, \phi(x_1)))^T$ and the transverse magnetic case $u(x) = (\phi_1(x_1), \phi_2(x_1), 0)^T$ are studied. First, we discuss bifurcations of nonlinear SPPs from linear ones given by simple isolated real eigenvalues of the operator pencil. Real eigenvalues occur in PT symmetric settings, where loss and gain are perfectly balanced. The bifurcation and the asymptotic expansion of the solutions are proved via a fixed point argument. Numerical computations are provided to support the analysis.

In the case of a complex frequency ω we find solutions of the nonlinear time dependent Maxwell equations in the form of series of odd harmonics multiplied by spatial profiles localized to the interface. The existence problem reduces to a sequence of linear inhomogeneous equations.

[1] T. Dohnal and G. Romani. Eigenvalue Bifurcation in Doubly Nonlinear Problems with an Application to Surface Plasmon Polaritons. *Nonlinear Differ. Equ. Appl.* **28** (9) (2021)

[2] T. Dohnal and R. He. Bifurcation in Nonlinear Eigenvalue Problems Modelling Transverse Magnetic Surface Plasmon Polaritons. arXiv:2311.17838, submitted.

[CT26] Peturbations of Embedded Eigenvalues for the Magnetic Laplacian

Wilhelm Treschow

LTH, Lund University

Friday, 9:45, 1D328

Perturbation problems for operators with embedded eigenvalues are typically difficult as their persistence is conditional on the type of perturbation considered. In this paper, we study a perturbation problem for embedded eigenvalues for a magnetic Schrödinger operator, when the underlying domain is an infinite cylinder. The magnetic potential is C^2 and asymptotically periodic, and the perturbations decay algebraically along the length of the cylinder. We additionally impose the non-degeneracy condition that the embedded eigenvalue of the unperturbed problem is not an eigenvalue of the magnetic laplacian on the fundamental domain of periodicity. Our main result is that the set of perturbations, for which a simple embedded eigenvalue persists, forms a smooth manifold with a finite co-dimension. This is acheived using methods from Floquet theory for elliptic systems for the spatial dynamics formulation, exponential dichotomies and their roughness properties, as well as Lyapunov-Schmidt reduction.

The result is based on joint work with Jonas Jansen and Sara Maad Sasane.

[CT26] Existence of spike solutions for the Schnakenberg model with advection term on Y-shaped metric graph

Yuta Ishii

National Institute of Technology, Ibaraki College

Friday, 10:15, 1D328

In this talk, we consider the Schnkenberg reaction-diffusion model with advection term on the Y-shaped metric graph. Here the Y-shaped metric graph is a domain consisting three finite segments joined at a junction. Moreover, we set that the advection velocity is different for each segment. We show the existence of one-peak stationary solutions on the Y-shaped graph. In particular, the location and amplitude of the spike is determined by the interaction of the advection velocity with the geometry of the graph. Moreover, the spike is shifted to the junction point or the boundary point depending on boundary (vertex) condition and the size of the advection velocity. This work was supported by JSPS KAKENHI Grant Numbers 21K20341 and 24K16958.

[CT27] The finite element method with neural networks to reconstruct the mechanical properties of an elastic medium

Rafael Henriques

University of Coimbra

Friday, 8:45, 1D341

In this work we investigate a mathematical model to reconstruct the mechanical properties of an heterogeneous elastic medium for the optical coherence elastography imaging modality. To this end, we propose machine learning tools by exploring neural networks to solve the inverse problem of elastography. In our framework, we analyze the theoretical relative error between the exact solution and the neural network for the case of noise free data and noisy data. Our algorithm updates the parameters combining the backpropagation technique with the ADAM optimizer to minimize a cost function which is defined using a fully discretized scheme of the direct problem. We report several computational results using fabricated data with and without noise.

Acknowledgements This work was partially supported by the Centre for Mathematics of the University of Coimbra (funded by the Portuguese Government through FCT/MCTES, DOI 10.54499/UIDB/00324/2020); the FEDER Funds through the Operational Program for Competitiveness Factors - COMPETE and by Portuguese National Funds through FCT - Foundation for Science and Technology, DOI 10.54499/2021.06672.BD (https://doi.org/10.54499/2021.06672.BD).

[CT27] Can we create (pseudo)randomness with polygons?

Sandeep Kumar

CUNEF UNIVERSITY

Friday, 9:15, 1D341

In this talk, we will discuss the Schrödinger map equation, a geometric partial differential equation by considering its evolution for polygonal curves in different geometrical settings. The equation is a special case of the famous Landau–Lifshtiz equation for ferromagnetism and its equivalent form in Euclidean space describes the evolution of a vortex filament in a real fluid, known as the vortex filament equation.

The dynamics of these equations for polygonal initial data when solved numerically exhibit several interesting characteristics of real fluids, e.g., the axis switching phenomenon and multifractality, often associated with turbulence. On the other hand, the algebraic construction of these solutions not only supports the numerical evolution but also indicates randomness, often observed in several natural phenomena. I will present some recent results, in particular, the case of helical-shaped vortices and curves in the hyperbolic space, and show that this unusual behaviour (randomness) resulting from a differential equation is indeed appears as a generic phenomenon [1–3].

[1] S. Kumar, Pseudorandomness of the Schrödinger map equation. Preprint arXiv:2311.01611.

[2] S. Kumar, *On the Schrödinger map for regular helical polygons in the hyperbolic space.* Nonlinearity **35(1)** (2022), 84–109.

[3] F. de la Hoz, S. Kumar and L. Vega, *Vortex Filament Equation for a regular l-polygon in the hyperbolic plane.* J. Nonlinear Sci. **32(9)** (2022).

[CT27] Error analysis for the Discontinuous Galerkin semidiscretization of degenerate parabolic equations

Sunčica Sakić

Charles University

Friday, 9:45, 1D341

This is a joint work with Vít Dolejší (Charles University, Prague, Czech Republic) and Scott Congreve (Charles University, Prague, Czech Republic).

In this talk, we present an error analysis of a semidiscrete scheme for a doubly nonlinear parabolic partial differential equation (PDE), admitting the fast-diffusion type of degeneracy. The solution of this equation is usually not smooth; namely, this type of equation has been studied in papers on the existence, uniqueness, and regularity of solutions to elliptic-parabolic differential equations (see, e.g., [1] and [4]). A well-known representative of such a PDE is Richards' equation, which is commonly used in porous media flow modeling.

A wide variety of numerical methods have been proposed for solving such problems. We mention [5], which showed that the higher-order space-time discontinuous Galerkin finite element method represents a promising tool for solving Richards' equation in an efficient, robust, and accurate way. However, the corresponding rigorous mathematical theory is still missing.

This talk focuses on the error analysis for the time-continuous scheme done in our paper [3]. Due to nonlinearities, we choose the local discontinuous Galerkin method [2] to discretize the spatial variable. We pay special attention to the estimation of the accumulation term, which can vanish. Moreover, we use continuous mathematical induction to derive a priori error estimates in the L^2-norm and the jump form with respect to the spatial discretization parameter and the

Hölder coefficient of the accumulation term derivative. Numerical experiments accompany the proposed theory. In addition, we present a simulation of a practical example that uses adaptive mesh refinement.

[1] H.W. Alt, S. Luckhaus, *Quasilinear elliptic-parabolic differential equations.* Math. Z. **183** (1983), 311–342.

[2] D.N. Arnold, F. Brezzi, B. Cockburn L.D. Marini (2002). *Unified analysis of discontinuous Galerkin methods for elliptic problems.* SIAM J. Numer. Anal., 39 (5), 1749–1779.

[3] S. Congreve, V. Dolejší, S. Sakić, Error analysis for local discontinuous Galerkin semidiscretization of Richards' equation, IMA J. Numer. Anal. (accepted for publication)

[4] F. Otto , L^1*-contraction and uniqueness for quasilinear elliptic-parabolic equations.* J. Differ. Equ. **131**(1) (1996), 20–38.

[5] V. Dolejší, M. Kuraz, P. Solin, *Adaptive higher-order space-time discontinuous Galerkin method for the computer simulation of variably-saturated porous media flows.* Appl. Math. Model. (2019), 276–305

[CT27] Diffusive wave model in a channel of finite length with a concentrated lateral inflow

Swaroop Nandan Bora

Indian Institute of Technology Guwahati

Friday, 10:15, 1D341

The diffusive wave model is one of the simplified forms of the well-known Saint-Venant equations and it is frequently used instead of the full model. We present an analytical solution for the linearized diffusive wave model represented by a simultaneous system of two first-order partial differential equations focused on spatial variation of the lateral inflow in a finite channel. A concentrated lateral inflow from a small-width tributary is considered through Dirac delta function. We use the Laplace transform method to solve these equations analytically. Two types of upstream boundaries are considered in the form of a flow-discharge hydrograph and a flow-depth hydrograph, while keeping a flow-depth hydrograph as the downstream boundary. Using unit-step responses of the lateral inflow, the effect of different boundaries on the flow-depth responses and the flow-discharge responses is studied for different values of the Peclet number

(P_e). The flow depth is observed to be more sensitive to the downstream boundary and other parameters. Consideration of the flow depth as the upstream boundary reflects the effect of all the parameters on the step responses presented. These responses are compared with the available semi-infinite channel responses. The semi-infinite channel responses are found to be an inappropriate substitute for the finite channel responses for $P_e < 5$ which implies that the downstream boundary cannot be ignored for these cases. However, for the case $P_e > 5$, although the semi-infinite channel responses are found to satisfactorily estimate the discharge along the entire channel, they estimate the flow depth at the locations closer to the upstream boundary only.

Posters

Asymptotics and stability of large L^∞-modulations of wave trains in reaction-diffusion systems

Joannis ALEXOPOULOS
Karlsruhe Institute of Technology

This is a joint work with Björn de Rijk (Karlsruhe Institute of Technology).

We study reaction-diffusion systems on the extended real line admitting a periodic traveling-wave solution ϕ_0 which is diffusively spectrally stable. Having a solution u and a phase modulation γ at hand, we focus on the long-standing question of how the modulated perturbation $u(\cdot,t) - \phi_0(\cdot + \gamma(\cdot,t))$ with respect to suitable norms and initial conditions evolves in time.

In my poster presentation, I will give an overview of existing answers and present our new result, outlining the main aspects and challenges of its proof. The essential extension consists of lifting any localization requirement on the phase modulation, the initial perturbation and their derivatives, as well as removing the smallness assumption on $||\gamma(\cdot,t)||_{L^\infty}$. Our method is robust and we expect that it can be applied to other semilinear (non-parabolic) systems.

Large-time existence and asymptotic stability of the generalized solution to the real micropolar reactive gas model

Angela Bašić-Šiško
University of Rijeka, Faculty of Engineering

We study a model that describes the dynamics of flow and thermal reaction in a mixture of compressible micropolar real reactive gasses. This model builds on the classical Navier-Stokes fluid flow model by incorporating thermodynamic effects arising from the variable temperature and the law of conservation of energy, microeffects arising from the local microrotations of the particles dispersed in the fluid through a microrotational velocity vector field and the law of conservation of angular momentum, and chemical aspects arising from the combustion process of the fuel within the gas mixture via a corresponding reaction dynamics equation. Additionally, we use a real gas model rather than an ideal one, where the equation of state accounts for a more general dependence of pressure on mass density and temperature. The governing system is parabolic and consists of quasilinear pde-s in terms of 5 unknown variables, the mass density ρ, the velocity vector field v, the microrotation velocity vector field ω, the absolute temperature θ and the fraction of unburned fuel z. The system is accompanied by suitable initial and homogeneous boundary conditions [1].

In our research, we prove the global existence of a solution to the described initial–boundary problem in a generalized sense in one spatial dimension. Moreover, we study the long-term behavior of the generalized solution and show that it stabilizes over time either by exponential or power decay towards a stationary solution, depending on certain parameters of the system [2–5]. Furthermore, we validate our theoretical conclusions by numerical simulations performed for different parameter values and discuss possibilities for improving the obtained results.

[1] A. Bašić-Šiško, I. Dražić, *One-dimensional model and numerical solution to the viscous and heat-conducting reactive micropolar real gas flow and thermal explosion*, Iran. J. Sci. Technol. Trans. Mech. Eng. (2022), doi: 10.1007/s40997-022-00498-w

[2] A. Bašić-Šiško, I. Dražić, *Local existence for viscous reactive micropolar real gas flow and thermal explosion with homogeneous boundary conditions*, J. Math. Anal. Appl., 509 (2), 125988 (2022), doi: 10.1016/j.jmaa.2022.125988

[3] A. Bašić-Šiško, I. Dražić, *Uniqueness of a Generalized Solution for a One-Dimensional Thermal Explosion Model of a Compressible Micropolar Real Gas*, Mathematics 12 (5), 717 (2024), doi: 10.3390/math12050717

[4] A. Bašić-Šiško, I. Dražić, *Global existence theorem of a generalized solution for a one-dimensional thermal explosion model of a compressible micropolar real gas*, Math. Methods Appl. Sci., (2024), doi: 10.1002/mma.10108 (accepted for publication)

[5] A. Bašić-Šiško, *Large time existence and asymptotic stability of the generalized solution to flow and thermal explosion model of reactive real micropolar gas* Math. Methods Appl. Sci. (2024), doi: 10.1002/MMA.10139 (accepted for publication)

Optimal control for maturity-structured systems with an application to *Chlamydia* treatment

Bornali Das

University of Szeged

This is a joint work with Prof Gergely Röst and István Balázs (University of Szeged, Hungary). This study addresses an optimal control problem within a nonlinear compartmental model, featuring a combination of real-valued variables and maturity-structured components governed by transport equations with nonlinear boundary conditions. Demonstrating the optimality of the control, the Pontryagin minimum principle serves as a necessary condition for this new type of system of differential equations. The practical implications of this theoretical framework are illustrated through an application focused on optimizing the treatment strategy for *Chlamydia* infection.

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Turing instability and dynamic bifurcation for the one dimensional Gray-Scott model

Taeyoung Ha

National Institute for Mathematical Sciences

We study the dynamic bifurcation of the one dimensional Gray-Scott model by taking the diffusion coefficient λ of the reactor as a bifurcation parameter. We define a parameter space Σ of (k, F) for which the Turing instability may happen. Then, we show that it really occurs below the critical number λ_0 and obtain rigorous formula for the bifurcated stable patterns. When the critical eigenvalue is simple, the bifurcation leads to a continuous (resp. jump) transition if $A_m(k, F)$ is negative (resp. positive). We prove that $A_m(k, F) > 0$ when (k, F) lies near the Bogdanov-Taken point $(\frac{1}{16}, \frac{1}{16})$. When the critical eigenvalue is double, we have a subcritical bifurcation that produces an S^1-attractor $\mathcal{A}_m$. We prove that $\mathcal{A}_m$ consists of four asymptotically stable static solutions, four saddle static solutions, and orbits connecting them. We also provide numerical results that illustrate the main Theorems.

Preserving correlations: A statistical method for generating synthetic data

Nicklas Jävergård

Karlstad Universitry

This is joint work with Adrian Muntean and Rainey Lyons from Karlstad University, Karlstad, Sweden, together with our Industrial partner Jonas Forsman from CGI, Data Advantage, Karlstad, Sweden.

We propose a method to generate statistically representative synthetic data. The main goal is to be able to maintain in the synthetic dataset the correlations of the features present in the original one, while offering a comfortable privacy level that can be eventually tailored on specific customer demands. We describe in detail our algorithm used both for the analysis of the original dataset and for the generation of the synthetic data points. The approach is tested using a large energy-related dataset. We obtain good results both qualitatively (e.g. via vizualizing correlation maps) and quantitatively (in terms of suitable ℓ^1-type error norms used as evaluation metrics). The proposed methodology is general in the sense that it does not rely on the used test dataset. We expect it to be applicable in a much broader context than indicated here.

Resolvent estimates and maximal regularity estimates for the Stokes equations

Naoto Kajiwara

Gifu University

We consider the Stokes equations in the half space $\mathbb{R}^n_+$, and prove resolvent L_q estimates and maximal L_p-L_q estimates with $1 < p, q < \infty$. The boundary condition is inhomogeneous, then the solution is given by Fourier multiplier operator. We show a sufficient condition of L_q-boundedness for an integral operator

$$T[m]f(x) = \int_0^\infty [\mathcal{F}^{-1}_{\xi'} m(\xi', x_n + y_n)\mathcal{F}_{x'} f] dy_n,$$

where $f : \mathbb{R}^n_+ \to \mathbb{C}$. The essential assumption is H^∞-property of m. Moreover, *operator-valued*

Fourier multiplier theorem is also proved. This implies maximal L_p-L_q regularity estimates.

Pattern formation in reaction diffusion systems with spatially varying coefficients

Jolien Kamphuis

Leiden University

This is a joint work with Dr. Martina Chirilus-Bruckner (Leiden University, Leiden, The Netherlands).

We are interested in the formation of patterns, which arise as solutions to nonlinear partial differential equations with spatially varying coefficients. We focus on the evolution of modulated periodic orbits that bifurcate from a background state in the presence of a spatially varying heterogeneity. Nonlinear PDEs with spatially varying coefficients emerge from application based models, such as ecology models, where the spatially varying coefficients represent a varying vegetation terrain. In this case, the corresponding solutions can be interpreted as (possibly moving) vegetation patches.

For various reaction-diffusion systems with spatially varying parameters, we first consider small spatially varying coefficients. For these systems, we investigate the evolution of periodic orbits using perturbation analysis to derive a modulation equation, of which the solutions serve as envelopes for the solutions of the original system. The spatially varying coefficients emerge in these amplitude equations in different ways. Thereafter, we consider spatially varying coefficients that are not necessarily small, and use the technique of Bloch waves to investigate the solutions at pattern onset.

A weighted particle approach to non-linear diffusion equations

Erik Lieback

Karlstad University

We design and study a particle method that can be used to numerically approximate solutions to the quadratic porous medium equation. The idea consists of first approximating the porous medium equation using a non-local transport equation, to which we approximate the solution with a particle method. We prove that the particle method converges, in a suitable norm, to the solution to the non-local transport equation. We provide numerical simulations to illustrate this convergence and estimate the order of convergence. In particular, we use the particle method to approximate the Barenblatt solutions to the quadratic porous medium equation. The analysis of the partial differential equations is to a large extent carried out in the sense of integrable functions, while the analysis of the particle method relies on a duality approach on the space of finite signed Radon measures.

On the Turing-fold bifurcation

Dock Staal

Leiden University

This is a joint work with Prof Dr Arjen Doelman (University of Leiden, Leiden, The Netherlands).

A burgeoning area of study within climate ecology focuses on ecosystem resilience through pattern formation. In ecology, ecosystems are often mathematically investigated using reaction-diffusion equations. Our focus centers on reaction-diffusion equations with the following characteristics.

First, the system depends on a parameter μ and the zero-function is an equilibrium solution for all values of μ. This parameter may represent precipitation, while the zero-solution signifies an unfavorable state such as desert. Additionally, a saddle-node bifurcation occurs, creating an ODE-stable state that models a desirable state, such as fully covered grassland. Moreover, on the stable saddle-node branch a supercritical Turing bifurcation happens in the direction of the saddle-node bifurcation, representing spatial reorganization of nature.

Based on available evidence, the stable patterns arising from the Turing bifurcation may extend beyond the saddle-node point and thus prevent or delay the systems collapse into the undesirable state. This mechanism of spatial reorganization is ubiquitous in nature. Consequently, understanding whether and when supercritical Turing bifurcations improve system resilience is vital to understanding the effects of climate change.

In our pursuit of understanding the Turing-Fold dichotomy, we investigated the codimension two bifurcation where Turing and Tipping coincide. Following the standard procedure, we derived the Ginzburg-Landau equations to examine patterns near onset. Surprisingly, however, this approach falls short in describing pattern behavior beyond tipping. To address the limitations of the Ginzburg-Landau approach, we proposed an alternative Ginzburg-Landau-like Ansatz. This allows us to derive a coupled system of equations that describes the near-onset behavior beyond the fold-point.

We have established several key findings regarding the coupled system. Akin to the Landau coefficient, the coupled system has one parameter value which if positive represents a supercritical Turing bifurcation and if negative signifies a subcritical Turing bifurcation. Moreover, we have shown that this parameter has the opposite sign as the Landau coefficient. Additionally, this new coefficient is orders of magnitude easier to analyze. This may hopefully aid in the ecological understanding of effects that cause spatial reorganization to occur. Finally, we have shown that the region of stable patterns that emerges from the supercritical Turing bifurcation extends beyond the tipping point: indeed, the Turing bifurcation enables the system to evade tipping.

Periodic orbits for the Delayed Rössler System

Anna Gierzkiewicz

Jagiellonian University in Krakow

This is a joint work with Robert Szczelina (Jagiellonian University in Krakow, Poland).

We apply the method from [1] for finding a wide variety of periodic orbits for multidimensional maps with an attracting n-periodic orbit. The set of periods is induced by the Sharkovskii ordering '$\triangleleft$' of natural numbers:

$$3 \triangleleft 5 \triangleleft 7 \triangleleft \cdots \triangleleft 2 \cdot 3 \triangleleft 2 \cdot 5 \triangleleft \cdots \triangleleft 2^2 \cdot 3 \triangleleft 2^2 \cdot 5 \triangleleft \cdots \triangleleft 2^k \triangleleft 2^{k-1} \triangleleft \cdots \triangleleft 2^2 \triangleleft 2 \triangleleft 1.$$

We prove the existence of n-periodic orbits for all $n \in \mathbb{N}$ in the Rössler system with a 3-periodic orbit, perturbed by a small delayed term. The proofs are computer-assisted with the use of CAPD library for C++ and the algorithms introduced in [2].

Acknowledgments: This research was funded in whole or in part by National Science Centre, Poland, 2023/49/B/ST6/02801.

[1] A. G., P. Zgliczyński, *From the Sharkovskii theorem to periodic orbits for the Rössler system.* J. Differential Equations **314** (2022), 733–751.

[2] R. Szczelina, P. Zgliczyński, *Algorithm for Rigorous Integration of Delay Differential Equations and the Computer-Assisted Proof of Periodic Orbits in the Mackey–Glass Equation.* Found. Comput. Math.

18 (2018), 1299–1332.

A posteriori error estimates for ion transport model

Arne Berrens

Technical University of Darmstadt

This is a joint work with Jan Giesselmann (Technical University of Darmstadt, Germany).

Our goal is to obtain an a-posteriori error bound for the finite volume solution of the ion transport model

$$\partial_t u_i = \operatorname{div}(u_0 \nabla u_i - u_i \nabla u_0) \quad \forall i = 1, \dots, n, \tag{71}$$

where u_i describes the concentration of the i-th species and u_0 is the solvent concentration. Here we set no flux-boundary conditions and impose volume-filling, i.e. $\sum_{i=0}^n u_i = 1$. In essence, our goal is to construct an upper bound for the distance of a numerical solution to the real solution in some metric, that can be computed from the numerical solution. The cross-diffusion system (71) admits an entropy structure with the Boltzmann entropy $h(u) = \sum_{i=0}^n u_i(\log(u_i) - 1)$. From this entropy structure arises the Gajewski metric

$$d(u|v) = \int_\Omega h(u) + h(v) - 2h\left(\frac{u+v}{2}\right) dx. \tag{72}$$

With the help of this metric one can show weak uniqueness for the ion transport model (see [1]). Our aim is to use a similar technique, as in the uniqueness proof, to get an upper bound to the error. A classical approach to solve (71) numerically, is to use a cell centered finite volume method (see [2]). This method yields a piecewise constant approximate solution, but we need a weakly differentiable solution to exploit the techniques used in the uniqueness proof. To get such an approximate solution we use a Morley type reconstruction (similar to [3]).

One major step along the way is to get an upper bound for u_0 in the H^1-seminorm and maximum norm. For this we utilize that u_0 solves the heat equation with homogeneous Neumann boundary conditions. This can be seen by summing (71) up for $i = 1, \dots, n$. An upper bound can then be achieved by the properties of the Morley reconstruction and Green's function for Poisson's equation (similar to [4]).

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Convergence Results for a System of PDE

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This is a joint work with Prof. Paschalis Karageorgis (Trinity College Dublin, Ireland), based on the results in [1].

We study the asymptotic behavior of positive radial solutions for quasilinear elliptic systems that have the form

$$\begin{cases} \Delta_p u = c_1|x|^{m_1} \cdot g_1(v) \cdot |\nabla u|^{\alpha} & \text{in } \mathbb{R}^n, \\ \Delta_p v = c_2|x|^{m_2} \cdot g_2(v) \cdot g_3(|\nabla u|) & \text{in } \mathbb{R}^n, \end{cases}$$

where Δ_p denotes the p-Laplace operator, $p > 1$, $n \geq 2$, $c_1, c_2 > 0$ and $m_1, m_2, \alpha \geq 0$. For a general class of functions g_j which grow polynomially, we show that every non-constant positive radial solution (u, v) asymptotically approaches $(u_0, v_0) = (C_\lambda|x|^\lambda, C_\mu|x|^\mu)$ for some parameters $\lambda, \mu, C_\lambda, C_\mu > 0$. In fact, the convergence is monotonic in the sense that both u/u_0 and v/v_0 are decreasing. We also obtain similar results for more general systems.

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A dynamic approach to heterogeneous elastic wires

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The elastic energy of a bending-resistant interface depends both on its geometry and its material composition. We consider such a heterogeneous interface in the plane, modeled by a curve equipped with an additional density function. The resulting energy, which depends on material parameters, captures the complex interplay between curvature and density effects and resembles the Canham–Helfrich functional.

We study a family of planar curves with density evolving in time according to the steepest descent associated to this energy. Describing the curves by their inclination angle, the L^2-gradient flow is a nonlocal coupled parabolic system of second order. We shortly discuss local well-posedness via maximal regularity theory on time-dependent little Hölder spaces. Once global existence is established, convergence of solutions follows with a suitable constrained Łojasiewicz–Simon gradient inequality. We show that the (non)preservation of quantities such as convexity and positivity of the density depends delicately on the choice of material parameters. The same applies for the asymptotic behavior of the system. We derive parameter dependent conditions under which the limit and the rate of convergence can be determined.

This is based on joint work with Anna Dall'Acqua (Ulm University), Fabian Rupp (University of Vienna) and Gaspard Jankowiak (University of Graz).

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Synchronization Phenomenon in the Network of Bursting Oscillators with Multiple-Slow-Time Scale Approach

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This work is devoted to studying the synchronization of finite-dimensional systems with inherent properties of the Pancreatic $\beta-$cell dynamics. The material is based on Fenichel's seminal work on geometric singular perturbation theory [1] and the extended form of Fenichels's theory for multiple time scales [2,3]. For the network of $N-$systems, the diagonal coupling (strength) matrix ϵ with entries $\epsilon_1, \cdots, \epsilon_N$ are positive small parameters with $|\epsilon| := \max_{j\in K} \epsilon_j << 1$, $K = [1, N] \subset \mathbb{N},\ N < \infty$, the system can be written as

$$\frac{dX}{dt} = F(X(t)) + \epsilon G(X(t), \epsilon), \tag{73}$$

where $X \equiv X(t) = [X_1(t), \cdots, X_N(t)]^T$, $G(\cdot) = [G_1(\cdot), \cdots, G_N(\cdot)]^T$ with $G_j : \mathbb{R}^N \to \mathbb{R}$ for all $j \in K$ is sufficiently smooth in X and ϵ. $F(X(t)) = [QF_1(X_1(t)), \cdots, QF_N(X_N(t))]^T$, $Q = [\mathbf{1}_k\ 0;\ 0\ \mu]$ where $\mathbf{1}_k$ is $k \times k-$dimensional identity matrix and μ is a diagonal matrix with entries μ_i $(i = 1, \cdots, s)$, $s = n - k$, $2 \leq k < n$, $0 < |\mu| < 1$, and $F_j : \mathbb{R}^n \to \mathbb{R}^n$ for $j \in K$, is C^r $(1 \leq r)$ vector field with respect to X_j on U, an open subset of $\mathbb{R}^n$. In particular, we restricted the properties of the vector field $F_j(X_j(t), \mu)$ such that for $\epsilon = 0$ system (73) shows the fold/homoclinic (square-wave) bursting for each $j \in K$, and for $\epsilon \neq 0$, system (73) defines the network of bursting oscillators connected symmetrically to each other with one of their certain (fast) components. A local coordinate transformation takes (73) to a standard (fast) form

$$\begin{aligned} \frac{dY_1}{dt} &= H_1(Y_1(t), Y_2(t), \mu_1, \cdots, \mu_s, \epsilon_1, \cdots, \epsilon_N) \\ \frac{dY_2}{dt} &= \mathcal{E} H_2(Y_1(t), Y_2(t), \mu_1, \cdots, \mu_s, \epsilon_1, \cdots, \epsilon_N), \end{aligned} \tag{74}$$

where $Y_1 \in \mathbb{R}^{kN}$, $Y_2 \in \mathbb{R}^{sN}$ with smooth functions $H_1 : \mathbb{R}^{nN} \to \mathbb{R}^{kN}$ and $H_2 : \mathbb{R}^{nN} \to \mathbb{R}^{sN}$, and $\mathcal{E}$ is a positive definite diagonal matrix with entries $\mathcal{E}_1, \cdots, \mathcal{E}_{sN}$ which are function of $\mu_1, \cdots, \mu_s, \epsilon_1, \cdots, \epsilon_N$. The slow manifold of system (74) has preserved the stability properties of system (73). Also, the layer problem (reduced problem) of the system (74) carried the spike-synchronization (burst-synchronization) criterion for the network system (73).

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Planar Interfaces in Singularly Perturbed Three-Component Reaction-Diffusion Systems

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This is a joint work with Arjen Doelman from the University of Leiden and Paul Carter from the University of California at Irvine.

In the article [1] the authors proposed that pattern formation in ecosystems could be driven by counteracting feedback mechanisms on widely different spatial scales. In the context of systems of reaction-diffusion equations this can be translated to components which diffuse with widely different diffusion coefficients, whose associated stationary and travelling wave spatial dynamical systems can be written in a canonical form suitable for the application of Fenichel theory, introduced in [3]. In [2] a criterion for the sideband (in)stability of a planar moving interface is derived for two-component systems. In [2], both the construction and the linear stability analysis for the particular case of a moving front is set-up using the geometric approach developed in [3]. In the three-component case

$$\begin{cases} u_t = u_{xx} + f(u,v,w;\mu) \\ v_t = \frac{1}{\delta_1^2} v_{xx} + g(u,v,w;\mu) \\ w_t = \frac{1}{\delta_2^2} w_{xx} + h(u,v,w;\mu) \end{cases} \tag{75}$$

we shall use the practical characterisation of multiple scale Fenichel theory presented in [4] to construct some moving front solutions.The nature of our approach is such that it can naturally be generalized to the case of n-component systems. Moreover, it provides the starting point for the derivation of an sideband (in)stability criterion of (traveling) planar multi-component fronts in the spirit of [2].

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Pattern Formation in a 2D Thermocapillary Thin-Film Equation

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This is a joint work with Bastian Hilder (TU Munich, Germany) and Jonas Jansen (Lund University, Sweden).

It is experimentally known that thin films of viscous fluids on heated plates develop polygonal, spatially periodic patterns. This is due to a self-sustaining thermocapillary effect causing an instability of the trivial constant state. Building upon the 1D results in [1], we consider a two-dimensional thin-film equation. It can be retrieved as an asymptotic limit of the Boussinesq–Navier–Stokes model for small fluid height. The equation is of fourth order and degenerate parabolic but we consider the stationary problem, which we are able to reduce to a second-order equation amenable to analytic bifurcation theory. The constant solution destabilizes via a (conserved) long-wave instability and we prove existence of a global bifurcation branch of stationary solutions of fixed mass, which are symmetric and periodic with respect to a fixed square or hexagonal lattice. We also prove that monotonicity properties and extrema are preserved along the branch.

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Travelling fronts in a two component PDE with spatially dependent coefficients

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This is a joint work with Frits Veerman (Leiden University, Leiden, Netherlands), Martina Chirilus-Bruckner (Leiden University, Leiden, Netherlands).

We consider a two component singularly perturbed system of reaction-diffusion equations with spatially heterogeneous coefficients for small $0 < \epsilon \ll 1$

$$\begin{aligned} \partial_t U &= \epsilon^2 \partial_x^2 U + U - U^3 - \epsilon(\alpha V + \gamma), \\ \hat{\tau} \partial_t V &= \partial_x^2 V - (1 + \delta_1 f_1(x))V + (1 + \delta_2 f_2(x))U, \end{aligned} \tag{76}$$

First, we construct two equilibria (background states), which depend on space due to the heterogeneity. Constructing just the background (in the vicinity of stable equilibria of the homegeneous PDE) states is already a difficult challenge, since the nonautonomous system of differential equations forces to work with noncompact invariant manifolds. Consequently the work by Fenichel on invariant manifold theory and geometric singular perturbation theory ([1], [2], [3], [4], [5]) needs to be generalized to this noncompact setting. Next, we construct stationary fronts connecting the background space. The interface of such fronts can only sit at very specific locations.

For travelling fronts we confine the scope of our study to $\delta_1 = 0$. A travelling front is an entire solution of the PDE (76) such that for each fixed time the profile is a sharp interface connecting the two background states asymptotically. Both the speed and the shape of the profile of such a front are varying in time which renders a comoving frame approach useless. We manage to prove existence of travelling fronts using an intricate spatio-temporal approach for a certain range of parameters (most notably $\alpha < 0$). Lastly we show (still $\delta_1 = 0$) that the position of the travelling front can be described in the singular limit by means of a delay equation. Numerically this delay equation seems to provide a very precise approximation (and irrespective of the sign of α) for the position of the front when $0 < \epsilon \ll 1$.

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Long time behaviour of solutions to non-local and non-linear dispersion problems

Maciej Tadej

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This work explores a non-linear, non-local model describing the evolution of a single species. We investigate scenarios where the spatial domain is either an arbitrary bounded and open subset of the n-dimensional Euclidean space or a periodic environment modeled by n-dimensional torus.

The analysis includes the study of spectrum of the linear, bounded operator in the considered equation, which is a scaled, non-local analogue of classical Laplacian with Neumann boundaries. In particular we show the explicit formulas for eigenvalues and eigenfunctions. Moreover we show the asymptotic behaviour of eigenvalues. Within the context of the non-linear evolution problem, we establish the existence of an invariant region, give a criterion for convergence to the mean mass, and construct spatially heterogeneous steady states.

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Weak and Perron's Solutions, Dirichlet problem for Linear Kinetic Fokker-Planck Equations

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This is a joint work with Prof Benny Avelin (Uppsala University, Uppsala, Sweden).
We investigate weak solutions and Perron-Wiener-Brelot solutions to the linear kinetic Fokker-Planck equation in bounded domains. We establish the existence of weak solutions by applying the Lions-Lax-Milgram theorem and the vanishing viscosity method in product domains. Additionally, we demonstrate the regularity of weak solutions and establish a strong maximum principle. Furthermore, we construct a Perron solution and provide examples of barriers in arbitrary bounded domains. Our findings are based on recent advancements in the theory of kinetic Fokker-Planck equations with rough coefficients, particularly focusing on the characterization of a weaker notion of trace defined through convolution-translation.

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